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The analysis of behaviour of thin-walled elements subjected to dynamic loads

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The methods of calculation and analysis of thin-walled elements with variable and different shapes of the cross sections subjected to dynamic loads are presented in this work. A particular attention is paid to the influence of the stresses caused by bimoments and specific elastic and inertial loads during the oscillations of the elements. The comparative analysis of the behavior of the elements of different cross section shapes is done. The methods of the frequent analysis of the behavior of the elements and possibilities of improving the structure from the point of view of its strength are presented. All calculations are done using numerical methods with either self made or existing algorithms for the numerical analysis.

Key words: **design resistance, dynamic load, thin – walled element, complex load, statical analysis, dynamic analysis, bimoment, moment of inertia, system of diferential equations, numerical methods.**

Introduction

IN the work is solved problem of behavior complex loaded thin-walled elements with variable cross-section under dynamic loads and possibilities of improving the structure from the point of hypotheses resistance. The basic hypoteses are:

a) thin-walled sticks theory of opened and closed one-cell and many-cell linear variable cross-sections;
b) behavior of thin-walled structures under time variable concentric loads;
c) neglecting the influence of transverse forces and inertia of rotation cross-section on the oscillations of a structural element;

Established system of differential equations are solved using numerical methods. The numerical methods which are used to solve the system are:

- Finite difference method with backward scheme;
- Runge-Cutta method;
- Various interpolation methods (spline etc.).

The influence of specific parts in the system of differential equations loaded oscillations is analyzed in the work. Also, the influence of deplinacy of cross-section on the stress of the elements for characteristic shape of profile is shown. The ways for improving structures from the base of resistance are given.

Statical analysis of thin-walled structures

The analysis of thin-walled sticks is based on the following hypotheses about the deformation of a stick [1]:

a) the shape of a cross-section remains the same during the deformation;
b) the slide in the middle plain is minor.

In order to analyse the deformations of the middle line points of profile, the following system of coordinates was adapted:

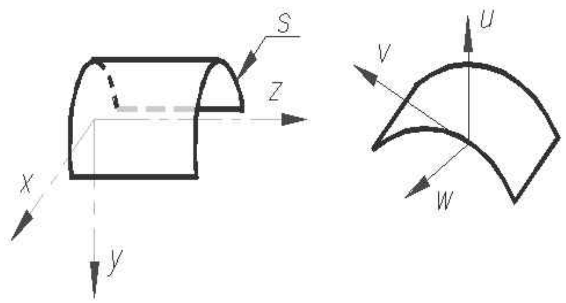

Figure 1. Definition of the coordinate system

Also, the system of main coordinates ξ and η and the coordinates of the pole ξ_p and η_p has to be established. For the analysis of the stress, the following hypotheses apply [5]:

c) the perpendicular stress $\sigma_z = \sigma$ is equally distributed through the thickness of the wall;
d) the tangent stress $\vec{\tau}_z$ is equal to the sum of the stress $\vec{\tau}_z = \vec{\tau}_s + \vec{\tau}_w$.

Tanget stress $\vec{\tau}_s$ on the cross-section is distributed as in torsion. The tanget stress $\vec{\tau}_w$ (Fig.2) is a stress caused by curving of the cross-section, and it can be assumed from the relation (1).

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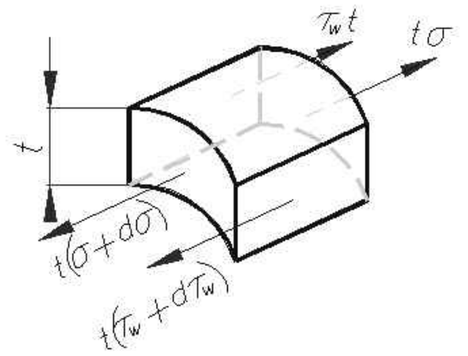


Figure 2. The tangent stress $\vec{\tau}_w$

$$d(\sigma \cdot t)ds + d(\tau_w \cdot t)dz + p_z dzds = 0$$

$$\frac{\partial(\sigma \cdot t)}{\partial z} + \frac{\partial(\tau_w \cdot t)}{\partial s} + p_z = 0 \quad (1)$$

Diferential equations of equilibrium of the stick in any system of coordinates is obtained from the conditions of equilibrium of the elements of the stick (Fig.3).

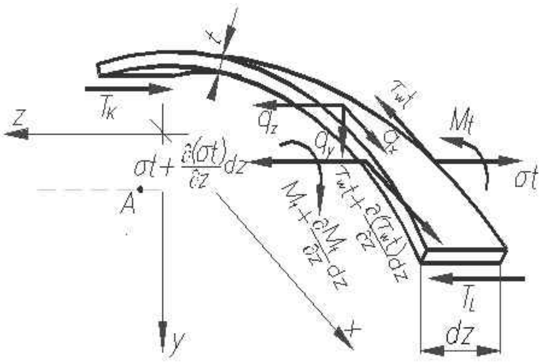


Figure 3. Inside forces on the element of a stick

System of differential equations of equilibrium of the stick in the system of the main coordinates has the following form [1]:

$$\begin{aligned} & EAw'' + \int_L p_z ds = 0 \\ & EI_x \eta^{IV} - \int_L \frac{\partial p_z}{\partial z} y ds - q_y = 0 \\ & EI_y \xi^{IV} - \int_L \frac{\partial p_z}{\partial z} x ds - q_x = 0 \\ & EI_\omega \theta^{IV} - GI_t \theta'' - \int_L \frac{\partial p_z}{\partial z} \omega ds - m = 0 \end{aligned} \quad (2)$$

The first equation determines the longitudinal movements $w(z)$ due to the axial force, the second and the third describe the movements $\xi(z)$ and $\eta(z)$ of the main pole on area sectorial. The fourth equation describes the torsion of the stick around the main pole, caused by the transversal forces and the torsion moment (m) distributed along the stick.

