UDK: 629.115.8:621.3.018.8:531.553 COSATI: 19-03, 13-06

Ride motion of non-linear suspension characteristics tracked vehicle on wavy surface

Rade Stevanović, BSc (Eng)¹⁾

The relation between oscillation amplitude and vehicle velocity is determined graphically for the tracked vehicle with extremely non-linear suspension characteristics in the case of motion on the wavy surface whose wave length is double than the track length on the ground. An approximate numerical procedure for solving non-linear oscillations is used to correct the graphical method. The vehicle oscillations with non-linearity mediocre characteristics were discussed.

Key words: tracked vehicle, vehicle suspension, suspension, vehicle oscillations, non-linear oscillations.

Principals symbols

$J_{y} = \text{vehicle hull moment of inertia about transverse axis}$ $I_i = \text{horizontal distance between i-spring and sprung weight centre}$ $n_i(k) = \text{spring number (damper number) per side}$ $F_{si} = \text{damper resistance force}$ $v_i = \text{relative velocity damper piston}$ $\mu = \text{friction coefficient}$ $\rho = \text{damping coefficient}$ $\sigma = \text{damping coefficient}$ $\alpha = \text{wave length of wavy surface}$ $h/2 = \text{amplitude of wavy surface}$ $A_{\varphi}, \alpha_{\varphi} = \text{amplitude and phase shift forced pitch oscillations}$ $e = \text{swing arm length}$ $c = \text{torsion bar spring rate}$ $C_{rs} = \text{reduced torsion bar spring rate in static wheel position}$ $d = \text{torsion bar diameter}$ $f_s, f_d, f = \text{rebound, positive vertical road wheel travel and current travel}$ $m_0 = \text{vehicle sprung weight}$ $\theta_s, \theta_m = \text{torsion bar twist angle in static wheel position and bump stop position}$ $v_z = \text{frequency of natural bounce oscillations of the vehicle}$ $\tau_{dm} = \text{nominal shear stress of torsion bar}$	$\omega, \omega_{\! \phi}$	 angular frequency of forced and free pitch oscillation with viscous damping
$l_i = - \text{horizontal distance between i- spring and} \\ \text{sprung weight centre} \\ n,(k) = - \text{spring number (damper number) per side} \\ F_{si} = - \text{damper resistance force} \\ v_i = - \text{relative velocity damper piston} \\ \mu = - \text{friction coefficient} \\ \rho = - \text{damping coefficient} \\ \sigma = - \text{damping ratio} \\ a = - \text{wave length of wavy surface} \\ h/2 = - \text{amplitude of wavy surface} \\ A_{\varphi}, \alpha_{\varphi} = - \text{amplitude and phase shift forced pitch} \\ \text{oscillations} \\ e = - \text{swing arm length} \\ c = - \text{torsion bar spring rate} \\ C_{rs} = - \text{reduced torsion bar spring rate in static wheel} \\ \text{position} \\ d = - \text{torsion bar diameter} \\ f_s, f_d, f = - \text{rebound, positive vertical road wheel travel and} \\ \text{current travel} \\ m_0 = - \text{vehicle sprung weight} \\ \theta_s, \theta_m = - \text{torsion bar twist angle in static wheel position} \\ nambut and bump stop position \\ v_z = - \text{frequency of natural bounce oscillations of the} \\ \text{vehicle} \\ \tau_{dm} = - \text{nominal shear stress of torsion bar} \\ \psi, \psi_s, \psi_m = - \text{swing arm angle in current, static wheel position} \\ \varepsilon_p = - \text{specific work load capacity} \\ \end{cases}$	J_y	 vehicle hull moment of inertia about transverse axis
$n_{s}(k) = \text{spring number (damper number) per side}$ $F_{si} = -\text{damper resistance force}$ $v_{i} = \text{relative velocity damper piston}$ $\mu = -\text{friction coefficient}$ $p = -\text{damping coefficient}$ $\sigma = -\text{damping ratio}$ $a = -\text{wave length of wavy surface}$ $h/2 = -\text{amplitude of wavy surface}$ $h/2 = -\text{amplitude and phase shift forced pitch}$ $oscillations$ $e = -\text{swing arm length}$ $c = -\text{torsion bar spring rate}$ $C_{rs} = -\text{reduced torsion bar spring rate in static wheel}$ $position$ $d = -\text{torsion bar diameter}$ $f_{s}, f_{d}, f = -\text{rebound, positive vertical road wheel travel and current travel}$ $m_{0} = -\text{vehicle sprung weight}$ $\theta_{s}, \theta_{m} = -\text{torsion bar twist angle in static wheel position}$ $v_{z} = -\text{frequency of natural bounce oscillations of the vehicle}$ $\tau_{dm} = -\text{nominal shear stress of torsion bar}$ $\varphi, \psi_{s}, \psi_{m} = \text{swing arm angle in current, static wheel position}$ $\varepsilon_{p} = -\text{specific work load capacity}$	l_i	 horizontal distance between i- spring and spring weight centre
