

## Applicability of average values of meteorological elements

Dragoslav R. Petrović, PhD (Eng)<sup>1)</sup>

**In practice it is important to know the statistical characteristics of meteorological elements, their nature being that of random variables characterized by average values and dispersion. Results of investigation of meteorological elements' nature are given and optimum measuring time for calculating average values is determined.**

*Key words:* meteorological measuring, meteorological elements, average value, statistical characteristics, projectile flight, artillery fire.

### Introduction

**W**EATHER (in synoptic sense) is characterised by physical atmosphere processes that are mutually dependent for a certain time interval in some domain of space. Atmosphere characteristics can be measured and are called meteorological elements. In these measurements air temperature, pressure, moisture, wind speed and direction, cloudiness, etc: can be determined taken into account. All meteorological elements can't be taken into account in the study of projectile dynamics and only the nature of temperature, air pressure, wind magnitude and its direction will be considered. Of all the calculated meteorological elements, air temperature is the most important, for its pressure depends on height variation, i.e. the generated wind magnitude and its direction. From the moment when meteorological elements are determined by measuring, until the time of processing and forming meteorological message, time pass, and making the measuring out-of-date. Space and time variation of meteorological elements can be observed through meteorological message accuracy: accuracy decreases with time increasing from the moment of forming the meteorological message (long-lasting meteorological message) and with increasing the distance from firing position. In this article the nature of average values of meteorological elements and time of validity will be demonstrated. In practical applications, some relations concerning the nature of meteorological elements must be observed in order to avoid making huge errors.

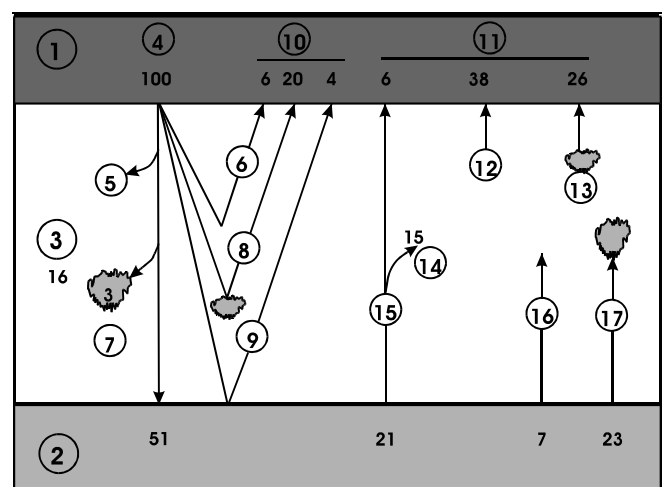
### Atmosphere and its properties

On its way from the sun to the target a projectile passes through the atmosphere enveloping the Earth. Air is a mixture of various gases and water vapour. Composition of gases in the air varies little with altitude. However mass variation and density are considerable [5–8]. On the altitude of 5.5km, total air mass is 50%, at 11 km 7.5%, and at 22km only 0.96%. Relative air density  $\gamma$  distribution dependence on altitude in percents is given in Table 1.

**Table 1.** Relative air density distribution on altitudes

| Altitude (km) | $\gamma$ (%) | Altitude (km) | $\gamma$ (%) |
|---------------|--------------|---------------|--------------|
| 0             | 100          | 33            | 1            |
| 6             | 50           | 48            | 0,1          |
| 18            | 10           | 93            | 0,0001       |

Physical atmosphere processes are mutually dependent and for a certain time interval in the predefined place or area are characterized as weather (in synoptic sense). Atmosphere characteristics can be measured and are called meteorological elements. The most significant element is temperature, for its variations dependence on the pressure. It is followed by wind magnitude and direction. Air temperature is the degree of its warming for which heat (energy) is used. Sun is a heat source which warm the surface from which the Sun beams are reflected and transferring heat to the air. Heat transfer to the air is done by radiation, conductivity and convection. Heat balance Earth – Sun in percents is given on Fig.1, where:



**Figure 1.** Earth – Sun heat balance

<sup>1)</sup> Military Technical Institute (VTI), Ratka Resanovića 1, 11132 Belgrade