Characteristic of the stick with closed many-cell cross-section (Fig.4)

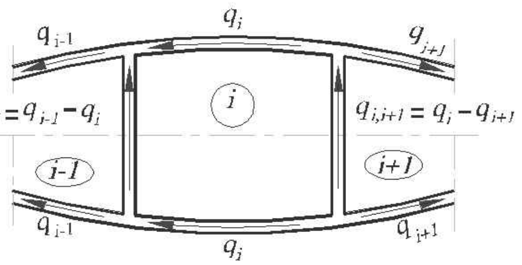


Figure 4. "i-cell" separeted

The stream of tangential stress through the walls of many-cell cross-section is calculated on the bases of the algebric equations [2]:

$$\frac{1}{2A_i^*}\left(\eta_{ii}\bar{q}_i - \sum_k \eta_{ik}\bar{q}_k\right) = 1 \begin{pmatrix} i = 1,2,...,n \\ k = 1,2,...,m \end{pmatrix} \quad (3)$$

n - entire number of cells in the cross-sections,

A_i^* - the area surrounded by the i-cell contour.

The coeficients η_{ii} and η_{ik} are:

$$\eta_{ii} = \oint_{s_i} \frac{ds}{t(s)} (i = 1,2,...,n)$$

$$\eta_{ik} = \int_{s_{i,k}} \frac{ds}{t(s)} (k = 1,2,...,m) \quad (4)$$

s_i - entire closed countor of the *i*-cell

$s_{i,k}$ - part of the contour s_i which is the same for i and neighboring *k*-cell (Fig.4.).

The unknown $\bar{q}_i$ has the form:

$$\bar{q}_i = \frac{q_i}{G\theta'} \quad (5)$$

where are:

q_i - the course of sliding down the i-cell countor

θ' - unique angle of torsion

G - modul of sliding

The torsion moment of inertia is:

$$I_t = 2\sum_{i=1}^{n} A_i^* \cdot \bar{q}_i \quad (6)$$

When $\bar{q}_i$ and I_t are known, the course of sliding can be determined [2]:

$$q_i = \frac{M_t}{I_t}\bar{q}_i \quad (i = 1,...,n)$$

$$q_{i,k} = q_i - q_k \quad (i = 1,...,n), \quad (k = 1,...,m), \quad (7)$$

as well as the stress of sliding

$$\tau_i = \frac{q_i}{t_i} (i = 1,...,n)$$

$$\tau_{i,k} = \frac{q_{i,k}}{t_{i,k}} \; (i=1,...,n), \; (k=1,...,m) \tag{8}$$

Dynamical analysis of thin-walled structures

To solve the problem of oscillations of elements with thin-walled profile, the starting point is D'Alambert's method, dynamical problem regarded as statical, with forces of inertia added to elastic forces. The movements w, ξ, η and θ should be shown as functions of two variables: axis *z* and time *t*. Appropriate mathematical operations applied:

$$EA\frac{\partial^2 w}{\partial z^2} - \rho \cdot A\frac{\partial^2 w}{\partial t^2} = 0 \;;$$

$$EI_y\frac{\partial^4 \xi}{\partial z^4} - \rho I_y\frac{\partial^4 \xi}{\partial z^2 \partial t^2} + \rho A\frac{\partial^2 \xi}{\partial t^2} + \rho A a_y\frac{\partial^2 \theta}{\partial t^2} = 0 \;;$$

$$EI_x\frac{\partial^4 \eta}{\partial z^4} - \rho I_x\frac{\partial^2 \eta}{\partial z^2 \partial t^2} + \rho A\frac{\partial^2 \eta}{\partial t^2} - \rho A a_x\frac{\partial^2 \theta}{\partial t^2} = 0 \;;$$

$$EI_\omega\frac{\partial^4 \theta}{\partial z^4} - GI_t\frac{\partial^2 \theta}{\partial z^2} - \rho I_\omega\frac{\partial^4 \theta}{\partial z^2 \partial t^2} + \rho A r^2\frac{\partial^2 \theta}{\partial t^2} + \tag{9}$$

$$+\rho A \cdot a_y\frac{\partial^2 \xi}{\partial t^2} - \rho A a_x\frac{\partial^2 \eta}{\partial t^2} = 0$$

Where:

a_x, a_y - the coordinates of main pole, in a special case when the main pole conforms with the center of inertia of the cross-section of the stick $(a_x = a_y = 0)$,

A - the area of the cross section of profile,

E, G - modules of elasticity and sliding of the material,

I_x, I_y - moment of flexion,

I_t - the torsion moment of inertia,

I_ω - the sector's moment of the profile of inertia,

r - distance, $r = \sqrt{a_x^2 + a_y^2}$ and

ρ - specific gravity of material.

The first equation in this system describes free longitudinal oscillations of the stick, the second and the third describe the transversal oscillations in the main planes and the fourth describes the torsion oscillations around the main pole of the cross-section of the stick.

To further analyze the system we could adopt the following assumptions:

a) pressure force *P* acts on the stick,
b) special load effects the thin-walled stick, with components q_x, q_y and m (moment of torsion) distributed along the stick,
c) cross-section of the stick is variable along the coordinate *z*. The geometric values are variables such as $I_x(z)$; $I_y(z)$; $I_\omega(z)$; $A(z)$); $a_x(z)$; $a_y(z)$ and $r(z)$.

$$\frac{\partial^2}{\partial z^2}\left[EI_x(z)\frac{\partial^2 \eta}{\partial z^2}\right] - \frac{\partial^2}{\partial z^2}\left[\rho \cdot I_x(z)\frac{\partial^2 \eta}{\partial t^2}\right] +$$

$$+\rho A(z)\frac{\partial^2 \eta}{\partial t^2} + \rho A(z)a_x(z)\frac{\partial^2 \theta}{\partial t^2} - P(z,t)\frac{\partial^2 \eta}{\partial z^2} = q_y(z,t)$$

$$\frac{\partial^2}{\partial z^2}\left[EI_y(z)\frac{\partial^2 \xi}{\partial z^2}\right] - \frac{\partial^2}{\partial z^2}\left[\rho \cdot I_y(z)\frac{\partial^2 \xi}{\partial t^2}\right] +$$

$$+\rho A(z)\frac{\partial^2 \xi}{\partial t^2} + \rho A(z)a_y(z)\frac{\partial^2 \theta}{\partial t^2} - P(z,t)\frac{\partial^2 \xi}{\partial z^2} = q_x(z,t)$$

$$\frac{\partial}{\partial z}\left[EI_\omega(z)\frac{\partial^3 \theta}{\partial z^3}\right] - \frac{\partial}{\partial z}\left[G \cdot I_t(z)\frac{\partial \theta}{\partial z}\right] -$$

$$-\frac{\partial^2}{\partial z^2}\left[\rho I_\omega(z)\frac{\partial^2 \theta}{\partial t^2}\right] + \rho A(z)r^2(z)\frac{\partial^2 \theta}{\partial t^2} + \tag{10}$$