$\begin{array}{lll} F_{si} & - \text{ damper resistance force} \\ v_i & - \text{ relative velocity damper piston} \\ \mu & - \text{ friction coefficient} \\ p & - \text{ damping coefficient} \\ \sigma & - \text{ damping ratio} \\ a & - \text{ wave length of wavy surface} \\ h/2 & - \text{ amplitude of wavy surface} \\ A_{\varphi}, \alpha_{\varphi} & - \text{ amplitude and phase shift forced pitch} \\ \text{ oscillations} \\ e & - \text{ swing arm length} \\ c & - \text{ torsion bar spring rate} \\ C_{rs} & - \text{ reduced torsion bar spring rate in static wheel} \\ \text{ position} \\ d & - \text{ torsion bar diameter} \\ f_s, f_d, f & - \text{ rebound, positive vertical road wheel travel and} \\ \text{ current travel} \\ m_0 & - \text{ vehicle sprung weight} \\ \theta_s, \theta_m & - \text{ torsion bar twist angle in static wheel position} \\ v_z & - \text{ frequency of natural bounce oscillations of the} \\ \text{ vehicle} \\ \tau_{dm} & - \text{ nominal shear stress of torsion bar} \\ \varphi, \psi_s, \psi_m & - \text{ swing arm angle in current, static wheel position} \\ \varepsilon_p & - \text{ specific work load capacity} \end{array}$	n,(k)	– spring number (damper number) per side
$\begin{array}{lll} v_i & - \mbox{relative velocity damper piston} \\ \mu & - \mbox{friction coefficient} \\ p & - \mbox{damping coefficient} \\ \sigma & - \mbox{damping ratio} \\ a & - \mbox{wave length of wavy surface} \\ h/2 & - \mbox{amplitude of wavy surface} \\ A_{\varphi}, \alpha_{\varphi} & - \mbox{amplitude and phase shift forced pitch} \\ & \mbox{oscillations} \\ e & - \mbox{swing arm length} \\ c & - \mbox{torsion bar spring rate} \\ C_{rs} & - \mbox{reduced torsion bar spring rate in static wheel} \\ & \mbox{position} \\ d & - \mbox{torsion bar diameter} \\ f_s, f_d, f & - \mbox{rebund, positive vertical road wheel travel and} \\ & \mbox{current travel} \\ m_0 & - \mbox{vehicle sprung weight} \\ \theta_s, \theta_m & - \mbox{torsion bar twist angle in static wheel position} \\ v_z & - \mbox{frequency of natural bounce oscillations of the} \\ & \mbox{vehicle} \\ & \mbox{torsion} \\ & \mbox{whicle} \\ & \mbox{torsion bar} \\ & \mbox{weight} \\ \theta_s, \psi_m & - \mbox{swing arm angle in current, static wheel position} \\ & \mbox{and mounted position} \\ & \mbox{super specific work load capacity} \\ \end{array}$	F_{si}	- damper resistance force
$\begin{array}{llllllllllllllllllllllllllllllllllll$	v_i	- relative velocity damper piston
$\begin{array}{llllllllllllllllllllllllllllllllllll$	μ	 – friction coefficient
$\begin{array}{llllllllllllllllllllllllllllllllllll$	р	 damping coefficient
$\begin{array}{llllllllllllllllllllllllllllllllllll$	σ	 damping ratio
$\begin{array}{lll} h/2 & - \mbox{ amplitude of wavy surface} \\ A_{\varphi}, \alpha_{\varphi} & - \mbox{ amplitude and phase shift forced pitch oscillations} \\ e & - \mbox{ swing arm length } \\ c & - \mbox{ torsion bar spring rate } \\ C_{rs} & - \mbox{ reduced torsion bar spring rate in static wheel position} \\ d & - \mbox{ torsion bar diameter } \\ f_s, f_d, f & - \mbox{ rebound, positive vertical road wheel travel and current travel} \\ m_0 & - \mbox{ vehicle sprung weight } \\ \theta_s, \theta_m & - \mbox{ torsion bar twist angle in static wheel position and bump stop position} \\ v_z & - \mbox{ frequency of natural bounce oscillations of the vehicle } \\ \tau_{dm} & - \mbox{ nominal shear stress of torsion bar } \\ \psi, \psi_s, \psi_m & - \mbox{ swing arm angle in current, static wheel position } \\ \varepsilon_p & - \mbox{ specific work load capacity } \end{array}$	a	 wave length of wavy surface
$\begin{array}{lll} A_{\varphi}, \alpha_{\varphi} & - \mbox{ amplitude and phase shift forced pitch} \\ & \mbox{oscillations} \\ e & - \mbox{ swing arm length} \\ c & - \mbox{ torsion bar spring rate} \\ C_{rs} & - \mbox{ reduced torsion bar spring rate in static wheel} \\ & \mbox{position} \\ d & - \mbox{ torsion bar diameter} \\ f_s, f_d, f & - \mbox{ rebound, positive vertical road wheel travel and} \\ & \mbox{ current travel} \\ m_0 & - \mbox{ vehicle sprung weight} \\ \theta_s, \theta_m & - \mbox{ torsion bar twist angle in static wheel position} \\ v_z & - \mbox{ frequency of natural bounce oscillations of the} \\ v_{ehicle} \\ \tau_{dm} & - \mbox{ nominal shear stress of torsion bar} \\ \psi, \psi_s, \psi_m & - \mbox{ swing arm angle in current, static wheel position} \\ \varepsilon_p & - \mbox{ specific work load capacity} \\ \end{array}$	h/2	 amplitude of wavy surface