1 -universe, 2 –earth, sea, 3 -atmosphere, 4 –input sun radiation, 5 - H<sub>2</sub>O absorbed by the surface, O<sub>3</sub>, 6 –rejected by air, 7 –absorbed in clouds, 8 –rejected from clouds, 9 – rejected from ground, 10 –rejected sun radiation, 11 – output infrared radiation, 12 -radiation H<sub>2</sub>O, CO<sub>2</sub>, 13 – radiation of clouds, 14 –absorbed from H<sub>2</sub>O, CO<sub>2</sub>, 15 –pure infrared radiation from ground, 16 –variable heat flux, 17 - latent heat flux.

It can be seen from Fig.1 that out of 100% sun energy, 19% is absorbed on its way through the atmosphere, i.e.: 16% air admixture and 3% in the clouds, and the rest of 30% is reflected back: 20% from the clouds, 6% from the air admixture and 4% from the surface. Remaining 51% is absorbed by the surface and it is used for air heating: 21% for radiation, 7% for conductivity and 23% for convection. Not only temperature, but also pressure and wind depend on the surface heat balance. Meteorological elements magnitudes vary in horizontal and vertical direction. In vertical direction there is a pattern in the variation of meteorological elements, but in the horizontal direction there is no evidence of the surface influence. For the sake facilitating the study of meteorological elements variation, atmosphere is divided into layers. Partitions and names of the layers are given in Fig.2. First partition is troposphere, closest to the surface and in middle geographic latitudes varying 10-12 km in depth, but can reach 17-18 km above the equator. Inside the troposphere, three under layers can be noticed. Within the first one 1-2 km deep mostly mechanical and earth ground heat influence the meteorological elements. In the second, ranging from the upper limit of the first layer and spreading 6-7 km the influence the surface is smaller, but in the third, that influence is almost absent.

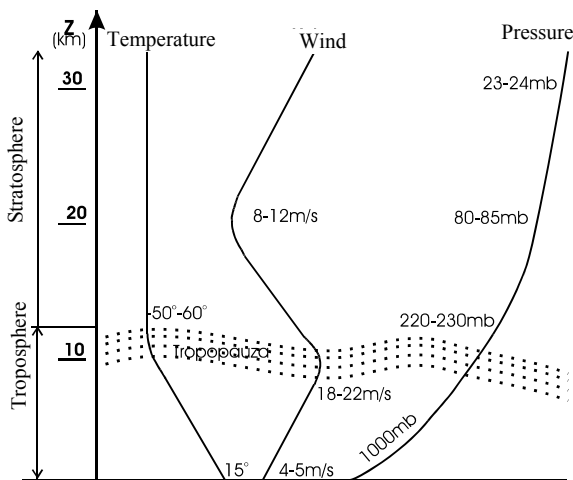


Figure 2. Atmosphere layers and meteorological elements variations

Second layer is named stratosphere. Between troposphere and stratosphere an underlayer, 100m to 2km deep, called tropopause is situated. Troposphere's characteristic is that in it the temperature variation with height is nearly constant, but the gradient is negative. In tropopause, the gradient is not constant, and in stratosphere isothermia exists.

**Meteorological elements variations**

If the measured values of the meteorological elements are known, their application is greatly facilitated. However, due to the turbulent nature of the atmospheric processes, they can be considered average values of their variations in a certain time interval. For the estimation of the values, the data given in Table 2 can be useful.

Table 2. Order of meteorological elements and its derivatives

|                           | $V_s$              | $V_v$             | $p$               | $l/p$               | $T$                |
|---------------------------|--------------------|-------------------|-------------------|---------------------|--------------------|
| 1                         | $10^0-10^1$        | $10^2-10^1$       | $10^5$            | $10^0$              | $10^2$             |
| $\partial/\partial_s$     | $10^{-5}-10^{-4}$  | $10^{-7}-10^{-6}$ | $10^{-4}-10^{-2}$ | $10^{-8}-10^{-7}$   | $10^{-5}-10^{-4}$  |
| $\partial/\partial_z$     | $10^{-3}-10^{-2}$  | $10^{-5}-10^{-4}$ | $10^1$            | $10^{-4}$           | $10^{-3}-10^{-2}$  |
| $\partial/\partial_t$     | $10^{-4}-10^{-3}$  | $10^{-6}-10^{-5}$ | $10^{-2}-10^{-1}$ | $10^{-7}-10^{-6}$   | $10^{-4}-10^{-3}$  |
| $\partial^2/\partial s^2$ | $10^{-10}-10^{-9}$ | $10^{-9}-10^{-9}$ | $10^{-9}-10^{-8}$ | $10^{-13}-10^{-12}$ | $10^{-10}-10^{-9}$ |