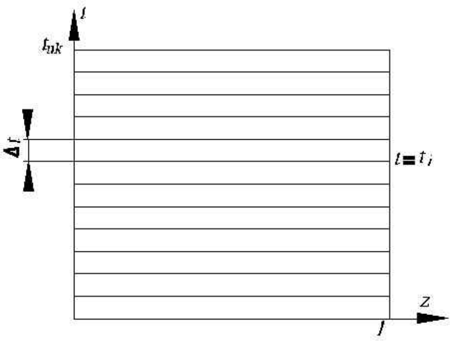
$$+\rho A(z)a_y(z)\frac{\partial^2 \xi}{\partial t^2} - \rho A(z)a_x(z)\frac{\partial^2 \eta}{\partial t^2} = m(z,t)$$

System of equations is nonlinear, non-homogenous and non-stationary system of partial differential equations with contour conditions depending on the loads and reliance. The solution of the shown system of equations for defined beginning and contour conditions gives the functions of movements (deformations): $\xi(z,t)$, $\eta(z,t)$ and $\theta(z,t)$. If the functions of movements and function of pressure force $P(z,t)$ are known, as well as the function of geometric characteristic along the stick change, the functions for the inside forces and stress depending on coordinate *z* and time *t* can be determined:

- moment of flexion around the main axes ξ and η (M_ξ, M_η),
- bimoment $B(z,t)$,
- moment of torsion and the torsion moment of deplination (M_t, M_ω),
- entire moment of torsion $(M_{tu} = M_t + M_\omega)$,
- the perpendicular stress from the longitudinal force and moment of deflection and the perpendicular stress from bimoment $\sigma_z(z,t)$, $\sigma_B(z,t)$,
- the transversal forces and entire transversal stress $(T_\xi, T_\eta), \tau$,
- the transversal stress from the transversal forces $\tau(T_\xi), \tau(T_\eta)$ and
- the transversal stress caused by torsion moment and the torsion moment of deplination $\tau(M_t), \tau(M_\omega)$.

Solving the system of differential equation of compulsory oscillations

The solution of the system of differential equations are functions of two variables in the system of coordinates z and t or 3-D areas. Dividing the domain of the function solution with the directions $t = t_i$ (Fig.5) families of curves with one variable are obtained:



$$\Delta t = \frac{t_{uk}}{n}$$

$$i = 3 : n$$

$$t_i = (i-3)\frac{t_{uk}}{n}$$

Function solutions are:

$$y(z,t_i) = Y_i(z)$$

$$x(z,t_i) = X_i(z)$$

$$\theta(z,t_i) = \theta_i(z)$$

Figure 5. The domain of system solution

The partial equations are approximated after Crank-Nichols explicit scheme with backward model [2]. Applying the given approximations on the system of differential equations $3x(n-3)$ ordinary differential equations are obtained. Solving them results in families of curves or functions of solution. Certain mathematical operations give the matrix differential equations for the small and bigger flexion-torsion characteristics. The flexion–torsion characteristics of thin-walled structures are defined in [3,4].

The small flexion-torsion characteristics:

$$\mathbf{X}_i^{IV} + \mathbf{A}(z)\cdot\mathbf{X}_i''' + \mathbf{B}(z)\mathbf{X}_i'' + \mathbf{C}(z)\mathbf{X}_i' + \mathbf{D}(z)\mathbf{X}_i = \\ = \mathbf{G}(z) - \mathbf{F}(z) \qquad (11)$$

The vectors: $\mathbf{X}, \mathbf{G}$ and $\mathbf{F}$, and matrix $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are defined in the folowing way:

$$\mathbf{X}_i = \begin{bmatrix} Y_i \\ X_i \\ \theta_i \end{bmatrix}; \quad \mathbf{A}(z) = \begin{bmatrix} A_y(z) & 0 & 0 \\ 0 & A_x(z) & 0 \\ 0 & 0 & A_\theta(z) \end{bmatrix};$$

$$\mathbf{B}(z) = \begin{bmatrix} B_y(z) & 0 & 0 \\ 0 & B_x(z) & 0 \\ 0 & 0 & B_\theta(z) \end{bmatrix};$$

$$\mathbf{C}(z) = \begin{bmatrix} C_y(z) & 0 & 0 \\ 0 & C_x(z) & 0 \\ 0 & 0 & C_\theta(z) \end{bmatrix}$$

$$\mathbf{D}(z) = \begin{bmatrix} D_y(z) & 0 & E_y(z) \\ 0 & D_x(z) & E_x(z) \\ F_\theta(z) & E_\theta(z) & D_\theta(z) \end{bmatrix}; \quad \mathbf{G}(z) = \begin{bmatrix} G_y(z) \\ G_x(z) \\ H_\theta(z)) \end{bmatrix};$$

$$\mathbf{F}(z) = \begin{bmatrix} F_y(z) \\ F_x(z) \\ G_\theta(z) \end{bmatrix}$$

$\mathbf{X}$ is the vector of the main pole movement, $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are matrices of geometric characteristics and stiffness of the stick, vector $\mathbf{F}$ defines the variability of loads, and $\mathbf{G}$ is the vector of loading in the given instant.

Similarly, the bigger flexion-torsion characteristics equationis:

$$\mathbf{X}_i^{IV} + \mathbf{A}_2(z)\cdot\mathbf{X}_i''' + \mathbf{B}_2(z)\mathbf{X}_i'' + \mathbf{C}_2(z)\mathbf{X}_i' + \mathbf{D}_2(z)\mathbf{X}_i + \\ + \mathbf{E}_2(z)\theta_i(z) + \mathbf{F}_2(z) = \mathbf{G}_2(z) \qquad (12)$$

$$\theta_i''(z) + C_{\theta_2}(z)\theta_i'(z) + D_{\theta_2}(z)\theta_i(z) + EF_2(z)X + \\ + G_{\theta_2}(z) = H_{\theta_2}(z)$$

This system of equations (11) and (12) is solved by using program language MATLAB 4.2.-C with Runge-Cutta method 5 (predictor-corrector). The entire algorithm of the solution for the system differential equations is organized acording to Fig.6.