$\begin{array}{llllllllllllllllllllllllllllllllllll$	$A_{\varphi}, \alpha_{\varphi}$	 amplitude and phase shift forced pitch oscillations
$\begin{array}{lll} c & - \mbox{torsion bar spring rate} \\ C_{rs} & - \mbox{reduced torsion bar spring rate in static wheel} \\ & \mbox{position} \\ d & - \mbox{torsion bar diameter} \\ f_s, f_d, f & - \mbox{rebound, positive vertical road wheel travel and} \\ & \mbox{current travel} \\ m_0 & - \mbox{vehicle sprung weight} \\ \theta_s, \theta_m & - \mbox{torsion bar twist angle in static wheel position} \\ & \mbox{and bump stop position} \\ v_z & - \mbox{frequency of natural bounce oscillations of the} \\ & \mbox{vehicle} \\ & \mbox{tordem} \\ & \mbox{whice} \\ & \mbox{many stop stress of torsion bar} \\ & \end{tabular} \\ \psi, \psi_s, \psi_m & - \mbox{swing arm angle in current, static wheel position} \\ & \end{tabular} \\ & $	е	- swing arm length
$\begin{array}{lll} C_{rs} & - \text{ reduced torsion bar spring rate in static wheel} \\ & \text{position} \\ d & - \text{ torsion bar diameter} \\ f_s, f_d, f & - \text{ rebound, positive vertical road wheel travel and} \\ & \text{current travel} \\ m_0 & - \text{ vehicle sprung weight} \\ \theta_s, \theta_m & - \text{ torsion bar twist angle in static wheel position} \\ & \text{v}_z & - \text{ frequency of natural bounce oscillations of the} \\ & \text{vehicle} \\ & \tau_{dm} & - \text{ nominal shear stress of torsion bar} \\ & \psi, \psi_s, \psi_m & - \text{ swing arm angle in current, static wheel position} \\ & \varepsilon_p & - \text{ specific work load capacity} \end{array}$	С	 torsion bar spring rate
$d = \text{torsion bar diameter}$ $f_s, f_d, f = \text{rebound, positive vertical road wheel travel and current travel}$ $m_0 = \text{vehicle sprung weight}$ $\theta_s, \theta_m = \text{torsion bar twist angle in static wheel position and bump stop position}$ $v_z = \text{frequency of natural bounce oscillations of the vehicle}$ $\tau_{dm} = \text{nominal shear stress of torsion bar}$ $\psi, \psi_s, \psi_m = \text{swing arm angle in current, static wheel position and mounted position}$ $\varepsilon_p = \text{specific work load capacity}$	C_{rs}	 reduced torsion bar spring rate in static wheel position
f_s, f_d, f_s - rebound, positive vertical road wheel travel and current travel m_0 - vehicle sprung weight θ_s, θ_m - torsion bar twist angle in static wheel position v_z - frequency of natural bounce oscillations of the vehicle τ_{dm} - nominal shear stress of torsion bar ψ, ψ_s, ψ_m - swing arm angle in current, static wheel position ε_p - specific work load capacity	d	– torsion bar diameter
$\begin{array}{ll} m_0 & - \text{ vehicle sprung weight} \\ \theta_s, \theta_m & - \text{ torsion bar twist angle in static wheel position} \\ v_z & - \text{ frequency of natural bounce oscillations of the vehicle} \\ \tau_{dm} & - \text{ nominal shear stress of torsion bar} \\ \psi, \psi_s, \psi_m & - \text{ swing arm angle in current, static wheel position} \\ \varepsilon_p & - \text{ specific work load capacity} \end{array}$	f_s, f_d, f	 rebound, positive vertical road wheel travel and current travel
$\begin{array}{lll} \theta_s, \theta_m & - \text{ torsion bar twist angle in static wheel position} \\ v_z & - \text{ frequency of natural bounce oscillations of the} \\ v_{dm} & - \text{ nominal shear stress of torsion bar} \\ \psi, \psi_s, \psi_m & - \text{ swing arm angle in current, static wheel position} \\ \varepsilon_p & - \text{ specific work load capacity} \end{array}$	m_0	 vehicle sprung weight
v_z - frequency of natural bounce oscillations of the vehicle τ_{dm} - nominal shear stress of torsion bar ψ, ψ_s, ψ_m - swing arm angle in current, static wheel position and mounted position ε_p - specific work load capacity	θ_s, θ_m	 torsion bar twist angle in static wheel position and bump stop position
$\begin{array}{ll} \tau_{dm} & - \text{ nominal shear stress of torsion bar} \\ \psi, \psi_s, \psi_m & - \text{ swing arm angle in current, static wheel position} \\ \varepsilon_p & - \text{ specific work load capacity} \end{array}$	v_z	- frequency of natural bounce oscillations of the vehicle
ψ, ψ_s, ψ_m – swing arm angle in current, static wheel position and mounted position ε_p – specific work load capacity	$ au_{dm}$	 nominal shear stress of torsion bar
ε_p – specific work load capacity	ψ, ψ_s, ψ_m	 swing arm angle in current, static wheel position and mounted position
	\mathcal{E}_p	- specific work load capacity