The values given in Table 2 are most common in practice for horizontal  $V_s$  and vertical  $V_v$  wind velocity, pressure  $p$ , specific volume  $l/\rho$  and temperature  $T(^{\circ}K)$  and their derivatives related to horizontal distance  $\partial/\partial_s$ , vertical height  $\partial/\partial_z$  and time  $\partial/\partial_t$ . All values refer to lower troposphere layers. Meteorological elements depend on the atmosphere heat balance. If stationary state in which heat is transferred by radiation is considered, i.e. rays of the determined wave length are the only energy sources. Then theoretically, temperature profile from which can be viewed the domain of temperature influence, can be seen as well as the domain of other heat transfer influences can be determined. A possible temperature profile is given by the following equation [5]

$$T(z) = \left[ \frac{1-A}{2\sigma} S_0 \cos \nu \left( 1 + 1.66 a_0 \rho H e^{-\frac{z}{H}} \right) \right]^{\frac{1}{4}} \quad (1)$$

and for  $a_0 \rho H = 1$  and  $(1-A) S_0 \cos \nu = 0.3 \text{ cal}/(\text{cm}^2 \cdot \text{min})$  is given on Fig.3.

In eq.1  $\sigma$  is Stefan-Bolcman constant,  $(1-A) S \cos \nu$  is related to parameters of short-wave radiation Earth-atmosphere. Model characteristic, given by curve 1, that it is fast temperature variation near the surface, for  $z = 0$ ,  $dT/dz = -19.5 (^{\circ}K)/\text{km}$ , for  $z = 3.2 \text{ km}$ ,  $dT/dz = -6.5 (^{\circ}K)/\text{km}$ , at  $z > 10 \text{ km}$ ,  $dT/dz = 0$ . Curve 2, 3, i 4 are references given by other authors. For curve 2 conditions are the same as for curve 1. Curve 3 is obtained at termical equilibrium and convection is taken into account in the atmosphere without clouds, and for curve 4 convection with clouds is taken into account. It is obvious that convection mostly changes the temperature profile in the troposphere, and a just little in the stratosphere. Models of temperature profile are smooth lines which is not the case in nature. Temperature measurements give only discreet points that can be connected by straight lines - [6].

Horizontal air dynamics parallel to the surface are called wind. Wind is defined by two quantities: magnitude and direction. From the point of view of macro-dynamics, wind stems from the pressure difference between two distinct points on the surface, which is in direct correlation with the atmospheric heat balance above the north and south hemispheres. Micro-dynamics depend on the surface shape and are characteristic for first underlayer of the tropopause. All dynamics of the air mass are of turbulent nature, regardless of the layer altitude.

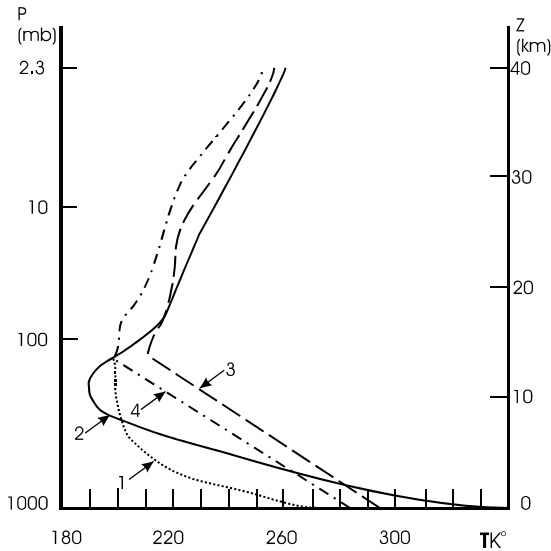


Figure 3. Temperature profile at radiation equilibrium and at termical equilibrium with convection