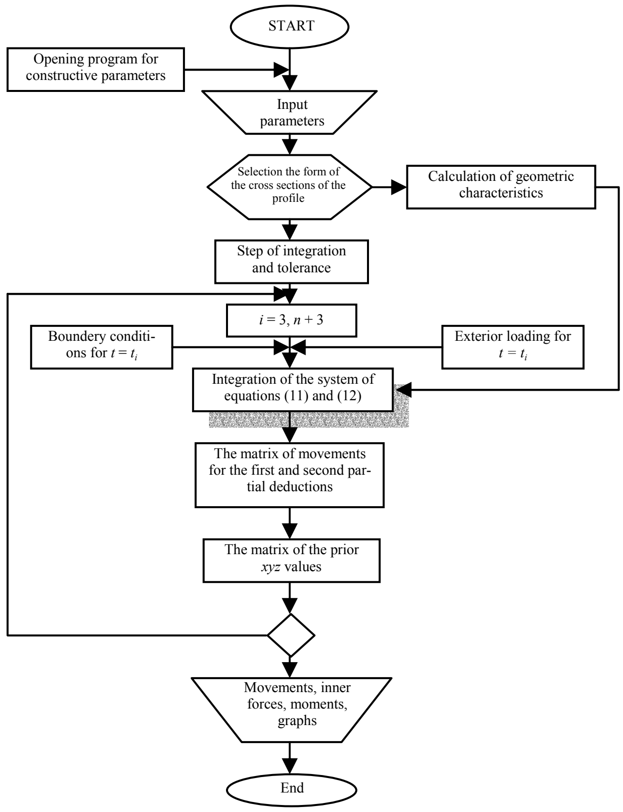


Figure 6. The global algorithm of the solution

Algorithm of the solution for the small and bigger flexion-torsion characteristic is shown in Figures 7 and 8.

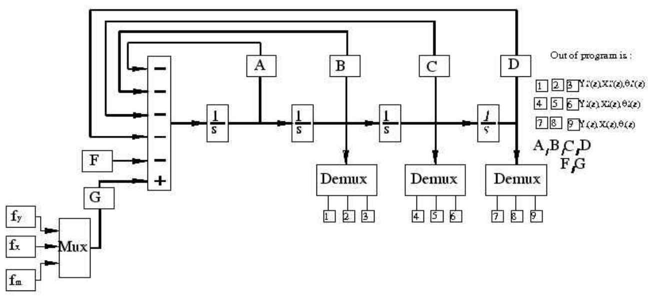


Figure 7. Algorithm for integration for the small flexion-torsion characteristic

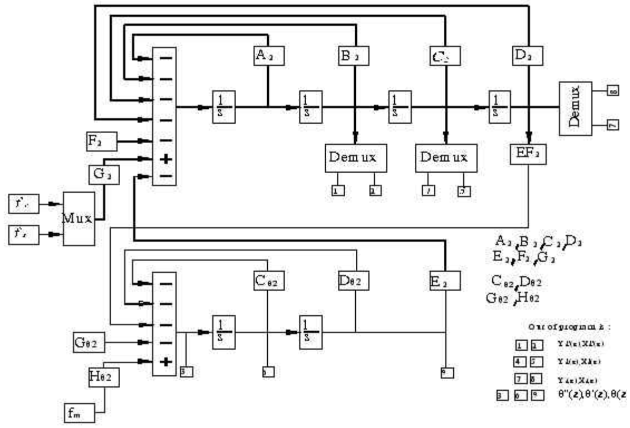


Figure 8. Algorithm for integration for the bigger flexion-torsion characteristic

Exterior load on thin-walled element is defined by functions $q_y(z,t)$, $q_x(z,t)$ and $m(z,t)$ representing specific loads on the element in a point depending on time t (Fig.9).

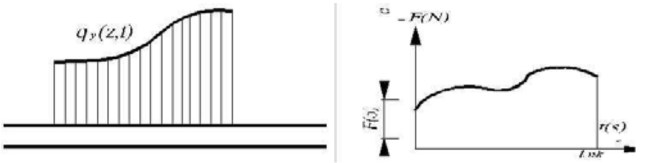


Figure 9. Exterior load (forms of loading)

The influence of exterior load in the form of bimoment has not been considered in this work. If thin-walled element is loaded by concentric forces and moments (in general case of time variable) then instead of them in surrounding area, a constant specific load can be assumed. If the load on the element is the concentric moment of flexion m_y and m_x, it is possible to represent them via specific loads around the point under the influence of the moments [5].

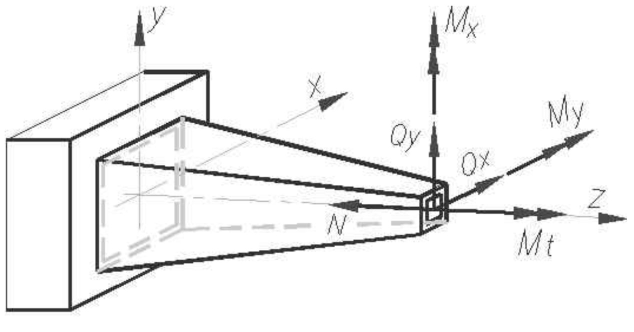


Figure 10. Loaded console with variable cross-section

The most frequent case is a console with variable cross-section loaded by space system of concentric forces and moments of flexion and torsion in the main plains (Fig.10). A typical example of a thin-walled element, dynamically loaded, are the legs of carriage of an artillery weapon (Fig.11).

The leg of the carriage is regarded as a space console with a variable cross-section which is loaded with space system of loads and moments that are variable in time on one end (Fig.12).