Introduction

THE tracked vehicle mechanical elastic system consists of the hull with its own inertia, springs with a certain spring rate and dampers with determined damping. The hull oscillation is caused by the impulse of external force action, which is the consequence of riding over a uneven terrain or the impulsive change of traction force, e.g. the impulsive braking.

The vehicle hull has three predominant degrees of freedom:

- vertical displacement (bounce),
- rotation about the transverse axis (pitch) and
- rotation about the longitudinal axis (roll).

There are three forms of the oscillation: the bounce, pitch and roll. The simultaneity of all three oscillation forms causes very complex figure of vehicle hull operating oscillation.

The smaller roll amplitude and faster damping are due to smaller vehicles moment of inertia about the longitudinal axis and higher value of frictional force between the road wheels and the tracks as well as the tracks and soil. This was proved by terrain tests. It can be concluded from the previous statement, that the vehicle hull motion during oscillations can be considered as the plane motion consisting of the bounce and pitch oscillations.

The suspension system calculation is based on the differential equations which make a connection between hull tracked vehicle oscillations and its design parameters and the motion conditions. Generally speaking, non-linear oscillations occur due to the fact that the force acting on the road wheel can not be described as a linear function by general coordinates. A non-linearity occurs due to the non-linear characteristics of the suspension, non-equality resistance forces in the dampers during jounce and rebound and the detachment of the wheels from the soil during oscillations.

¹⁾ Military Technical Institute (VTI), Ratka Resanovića 1, 11132 Belgrade

Due to problems related to non-linear differential equations integration, they were reduced in the past to linear by the linearity procedure by removing the second and upper exponent. In this way, the vehicle oscillation figure becomes deformed. However, the difference of oscillation graphs can not be neglected in the case where generalized coordinates and velocity value are still small. In the opposite case, in the cases when the oscillation of vehicle which moves with high velocity, which causes the detachment of the wheels from the soil or in the case when friction dampers were mounted, the oscillation must be considered as non-linear.

The exact mathematical definition even with the simplest non-linear suspension system, causes the complication of mathematical arrangement, so its use for the engineering calculation becomes unpractical. There are numerical solutions researched, which can approximately define non-linear oscillations. Since solutions with clearly defined procedure that determines the oscillations amplitude dependency on the tracked vehicle speed over wavy surface were not available; the subject of this paper was to find this dependence.

Forced damped linear angular tracked vehicle oscillations

A possible scheme of the vehicle hull position, the springs and the dampers are given in Fig.1, for the vehicle which is placed on the wavy surface whose wave length a is twice as long as the track length on the ground. The origin of the coordinate system is allocated to the sprung weight centre, positive direction of z-axis is routed downward (to the direction of increase the spring forces) and positive direction of lean angle φ is routed to degrade forward part of the vehicle hull. The unmoving coordinate system is introduced, and its origin is allocated to the point deflection of the wavy surface. The direction of z'-axis is upwards. The movable coordinate system follows the vehicle sprung weight oscillation and introduces the disturbance force equations by unmovable coordinate system.



Figure 1. Tracked vehicle springs and dampers position on wavy surface

The general solution of homogeneous differential equation of motion in the case of linear free oscillations tracked vehicle with viscous damping [1]:

$$\ddot{\varphi} + 2p\dot{\varphi} + \omega_{\varphi}^2\varphi = 0 \tag{1}$$

is the following:

$$\varphi_h = A e^{-pt} \cos(\omega t - \alpha) \tag{2}$$

Parameters from equation (1) and (2) were determined by the following relations:

$$\omega_{\varphi} = \sqrt{\frac{2\sum_{i=1}^{n} C_{rsi} l_i^2}{J_y}}$$
(3)

$$2p = \frac{2\mu \sum_{i=1}^{k} l_i^2}{J_v}$$
(4)

This relation can be applied when the damper resistance force is proportional to the relative velocity of the damper piston related to the damper, where directions of their motions are opposite.

$$\mu = -\frac{F_{ai}}{v_i} \tag{5}$$

Relation between the damping coefficient and upwards used damping ratio is the following:

$$\sigma = \frac{2p}{\omega_{\varphi}} \tag{6}$$

Angular frequency of free pitch oscillation with the influence of viscous damping coefficient is the following:

$$\omega = \sqrt{\omega_{\varphi}^2 - p^2} \tag{7}$$

The real solution of angular frequency ω exists for the condition $\omega_{\varphi} > p$. In the case when there is no a real solution of $\omega(p > \omega_{\varphi})$, the motion will not be oscillatory.