**Average values of meteorological elements**

Turbulent nature of meteorological elements variations, in particular temperature and wind as values most interesting in practice, particular mathematical methods are required for studying and recording them in the form suitable for application. When turbulent processes are in question, small perturbation of initial state can not be taken into account for forecast. For it, meteorological elements have random variable attributes with a certain law of distribution, mean value - average and dispersion  $\sigma$ . Some turbulent processes, with value  $a$  can be represented in form

$$a = \bar{a} + a' \tag{2}$$

where  $a$ ,  $\bar{a}$ ,  $a'$  are related to current, average and pulsation value variable in question, and then  $V_{i,i=1,2,3}, p, \rho, T$ . Meteorological elements  $V_{i,i=1,2,3}, p, \rho, T$  are variables that vary daily and randomly. The average values can be calculated for infinite time interval and large number of measured data. Intervals of interes are finite, and number of sequences is one. Ergodic hypothesis enable the substitution of average values obtained from large number of measured data with average values obtained in finite time interval  $T$ . For stationary process the following expression for  $\bar{a}$

$$\bar{a}(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} a(t + \tau) d\tau = const \tag{3}$$

In case of finite time interval  $T$ , non-infinite average values depend on size  $T$ . In Fig.4 possible situations are illustrated.

Curve in Fig.4: a) illustrates small variation of average values in  $T$ ; b) interval  $T_3$  or  $T_4$  is chosen so that it is not a part of average, but for  $T_1$  and  $T_2$  it is. In the close energy spectre, most energy consumption is in lower atmosphere layers for creating fast variation - pulsation of meteorological elements. In Fig.5 results of amplitude  $E_\omega$  distribution of wind pulsation near surface in frequent domain  $\omega = 0.0007 - 900 (Hz * 3600)$  are given.

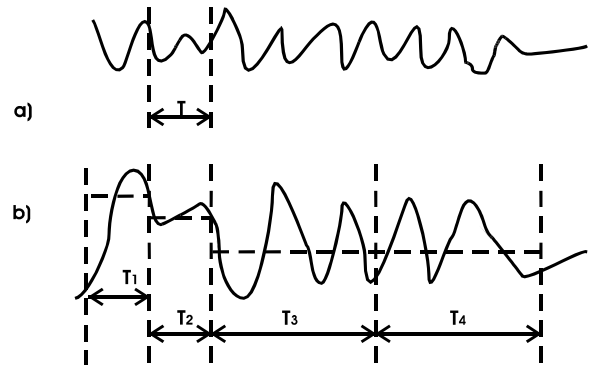


Figure 4. Typical variation of meteorological elements in time at various intervals

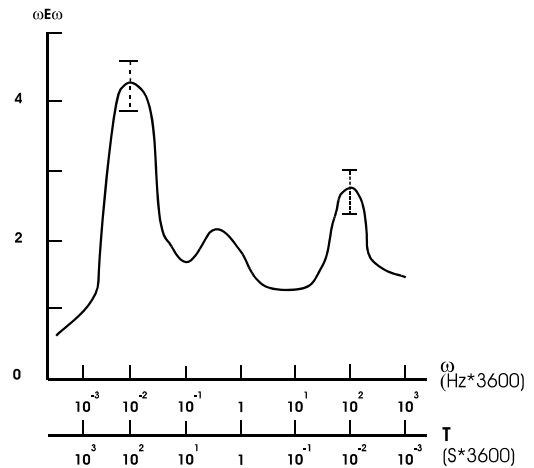


Figure 5. Wind speed spectre near the surface

On apcise in Fig.5  $\ln \omega$ , on ordinate is  $\omega E_\omega$  is given. It can be seen from the figure that pulsation energy is concentrated in two domains: in pulsation domain of large scale (synoptic domain) with centre near  $\omega_1 = 0.01 (Hz * 3600)$  and in pulsation domain of small scale with centre near  $\omega_2 = 80 (Hz * 3600)$ . An aproximate expression for wind speed as function of two members can be obtained.