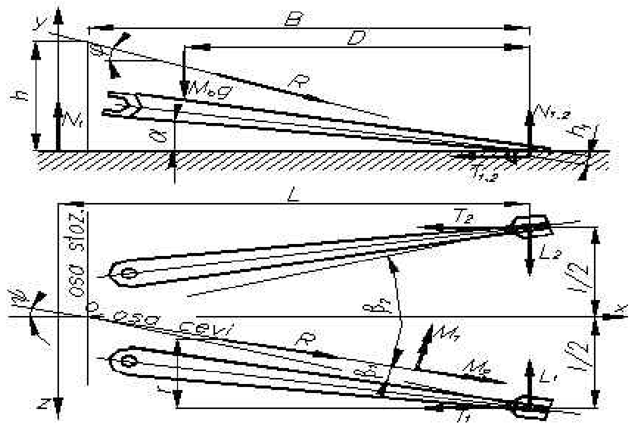


Figure 11. Scheme of legs loading

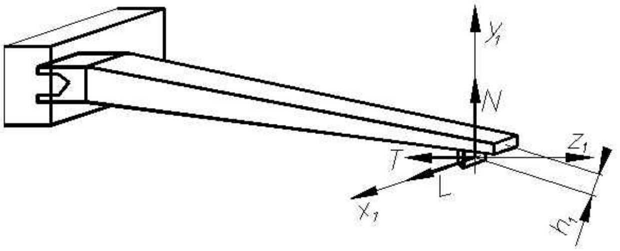


Figure 12. Loaded space console

Results of calculation and comparative analysis of compulsory oscillations of thin-walled console with variable cross-section

The solution of the system of differential equations results in the vector of movement of the center of inertia in the system of main coordinates and the angle of rotation of the cross-section profile depending on the coordinates *z* and *t*. Further more, inner forces, moments and values of perpendicular stress are obtained as exits. The exit value are in the shape of the surface in the space (Fig.13).

Diagram of inner forces and stress result from the calculated function of the yield (Fig.14), for e.q., the moment of flexion in the main plain, the moment of torsion and bimoment (Fig.15) and, in (Fig.16), the perpendicular stress and the stress from bimoment.

The moment of torsion along the time axis, follows the form of alternation by the moment of bending. Along the longitudinal coordinate, the moment of torsion is approximatively constant. The value of bimoment is negligible due to the absence of curving of the cross-section.

The perpedicular stress is calculated for every cross-section (coordinate *z* in function of *t*). The stresses are calculated for a chosen point on the contour of the cross-section.

The maximum value of stress due to of the bimoment is $\sigma_B(z,t)_{\max} \le 0{,}1\%$ from the perpedicular stress σ caused by other effects.

In case of bigger flection-torsion characteristies, the stress caused bimoment can be neglected.

In $z = z_m = 9.5\,\text{m}$ the perpendicular stress with the strain gages on the real structure was measured. Four measurements in the same conditions and the average experimental curve (Fig.17) were obtained.

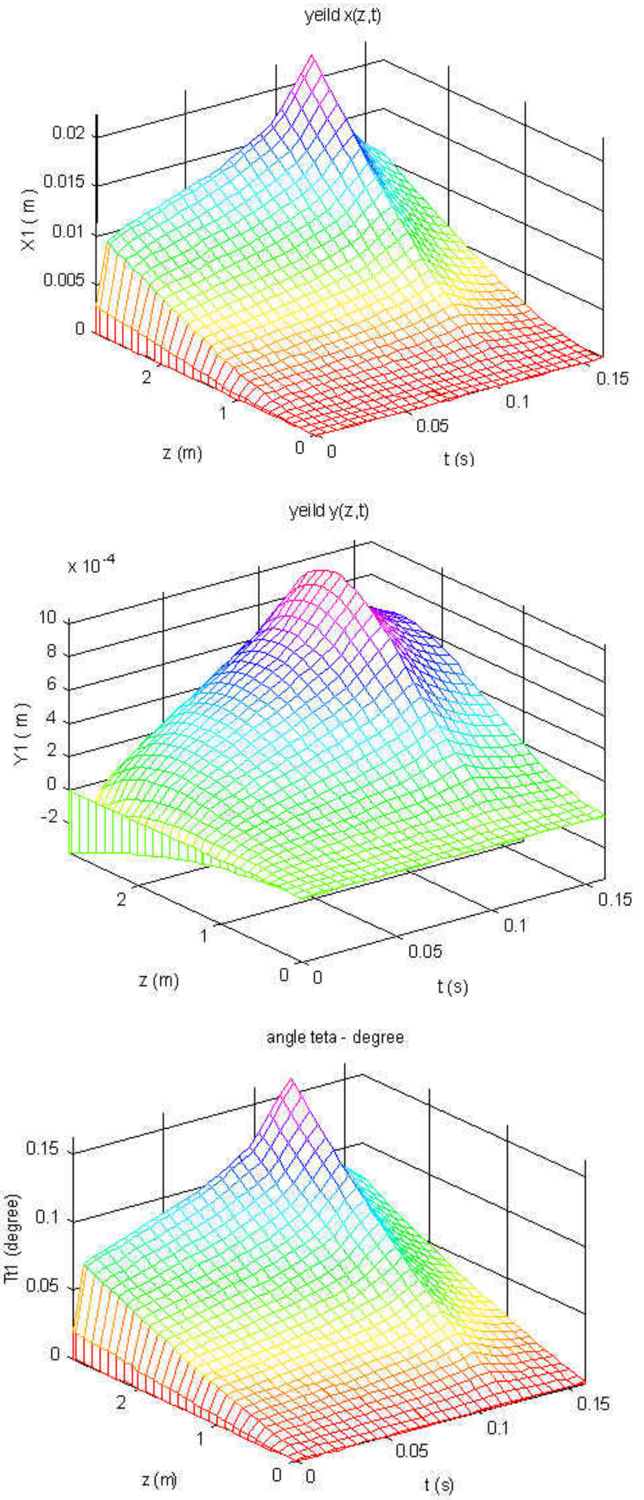


Figure 13. Vector of movement of the center of inertia and the rotation of profile