The general solution of the differential equation of motion in case of linear forced oscillations tracked vehicle with viscous damping is the following:

$$\ddot{\varphi} + 2p\dot{\varphi} + \omega_{\varphi}^{2}\varphi = B\cos qt + D\sin qt \tag{8}$$

The obtained equation consist of the sum of the solution of the homogeneous equation (2) and particular solution, and have to be obtained in the following form:

$$\varphi_p = M \cos qt + N \sin qt \tag{9}$$

Due to the factor e^{-pt} , the free oscillations are quickly damped and have a minor role during the vehicle oscillations, and the forced oscillations are iterated by the constant intensity. Therefore the homogeneous solution, which depends on the initial condition, can be neglected, and the oscillation can be presented by the equation:

$$\varphi = A_{\varphi} \cos(qt - \alpha_{\varphi})$$

The amplitude A_{φ} is calculated by expression:

$$A_{\varphi} = \sqrt{M^2 + N^2} = \sqrt{\frac{B^2 + D^2}{\left(\omega_{\varphi}^2 - q^2\right)^2 + 4p^2q^2}}$$
(10)

where are:

$$B = \frac{h}{J_y} \sum_{1}^{n} C_{rsi} l_i \sin \frac{2\pi l_i}{a}$$
(11)

$$D = -\frac{hq\mu}{J_y} \sum_{1}^{k} l_i \sin \frac{2\pi l_i}{a}$$
(12)

The following ratio between the frequencies forced oscillations q and the vehicle velocity v exists:

$$q = \frac{2\pi v}{a} \tag{13}$$

It can be seen from the equation (10) that in the case of the absence of damping (p = 0) and the equality of the frequency of free and forced oscillations, the amplitude aspires to infinity.

The relation between the amplitude of the forced oscilla tions and the vehicle velocity is determined for vehicle shown in Fig.2, with torsion bar suspension shown in Fig.3. The vehicle suspension characteristics are the following:

- swing arm which is installed under the same mounted angle and with the symmetrical arrangement of the road wheels according to the sprung weight centre,
- the mutual road wheel distance 0,8 m,
- the sprung weight $m_0 = 12000 \text{ kg}$,
- the frequency of the free bounce oscillation $v_z = 1.2$ Hz,
- the swing arm length $e = 0.25 \,\mathrm{m}$.
- number of the dampers k = 4, on front and rear road-wheels.
- for the rolled surface and presetted torsion bar from nickelchromium allow steel by using electro slag refining, nominal shear stress of $\tau_d = 1250$ MPa was accepted.



Figure 2. Tracked vehicle profile



Figure 3. Individual suspension on torsion bars

The condition that the minimum reduced torsion bar spring rate is in the static wheel positions was introduced, in order to simplify the mathematical approximation suspension characteristics. In this case, the swing arm angle in the static wheel position is obtained by the following equation:

$$\psi_s = \left(12\frac{g}{\pi^2}ev_z^2\right)^{-1} \approx \left(11,9ev_z^2\right)^{-1}$$
(14)

The calculation of other parameters was performed by the procedure described in [2] and their values are shown in Table 1.

Table 1: Suspension parameters of the vehicle 1

Parameter	Ψs [°]	θ_s [°]	θ_m [°]	f_d [mm]	<i>f</i> _s [mm]	c [Nm/rad]	<i>C_r</i> [N/m]	<i>d</i> [mm]	ε_p [Nm/kg]
Value	13,3	34,8	111	280	130	3929	57015	31,42	3,895

By using relationships (3-7) and (9-13), the relation between the amplitude of the forced linear oscillations and the vehicle velocity for the case when the vehicle moves on the wavy surface with the amplitude of 0.05 m, without the damping in the suspension system, and with the damping ratio of $\sigma = 0.2$ was established. This relationship is graphically presented in Fig.4.



Figure 4. The relationship between the amplitude of the linear oscillations and the vehicle velocity, on wavy surface amplitude 0,05 m

It can be seen from the Fig.3 that in the case of the damping ratio $\sigma = 0.2$ and the corresponding resonance velocity, the vehicle hull lean amplitude has such a value that the swing arms on forward road wheels contact the bump stops. Contact is possible under the velocity values of 6.07-7.41 m/s in the case where there is no damping.

Non-linearity characteristics of torsion bar suspension

The character of inter dependence between the force on the swing arm journal and its displacement, the so called suspension characteristics, could be approximated by straight line, and oscillations could be mathematically presented by the homogeneous and un-homogeneous differential equations of the second row, for the many elder tracked vehicle with the torsion suspension system. On the other side, in case of the high speed tracked vehicle, the quality of the ride predominantly influenced by the pitch oscillations. The relation between the torque and the lean angle is closer to the straight line, then the suspension characteristics in the zone of small and middle lean angle. However, there are the torsion suspension systems with the extremely non-linear suspension characteristics, where the calculation of non-linear oscillations should be based on non-linear theory of oscillations.

Suspension characteristics for the vehicle 1 are shown in Fig.2, with the parameters value given in Table 1, shown in Fig.5 (solid line), while the approximation with third exponent curve, determined by Maclaurin approximation is represented by the broken line. It is obvious that the approximation is nearly concurrent with the suspension characteristics in the range road wheel traveling from -0.1 m to +0.15 m.



Figure 5. Suspension characteristics of the vehicle 1

Graphical procedure solution of forced non-linear angle vehicle oscillations

Provided that the sum of the torque of all forces acting on the vehicle, apart from the damper resistance forces (Fig.1), is equal zero:

$$M_{\varphi} + M_{\gamma} + M_{h} = 0 \tag{15}$$

By division with the torque of inertia, the differential equation of the forced angular oscillations is obtained:

$$\ddot{\varphi} + \frac{M_y(\varphi)}{J_y} = B_n \sin qt \tag{16}$$

where:

Table 2: Calculation procedure

 $M_{\varphi} = J_y \ddot{\varphi}$ – torque of inertia, whose direction is opposite to the acceleration $\ddot{\varphi}$ direction,

 M_{ν} – torque of additional spring elastic forces,

 M_h – disturbance torque, i.e. torque at the horizontal position of the vehicle hull, when a front road wheel is on a prominence of the wavy surface, and rear road wheel is on the recession, with the height difference h.