$$v(t) = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t \tag{4}$$

and average value

$$\bar{v}(t) = A_1 \frac{2 \sin \omega_1 t \sin \omega_1 \frac{T}{2}}{\omega_1 T} + A_2 \frac{2 \sin \omega_2 t \sin \omega_2 \frac{T}{2}}{\omega_2 T} \tag{5}$$

Choosing interval  $T$  as

$$\omega_1 T = \frac{1}{\omega_2 T} \gg 1 \tag{6}$$

The following is obtained:

$$\bar{v}(t) = A_1 \sin \omega_1 t \tag{7}$$

It is obvious from (7) that  $v$  does not depend on  $T$  in domain of small scale spectre. Therefore, an interval  $T$  can be chosen such, that  $v(t) = \bar{v}(t)$  can be taken with sufficient ecuracy. Conclusion in the case of wind is valid for other meteorological elements. Under such circumstances, for finite interval  $T$ , expression for average values of meteorological elements will be



**Table 9.** Correlation coefficients  $r|W_{x,z}^{Y_i}, W_{x,z}^{Y_j}|$ 

| $Y_j$ (m) | $Y_i$ (m) |      |      |      |      |      |      |      |
|-----------|-----------|------|------|------|------|------|------|------|
|           | 200       | 600  | 1000 | 1400 | 1800 | 2200 | 2600 | 3000 |
| 200       | 1         | 0.93 | 0.91 | 0.89 | 0.89 | 0.88 | 0.88 | 0.88 |
| 600       |           | 1    | 0.97 | 0.95 | 0.93 | 0.91 | 0.91 | 0.90 |
| 1000      |           |      | 1    | 0.98 | 0.96 | 0.94 | 0.93 | 0.92 |
| 1400      |           |      |      | 1    | 0.98 | 0.96 | 0.95 | 0.94 |
| 1800      |           |      |      |      | 1    | 0.98 | 0.96 | 0.95 |
| 2200      |           |      |      |      |      | 1    | 0.98 | 0.97 |
| 2600      |           |      |      |      |      |      | 1    | 0.98 |
| 3000      |           |      |      |      |      |      |      | 1    |

Correlation coefficients  $r|W_{x,z}^{Y_i}, W_{x,z}^{Y_j}|$  that give degree of relations dependence among random variables that express wind components for different altitudes are given in Table 9.

### Discussion

Eq. (6) can be used for obtaining estimation of optimal period  $T$  for which average values - [9] ought to be calculated.

$$T = \frac{1}{\sqrt{\omega_1 \omega_2}} = \frac{1}{\sqrt{0.1 \cdot 80}} = 67 \text{ (mi nuts)} \quad (11)$$

Experimental determining of vertical air dynamics and turbulence can be carried out by considering the increasing in vertical air flow between isobar surfaces on levels  $\Delta\pi_k$  - [10]. If radio-sonde measurements are done every 30s, average values for 5 minute comprise 11 measurings, for which time layer depth grows to 1500m, which is approximately equal to isobar layer. Depth of layer  $k$ ,  $H_k$ , can be determined from streight line equation

$$H_k = H_{k-1} + \tau W_k \quad (12)$$

where  $\tau$  -time (in minutes),  $W_k$  - vertical radio-sonde speed, constant for given layer. Height dispersion  $H_k$ ,  $D_H$ , is calculated as

$$D_H = \frac{1}{n-1} \sum_{i=1}^n H_{kj} - \tau_j W_k \quad (13)$$

Values of standard deviation  $\sigma_H = \sqrt{D_H}$  will represent integral turbulent intensity in some frequent range  $\omega$ . If air masses oscilate with  $\omega$ , for 5min and wind speed  $W_k = 5\text{m/s}$ , under radar beam inclination approximately equal  $30^\circ$ , horizontal distance will be  $L_j=3000\text{m}$  and speed  $V=10\text{m/s}$ . If  $\omega$  is given as

$$\omega = 2\pi \frac{V}{L} \quad (14)$$

for given condition  $\omega_1=0.0209\text{rad/s}$ .

For sample frequencies 30s, Nikvist frequency is equal

$$\omega_N = \frac{\pi}{\Delta t} \quad (15)$$

meaning  $\omega_N=0.105\text{rad/s}$ .