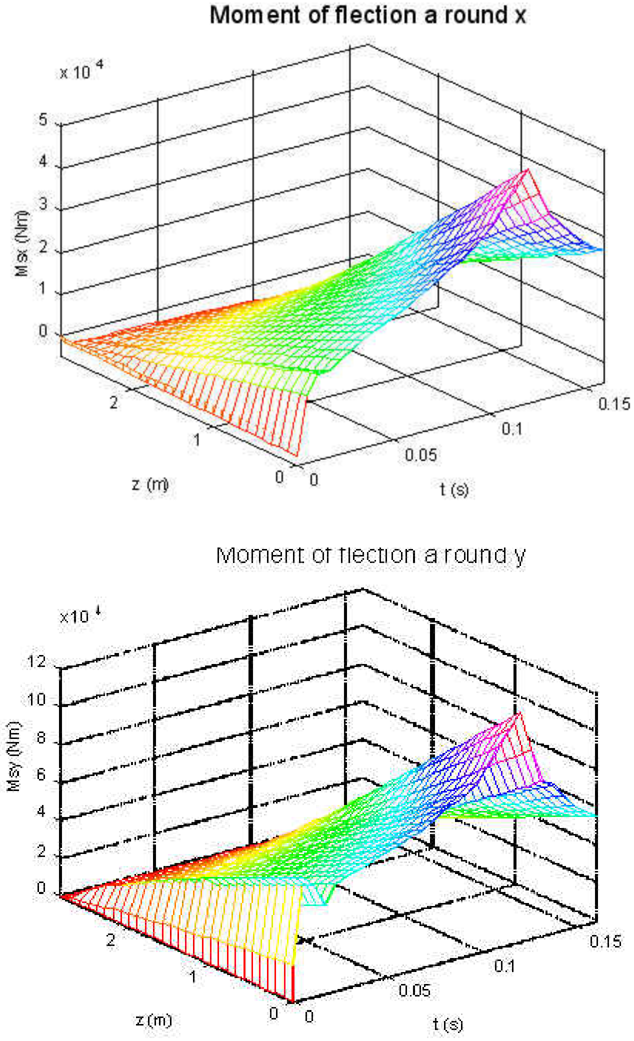


Figure 14. Diagram of the moment of flexion

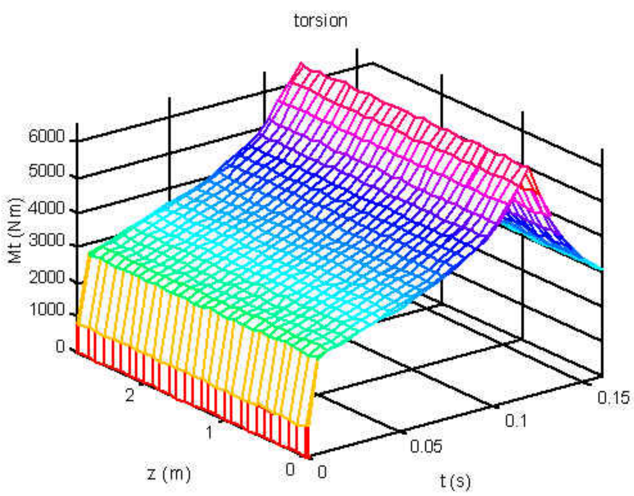


Figure 15. Moment of torsion and bimoment

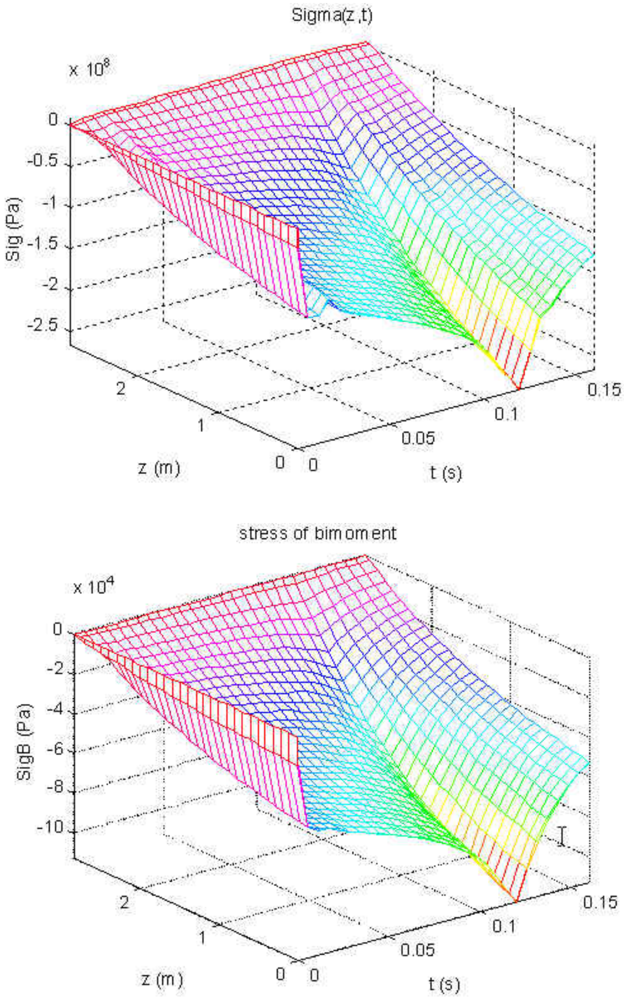


Figure 16. Perpendicular stress and the stress from bimoment

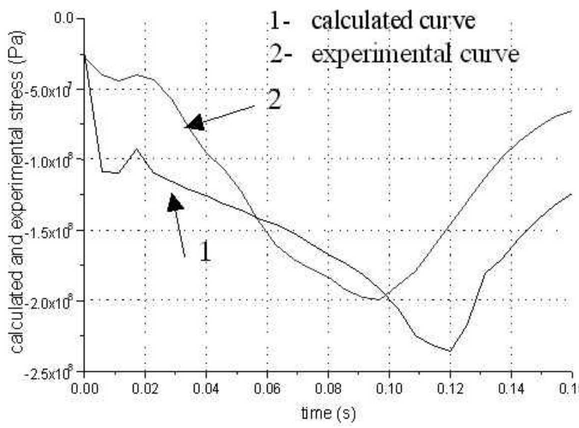


Figure 17. Calculated and experimental stress for $z = z_m$

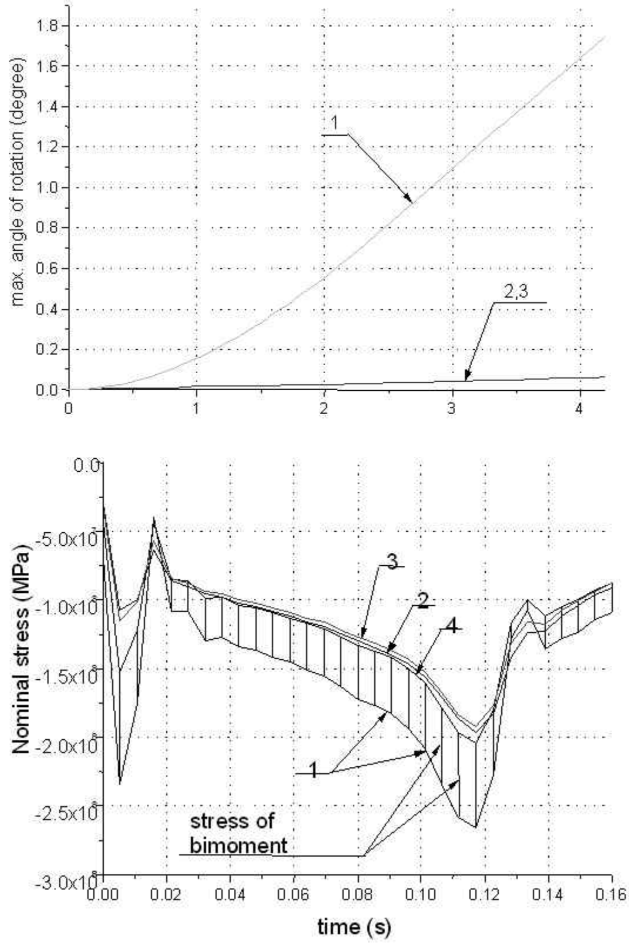
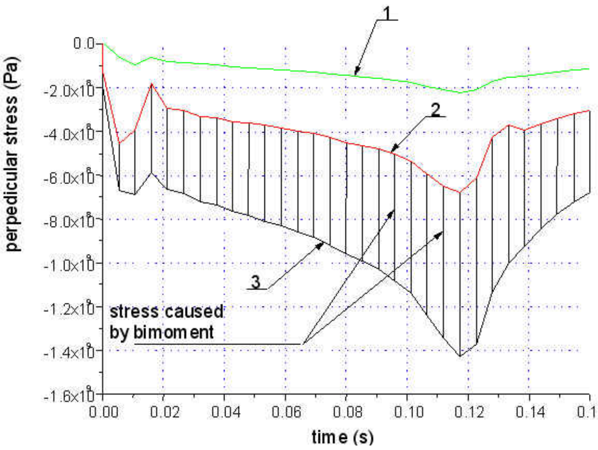


Figure 18. The comparative results for the stress for the all three profiles rectangles and $z = z_m$



1 - one-cell triangle; 2 - I profile-classical Sen-Venan; 3 - I profile-bimoment theory

Figure 19. The influence of bimoment on the perpendicular stress

The positive phase displacement of the calculated model compared to the experimental one is caused by disregarding the time-reactions of the elements connected to the observed element and oscillate with it, in the calculations. Displacements are less then 10% of the nominal values and are in the boundaries of engineering accuracy. The comparative results of the maximum functions of the angle of rotation of the cross-section and the stress are one-cell rectangle (opened - 1 and closed - 2) and the two-cell closed rectangle - 3 and the one cell opened rectangle - 4 (calculated by classical Sain-Venant theory), (Fig.18).

In the open rectangle an important part is the stress is caused by bimoment (bimoment theory). For the small flexion torsion characteristics (for I-profile) the influence of bimoment on the perpendicular stress is extensive (Fig.19).

For the I profile, the stress of bimoment is 30-40% of entire perpendicular stress. This percentage is much bigger in the points of maximum loads. For the profiles with small flexion-torsion characteristic, the Sen-Venan's torsion moment differs significantly from the external. For a console with variable cross-section, the influence of torsion moment of bending decreases at first and then increases. In thin-walled elements with variable cross-section the influence of bending is bigger than for the elements with constant cross-section, either for the maximum or minimum cross-section.