 $q = 2\pi v/a$ – frequency of the forced oscillations, during the vehicle motion v on a wavy surface with the wave length a.

Calculation procedure of torques M_y and M_h is presented in Table 2.

Dependence of torque originating from additional spring forces acting on the vehicle hull, on the vehicle hull lean angle $M_y(\varphi)$, is graphically shown in Fig.10 (for the second numerical example), while Fig.6 shows the dependence of the quotient $M_y(\varphi)/J_y$.

The coefficient B_n for non-linear oscillation, is obtained from expression:

$$B_{n1} \equiv \frac{M_{h=(0.05)}}{J_y} = 0.80121 (s^{-2}) \text{ and}$$
$$B_{n2} \equiv \frac{M_{h=(0.02)}}{J_y} = 0.31391 (s^{-2})$$

If the differential equation solution (16) assumes the of form:

$$\varphi = A_{\varphi} \sin qt \tag{19}$$

the following equation is obtained:

$$\frac{M_{y}(\varphi)}{J_{y}} = \left(B_{n} + q^{2}A_{\varphi}\right)\sin qt \qquad (20)$$

from which, by graphical procedure [3], the oscillation amplitude can be obtained, by finding the abscissa of section of the equation $M_y(\varphi)/J_y$ and the straight line of which the coefficient is equal to the square of the frequency of forced oscillations q while B_n section is on the positive direction of ordinate. With the example in question, the

	var	iety due to the hull	lean	variety due to the wheels encountering wavy surface					
$f_i = (m)$	$f_i = \varphi l_i$ $l_1 = l_6 = 2m$ $l_2 = l_5 = 1.2m$				$f_i = -\frac{h}{2}\sin\frac{2\pi l_i}{a}$ $a = 8 m h_i/2 = 0.05 m$				
	$l_3 = l_4 = 0.4 m$				$h_2/2=0.02 m$				
	$f_1 = f_6$	$f_1 = f_6 \qquad \qquad f_2 = f_5$		$f_1 = f_6$	$f_2 = f_5$	$f_3=f_4$			
	$2\varphi = f$	1.2 <i>φ</i> =0.6 <i>f</i>	$0.4\varphi = 0.2f$	h	hsin0.6π≈0.809h	hsin0.2π≈0.309h			
$F_i(N)$	$F_{i} = \frac{c}{e} \frac{(\psi_{mi} - \arcsin(\sin\psi_{mi} - \frac{f_{i} + f_{s}}{e})}{\sqrt{1 - \left(\sin\psi_{mi} - \frac{f_{i} + f_{s}}{e}\right)^{2}}} (17) f_{sl} = 0.13 \ m \ f_{sll} = 0.138 \ m \ F_{i} \ge 0$								
$F_i(N)$ (approx)	$F_{i} = F_{0} + C_{rs}f + C_{rs}\beta f^{3} (18) \qquad F_{i} \ge 0$								
$M_i(Nm)$	$M_y = \Sigma F_i l_i$ $M_h = \Sigma F_i l_i$								

torque line is symmetrical in relation to the origin $(M(\varphi) = -M(\varphi))$, which is a necessary condition for the graphical method to be applied [3,4].

The graphical procedure for determining the dependence of the oscillation amplitude upon the amplitude of wavy surface as of 0.05 m is shown in Fig.6.



Figure 6. Graphical procedure determination of relationship between the amplitude of the linear oscillations and the vehicle velocity

Dependence of the oscillation amplitude on the vehicle velocity is shown as a solid line for the amplitude of wavy surface 0.05 m (Fig.7) as for the amplitude it is 0.02 m (Fig.8).



Figure 7. Relationship between the amplitude of the linear oscillations and the vehicle velocity on the wavy surface amplitude of 0.05 m



Figure 8. Relationship between the amplitude of the linear oscillations and the vehicle velocity on the wavy surface amplitude of 0.02 m

Dependence of the resonant velocities on the vehicle velocity and the non-linear oscillations is shown by broken-dotty line. Final dependence can be obtained by graphical method, in the section of torque line and straight lines drawn from the origin, whose line coefficient corresponds the square frequency of forced oscillations, or analytically on the basis of torque dependence on the angle of vehicle hull leaning.

Dependence of the torque on the leaning angle is not a continual line but has two points in the first and third quadrants in which the curve inclination changes by leaps that correspond to the angles when road wheels disconnect from the soil. For this reason, the dependence of the oscillation amplitude on the pass velocity over the given wavy surface (frequency of forced oscillations), has its singular point f and h (Fig.7). The same is valid for the resonant curve that breaks in the point g.

According to the non-linear oscillations theory [5], it is impossible to realize the oscillations in the zone to the right of the resonant line and to the right of the line BC (Fig.7). Oscillations in the zone to the left of the resonant line and to the right of the line BC, can be realized only by gradual increase of the forced oscillations frequency. Since this condition can hardly be realized while driving on real paths, dependence of forced amplitude oscillations on the vehicle velocity is along the ABCD line.