It can be seen that frequencies are in the range of  $0.0209\text{rad/s} < \omega < 0.105\text{rad/s}$ . Frequencies greather than Nikvists will produce considerable numerical errors. Time range diminishing in averaging is possible in high frequency domain, which is important during the study of near surface phenomena. However time range of averaging increasing in low frequencies domain is possible for account of increasing time range, which is in accordance with (11).

### Conclusion

Projectile moving from the gun to the target passes throught atmosphere which envelopes the Earth. Physical atmosphere processes are mutually dependent and for a certain time interval in predefined surface are characterized as weather (in sinoptic sense). Atmosphere characteristics can be measured and they are called meteorological elements. The most important one is temperature, due to its variations depending on pressure, and is followed by wind magnitude and direction. Not only temperature, but pressure and wind magnitude and its direction as well, depend on the Earth heat balance. Meteorogical elements vary in altitude and horizontal. In vertical direction there is a patern in variation of meteorological elements, but in the horizontal direction there is no evidence of the surface influence due to the surrounding nature. For the sake of facilitating the study of meteorological elements variation, atmosphere is devided into layers. The first one - troposphere is closest to the surface and in middle geographic latitudes varying 10-12 km in depth, but reaching up to 17-18 km above the equator. Inside the troposphere, three underlayers can be noticed. In the first underlayer 1-2 km deep mostly mechanical and surface heat influence the meteorological elements. In the second underlayer, from the upper limit of the first layer 6-7 km the influence of the surface is smaller, but in the third that influence is almost absent. The second layer is called stratosphere. Between troposphere and stratosphere an underlayer called tropopause is situated, 100m to 2km deep. A characteristic of the troposphere is that the temperature gradient in it is negative and nearly constant. In the tropopause, the gradient is not constant, and in the stratosphere isothermia exists. In the troposphere the wind direction and magnitude are not predictable near the surface; the magnitude becomes constant with the positive gradient and unpredictable direction. In the stratosphere the wind magnitude and direction are also unpredictable, the direction is changed in relation to the previous layer as a rule, but the wind magnitude and direction are constant for a longer time period during the day.

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## Upotrebljivost srednjih vrednosti meteoroloških elemenata

Poznavanjem prirode i statističkih karakteristika merenih meteoroloških elemenata (vetar, pritisak, temperatura) koji imaju atribut slučajne veličine (karakterišu ih srednja vrednost i rasturanje), u praksi se mogu korisno upotrebiti prilikom obrade rezultata visinskog merenja. Najvažniji podatak koji se mora unapred znati je vreme važenja merenih meteoroloških podataka. Istražena je priroda meteoroloških elemenata i nadjen optimalan interval merenja pogodan za osrednjavanje.

*Ključne reči:* meteorološko merenje, meteorološki podaci, srednja vrednost, statističke karakteristike, let projektila, artiljerijsko gađanje.

## Les possibilités d'emploi des valeurs moyennes des éléments météorologiques

Quand on connaît les caractéristiques statistiques des éléments météorologiques mesurés (vent, pression, température) qui ont des attributs des valeurs occasionnelles (valeur moyenne et dispersion), on peut les utiliser en pratique lors du traitement de leur mesure d'altitude. La donnée la plus importante qu'il faut connaître à l'avance est le temps de validité des données météorologiques mesurées. On a examiné leur nature et on a trouvé l'intervalle de mesure.

*Mots clés:* mesurage météorologique, données météorologiques, valeur moyenne, caractéristiques statistiques, vol de projectile, tir d'artillerie.

## Употребительность среднего значения метеорологических элементов

Познанием статистических характеристик измеренных метеорологических элементов (ветер, давление, температура), которые считаются случайными величинами (среднее значение и рассредоточение), их в практике возможно употребить при обработке их высотных измерений. Важнейшее данное, которое нужно знать вперед, это срок важности измеренных метеорологических данных. Исследована их сущность и найден интервал измерения.

*Ключевые слова:* метеорологическое измерение, метеорологические данные, среднее значение, статистические характеристики, полет снаряда, артиллерийское ведение огня.