The elements with variable cross-section where moment of torsion cannot be disregarded and the flexion-torsion characteristicis small, bimoment theory aught to be used in calculations.

The frequent analysis of dynamic behaviour of an element

Thin-walled element can be regarded as multivariable inertial-elastic system with a certain number of entries and exits (Fig.20). Vector of exits in such systems consists of the time functions of the strain, stress etc. at a centain point $(z = z_0)$.

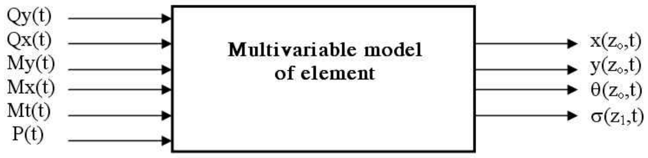


Figure 20. Multivariable model of an element

Assuming that this is a linear and stable multivariable system n-ordered, the matrix of transfer functions could be obtained on the bases of the corresponding enter-exit combinations using Fourie-transformation.

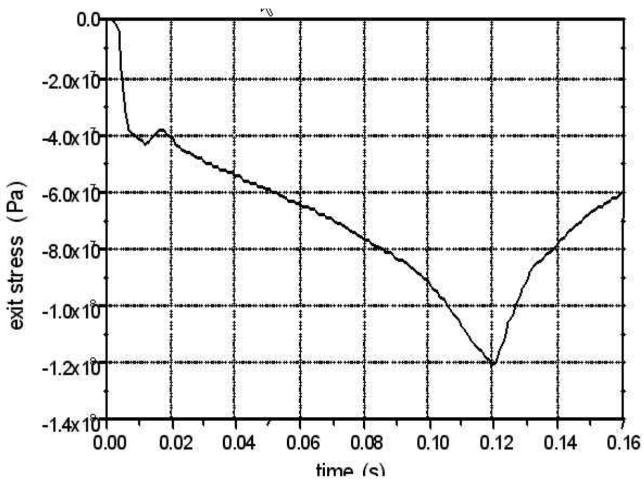


Figure 21. The exit signal $\sigma(z,t)$ in *t*-domen

Transforming the enterance vector functions in s-domain is carried out after the discretisation of the corresponding continous enterance signal and its Fourie transformation (Fig.21). In this way, the amplitude spectar for every enterance signal can be obtained. The vector of exit signals in s-domain is determined as multiplication of the matrix of transfer functions of the system and the vector of enterance signals in s-domain (Fig.22).

According to the assumption that the system is linear and knowing that for such system the principle of superposition applies, the transfer function $H_{ij}(s)$ can be determined regarding the element as a system with one entry and one exit, assuming that other enterance signals are equal zero, for all the combinations of enter once and exit signals.