Comparing the dependence of amplitude oscillation on the vehicle speed, or on the frequency of forced oscillations, it can be concluded that the maximum amplitude oscillations for the amplitude of wavy surface 0.05 m (Fig.7) is slightly higher compared to the amplitude 0.02 m (Fig.8). That means that the magnification factor is higher if the torque of disturbance is lower. Nevertheless, the interval of velocities in which the oscillations of high amplitudes might appear is considerably smaller in case of the low amplitude of wavy surface. On the other hand, the relation of amplitude oscillation outside the area of resonant velocities is approximate to the relation of the amplitudes of wavy surface.

The graphical procedure for defining the minimum amplitude of wavy surface where the contact of the swing arms with the bump stops is possible, is shown in Fig.6. A straight line is drawn from the point of torque and the angle of vehicle hull leaning at which the front vehicles road wheels contact the bump stops, thus making the line to tangent the torque line (a dotty-straight line). By multiplying the section on the ordinate with the moment of inertia, the torque value is obtained (case in question 47211 Nm); it is then used to determine amplitude of the wavy surface (case in question 64.4 mm).

Vehicle oscillations with low non-linearity suspension

To denominate the special features of oscillation of the vehicle 1 with the characteristic suspension, as in Fig.5, it is enough to determine the dependence of the torque on the angle of the vehicle hull lean for another vehicle 2 with normal non-linear suspension. In this example the swing arm length is e = 350 mm, as it is with vehicles of this class. The same value of positive vertical road wheel travel 280 mm, is obtained from nominal shear stress of torsion bar of 995 MPa. There is no demand of having the minimum reduced torsion bar spring rate in the static wheel position, because swing arms angle in static wheel position would be small and the road wheels large in diameter.

Suspension parameters of the vehicle 2 is presented in Table 3.

Table 3. Suspension parameters of the vehicle 2

Parameter	Ψs [°]	θ_s [°]	$ heta_m$ [°]	<i>f_d</i> [mm]	<i>f</i> _s [mm]	c [Nm/rad]	<i>C_r</i> [N/m]	<i>d</i> [mm]	€ _p [Nm/kg]
Value	15	25,72	75,45	280	137,7	7386	56837	36,79	2,9066

Suspension characteristics of low non-linearity are shown in Fig.9 (solid line) along with characteristics of the described third exponent equation (broken line) and characteristics from the first example (dotty line).



Figure 9. Suspension characteristics of the vehicle 2

It is obvious that the characteristics from the first example rapidly increase in the last third of displacement. It is also obvious that the characteristics from the second example can be successfully substituted with the Maclaurin approximate third exponent equation along the displacement.

Dependence of the torque on the vehicle 2 hull lean angle has been defined with regards to the real force value (Fig.10).



Figure 10. Dependence of the torque on the vehicle 2 hull lean angle

Deviation of this dependence from the straight line is minimal and with amplitude of wavy surface of 8 mm, it is possible to have increase amplitude of oscillation up to the dynamic displacement.

By drawing straight line from points on the ordinate line indicating static value of torque on the amplitudes of wavy surface of 50 mm and 20 mm to the extreme points of the diagram or to the singular point, and by dividing the direction coefficient with the torque of inertia, the vehicle velocity values were obtained, where an impact of the swing arms and bump stops is possible. Case in question, the velocity range for the amplitude of wavy surface 20 mm is 6.63-7.12 m/s while for the amplitude 50 mm it ranges from (6.15-7.5) m/s. Compared to the corresponding velocity ranges of the systems with the linear suspension characteristic and for the same amplitude (6.07-7.41 m/s), it is clear that the range of critical velocities is almost the same.

Numerical solution of forced non-linear oscillations procedure

Numerical procedure can be rationally applied to some well known and existing solutions, and in the case when the suspension characteristics can be presented as a polynomial with member of the first and third exponent, as in Fig.5.

$$\varphi = A_{\varphi} \sin qt \tag{21}$$

the differential equation of non-linear oscillations has the following form:

$$\ddot{\varphi} + \omega_{\varphi}^{2} \varphi + \beta \omega_{\varphi}^{2} \varphi^{3} = B_{n} \sin qt \qquad (22)$$

where are:

$$\omega_{\varphi}^{2} = 28.29668 (s^{-2})$$
$$\beta = 36.2942(-)$$
$$B_{n} = 0.80131 (s^{-2})$$

The first two parameters were determined by Maclaurin approximate on the bases of equation (M_y/J_y) of vehicle hull lean angle. The variation of exact torque value for angle $\varphi = \pm 0.045$ rad is less than 1%. The equation (22) was solved found by the double differentiation of the sine function assumed solution (19), so the first approximation is obtained [3]:

$$\varphi \approx \frac{(-1)}{q^2} \left[B_n - \omega_{\varphi}^2 A_{\varphi} - \frac{3}{4} \omega_{\varphi}^2 \beta A_{\varphi}^3 \right] \sin qt - \frac{\omega_{\varphi}^2 \beta A_{\varphi}^3}{36q^2} \sin 3qt$$
(23)

The equation for the amplitude A_{φ} was obtained by (19) and (23):

$$(q^{2} - \omega_{\varphi}^{2})A_{\varphi} + B_{n} = \frac{3}{4}\omega_{\varphi}^{2}\beta A_{\varphi}^{3}$$
(24)

Due to the fact that the graphical solution is derived from the differential equation (16), which for assumed approximate relationship $M_Y(\varphi)$ (21) becomes equation:

$$(q^2 - \omega_{\varphi}^2)A_{\varphi} + B_n = \omega_{\varphi}^2\beta A_{\varphi}^3$$

This is the evidence that the graphical procedure error occures because the coefficient with the cube member needs to be reduced by one fourth of the value.

The exact torque dependence on the vehicle hull lean angle and the dependence expressed by the third exponent equation are shown in Fig.11. Variation of these dependences are important only in the adjacent angles.

By solving the equation (24), relationship between oscillation amplitude and vehicle velocity was obtained and it is shown (by solid line) in Fig.12. This curve size differs from the one obtained by graphical method, particularly in the zone of the vehicle hull lean angle where real and assumed relation between the torque of spring elastic forces and vehicle hull lean angle differ significantly. This adduced to conclusion that graphical procedure gives rather score.



Figure 11. Approximation of the dependence of the torque on the vehicle 1 hull lean angle by third exponent equation



Figure 12. Numerical procedure of determination of the relationship between oscillation amplitude and vehicle velocity

In regards to this, it is proven that by numerical procedure more correct results are obtained for even less vehicle hull lean angle, so the numerical procedure can be used in the angle section to ± 0.045 rad.

These values of the oscillation amplitude obtained in the numerical procedure are at velocities 0.2 m/s lower than those obtained by the graphical procedure. For this reason, it can be accepted that the part of the diagram up to the amplitude of ± 0.045 rad was obtained by the analytical procedure, and over this value the authoritative amplitude was obtained by the graphical procedure, provided that the responsive velocity was reduced by the value of 0.2 m/s.

Conclusion

By the graphical procedure for solving the forced oscillations for the tracked vehicle with the extremely non-linear suspension characteristics which moves on a wavy surface, the reduction of maximum oscillation amplitude was concluded. The approximate numerical procedure can be used for obtaining more accurate results in the low amplitude. The forced oscillation of almost all light tracked vehicles with torsion suspension, can be solved by the forced linear oscillation method.

References

- ZABAVNIKOV, N.A.: Osnovy teorii transportnyh guseničnyh mašin, Mašinostroenie, Moskva, 1968.
- [2] STEVANOVIĆ,R.: Potencijalna energija kao jedan od kriterijuma za izbor parametara torzionog sistema oslanjanja borbenih guseničnih vozila, Beograd, Naučno tehnički pregled, Vol.LII, 2002, No.2, pp.26-32
- [3] RAŠKOVIĆ, D.: Teorija oscilacija, Naučna knjiga, Beograd, 1957.
- [4] Timošenko, S.P., Jang D.H.: *Teorija oscilacija*, Građevinska knjiga, Beograd, 1966.
- [5] DMITRIEV,A.A., ČOBITOK,V.A., TELJMINOV,A.V.: Teorija i rasčet nelinejnyh sistem podressorivanija guseničnyh mašin, Mašinostroenie, Moskva, 1976.

Received: 21.02.2005.

Kretanje guseničnog vozila, nelinearne karakteristike oslanjanja po talasastoj površini

Za gusenično vozilo s naglašeno nelinearnom karakteristikom oslanjanja, grafičkim putem je određena zavisnost amplituda oscilovanja od brzine vozila koje se kreće po talasastoj površini talasne dužine jednake dvostrukoj dužini naleganja gusenice. Približni numerički postupak rešavanja nelinearnih oscilacija korišćen je za korigovanje grafičkog postupka. Diskutovane su osobenosti oscilacija vozila s karakteristikom umerene nelinearnosti.

Ključne reči: gusenično vozilo, oslanjanje vozila, sistem oslanjanja, oscilacije vozila, nelinearne oscilacije.

Le mouvement d'un véhicule à chenilles à caractère non-linaire d'appui sur une route ondulante

Pour un véhicule au point d'appui accentué, par la voie graphique est déterminé la dépen - dance de l'amplitude d'oscillation de la vitesse du véhicule qui se déplace sur une route ondulante dont la longueur d'ondes est égale à la double longueur du chevauchement des chenilles. Pour corriger le procédé graphique on a utilisé le procédé numérique approximatif dans la résolution des oscillations non-linaires. Les caractéristiques des oscillations des véhicules à non-linéarité modérée ont fait l'objet des discussions.

Mots clés: vécule à chenilles, point d'appui de vécule, système d'appui, oscillation de vécule, oscillations non-linéaires.

Движение гусеничной машины нелинейной характеристики подвески по волнистой поверхносты

Для гусеничной машины с чрезвычайно нелинейной характеристикой подвески, графическим методом определена зависимость амплитуд колебания от скорости гусеничной машины которая движется по волнистой поверхносты, длиной неровности ревнолй удвоенной длине базы (опорной поверхности гусениц). Приближенный аналитическый метод решения нелинейных колебания исполъзован для корригирование графического метода. Рассматриваны особенности колебаний гусеничной машины с характеристикой умеренной нелинейности.

Ключевые слова: гусеничная машины, подвеска машины, подвеска, колебания гусеничной машины, нелинейные колебания.