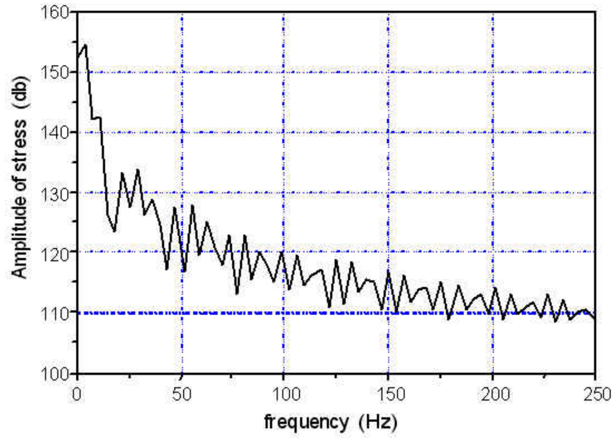


Figure 22. The amplitude spectar of the exit signal $\sigma(z,t)$

$$H_{i,j}(s) = Y_j(s)/U_i(s)$$

where:

$Y_j(s)$ - j-exit signal in s domain

$U_i(s)$ - i-enterance signal in s domain

This transfer function is a function of response of the system for j-exit signal on Dirach impulse with i entrance signal in s-domain. With the inverse Fourie transformation of the transfer function $H_{ij}(s)$, the time response of the element on the Dirach impulse corresponding enterance signal is obtained. This action is repeated for all the combinations of the entrance-exit signals. When the matrix of transfer function is known, a frequent analysis of behaviour of an element in any case of time-loads is possible by applying methods of multivariable convolution and deconvolution.

The matrix of transfer functions for an element, can also be obtained experimentally by measuring impulse responses of the element for every enterance signal and every observed exit. In the given example alghoritms for fast Fourie transformation in the program language MATLAB were used for solving transfer functions.

Conclusion

The solution of the system of differential equations for the static equilibrium of a differential element of thin-walled structure for the constant cross-section in the case of constant space system of forces and moments, the vector of movement of center inertia of cross-section in the system of the main coordinates and the angle of rotation of profile around the center of torsion are obtained. The obtained functions are the stationary functions of independent variable longitudinal coordinate. The position of the center of sliding influences the value of torsion moment of bending i.e. additional stress due to the bimoment. The influence of bending for the element with variable cross-section is bigger than in the case of an element with constant cross-section.

The solution of the system of differential equations for the dynamic equilibrium of a differential element in the case of variable external system of applied forces and moments, gives the time functions of the vector of movement and torsion for every cross-section along the element. In this case it is possible to examine the influence of variability of loads on the functions and the inside forces. The position of the main pole of sectoral area (for example), has a great influence on the external values.

If the main pole is further from the center of inertia of cross-section an important increase in time-function response of the observed variable is obtained. This increase is bigger if the function of load is close to impulse function. In this case the “balance” of profile is proposed. The shown program model make it possible to examine the influence of the dimensions of a profile on the behavior of an element (geometrical dimensions, thickness of walls etc.).

The program model should be annexed with the tests for the estimation of the stability and convergence of the system of compulsory vibrations of differential equations.

References

[1] RUŽIĆ,D.: *Otpornost konstrukcija*, Beograd 1995.

[2] SMITH,G.D.: *Numerical Solution of Partial Differential Equations*, Oxford, 1965.

[3] VLASOV,V.Z.: T*onkostennye uprugie sterzhni*, Moskva 1959.

[4] BIRGER,I.A., MAVLJUTOV,R.R.: *Soprotivlenie materialov*, Moskva 1986.

[5] DAVIDOVIĆ,M.: *The analysis of the behavior of thin-walled elements with variable cross section of a complex structure subjected to dynamic loads,* MR thesis, Beograd, 1998.

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Analiza ponašanja tankozidnih elemenata pri dinamičkom opterečenju

U radu su prikazane metode proračuna i analize tankozidnih elemenata promenljivih poprečnih preseka, različitih oblika profila pri dinamičkom opterećenju. Posebno je analiziran uticaj napona usled bimomenta i specifičnih elastičnih i inercijalnih opterećenja pri oscilovanju elemenata. Napravljena je uporedna analiza ponašanja elemenata različitih oblika profila poprečnog preseka. Prikazane su metode frekventne analize ponašanja elemenata kao i mogućnosti poboljšanja konstrukcije sa stanovišta otpornosti. Svi proračuni su rađeni numeričkim metodama, sa kreiranim ili postojećim algoritmima numeričke analize.

Ključne reči: **otpornost konstrukcije, dinamičko opterećenje, tankozidni element, složeno opterećenje, statička analiza, dinamička analiza, bimoment, moment inercije, sistem diferencijalnih jednačina, numeričke metode.**

Analyse du comportemenet des éléments aux parois minces pendant la charge dynamique

Ce papier présente les méthodes du calcul et les analyses des éléments aux parois minces et aux sections transversales variables avec de différentes formes de profils pendant la charge dynamique. L’influence de la tension, provoquée par le bimoment et les charges inertieles spécifiques et élastiques pendant l’oscillation des éléments a été particulièrem- ent analysée. Une analyse comparée du comportement des éléments aux différentes formes de profils de la section transversale a été faite. On a démontré les méthodes de l’analyse fréquente du comportement des éléments ainsi que les possibilités d’améliora- tion constructive en ce qui concerne la résistence. Tous les calculs ont été effectués au moyen des méthodes numériques, avec les algorithmes de l’analyse numérique, créés ou déjà existents.

Mots clés: **résistence de contruction, charge dynamique, élément aux parois minces, charge composé, analyse statique, analyse dynamique, bimoment, moment d’inertie, système d’équations différentielles, méthode numérique**

Анализ поведения тонкостеных элементов при динамической нагрузке

В этой работе показаны методы расчета и анализа тонкостеных элементов изменчивых поперечных сечений, различных форм профилей при динамической нагрузке. Особенно анализировано влияние напряжения вследствие бимомента и специфических эластичных и инерционных нагрузок при колебании элементов. Тоже проведен сравнительный анализ поведения элементов различных форм профилей поперечного сечения. В работе показаны и методы частотного анализа поведения элементов, как и возможности улучшения конструкции со стороны прочности. Все расчеты проведены численными методами, со добавлением доработанных или уже существующих алгоритмов численного анализа.

Ключевые слова: устойчивость конструкции, динамическая нагрузка, тонкостеный элемент, комплексная нагрузка, статический анализ, динамический анализ, бимомент, момент инерции, система дифференциальный уравнений, численные методы.