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New solution of linear regression equations derived from related plan experiment

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Dimensional reasoning is a very powerful tool for determining the exponents φ_i, not requaring any experimental work except for a coefficient K equation $f_1 = K p_1^{\varphi_1} p_2^{\varphi_2} p_3^{\varphi_3} ... p_n^{\varphi_n} = K\prod_{i=1}^{n} p_i^{\varphi_i}, (i = 1,...,n)$ where p_1 are mechanical system parameters. It has enabled us to deduce the parameter relationships even with very little knowledge, and certainly no mechanical analysis, of the mechanical system phisical problem. However,it is inadequate in problems where there are $n > 3$ parameters to be included on the right-hand side.

This paper is the result of an investigation process for determining exponents φ_i and coefficient K in the above equation putting no restriction as for the number of parameters and actual problems asociated with it. One of the significant results eliminating need for any numerical or analytical methods for solving a system of ordinary linear equations as a means of determining of the exponents φ_i and corrective factor -coefficient K. It is based on the new construction plan experiment, that is when the order and number of variables is concerned, in close relation with linear regression equations.

Key words: **linear regression, system linear equations, experiments plan.**

Introduction

EXPERIMENTAL data is often used for interpolation [1] in engineering practice. Instead of this data can be transferred and expressed in the form of any analytical function. It is usually used in polinom form:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + ... + a_n x^n \tag{1}$$

Polinom, for node points (x_i, y_i), $i = 1,...,n$ can be of n-th degree and then for $x = x_i$; will be $y = y$; Fig.1, curve 2, and if degree polinom is less than n, let n for example be $n = 3$, for $x = x_i$; will be $y \# y$; Fig.1, curve 1. In this case the least square method must be used for obtaining polinom coefficients a_i; for the number of node points (x_i, y_i) is greather than the number of coefficients.

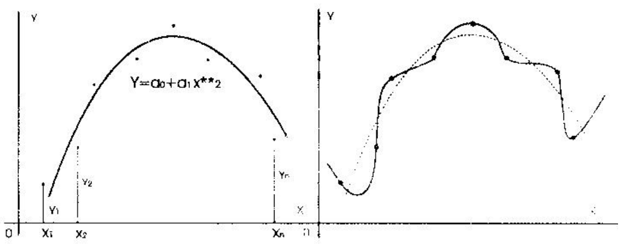

Figure 1. Least square approksimation with n -th degree polinom

For high polinom degree, values among node points will be unreal, esspecialy at the very beginning and end of the interval in question.

In the study of functional relationships of any mechanical system by means of mathematical analysis, any single-valued continuous function can be developed in the form of an infinite series like (1). The mechanical system equation f_i often takes the following form:

$$f_i = K p_1^{\varphi_1} p_2^{\varphi_2} p_3^{\varphi_3} ... p_n^{\varphi_n} = K\prod_{i=2}^{n} p_i^{\varphi_i}, (i = 1,...,n) \tag{2}$$

where: K - multiplying factor; p_i - system parametres - varables; φ_i - appropriate exponents.

One of the clasical methods for determining the exponents in eq.(2), if (2) is a physical equation, is by using dimensional analysis procedure under certain circumstances [2], but it is not adequate in all problems. If eq.(2) is a physical equation, it is necessary that every such term should contain the same combination of units. There is no means of developing such a series, but it is possible to find the form which a single term must take. Then by use of the proper multiplying factor, can be found a representative form for the desired function in terms of the quanties involved. Using of the multiplying factor K all terms in the series would be grouped into a single equivalent value. It is evident s that such multiplying factor can be determined only by experiment. The value of K will vary with varying physical parameters -values of variables or conditions of experiment.

Dimensional analysis method procedure is inadequat in all problems where there are $n > 3$ parameters to be included on the right-hand side. If the problem is in mechan-

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ics, all parametres can be expressed in terms of mass (*M*), lenght (*L*) and time (*T*) (if the problem is in thermodynamics temperature, is added to above parameters). In that case, the final form of the relationship will include determinate powers of three of these variables, along with arbitrary functions of $(n-3)$ non-dimensional parameters, each of which is a group of some or all of the n variables. To overcome these difficulties, for obtaining exponents in eq.(2) when $n>3$, least square method procedure is used. When logarithms are on both sides (2), the following is obtained

$$\ln f_i = \ln K + \varphi_1 \ln p_1 + \varphi_2 \ln p_2 + \varphi_3 \ln p_3 + \\ + \varphi_4 \ln p_4 + ... + \varphi_n \ln p_n \quad (3)$$

If logarithms from we neglected the above equation and its arguments are considered, the well known linear regression equation can be recognized. Solutions for φ_i can be obtained from (3) by least square procedure from the formed $n+1$ equations. This means that at least $n+1$ experiments must be performed. As a rule, the number of statistical processing varies from $n^2 - n^3$ as lower limit, but it is musc to extensive to do and rather expensive. A new method was the result of author's investigation concerning determining exponents in equations (2-3) without restriction as for the number of parameters involved. It is not necessary to use numerical methods that are used solving for systems of ordinary linear equations, due to the established procedure for deriving $n+1$ equations that are in concordance with the experiment plan.

New method for φ_i

Close relation between experiment plan and eq.(2) for determining φ_i, exponents and multiplying factor K [3] should be established, as it was shown in the experiment. One such experiment plan is given in Table 1.

Table 1. New plan experiment

p_1	p_2	p_3	...	p_n n	$(f_i)_e$
$(p_1)_1$	$(p_2)_1$	$(p_3)_1$	...	$(p_n)_1$	$(f_0)_e$
$(p_1)_2$	$(p_2)_1$	$(p_3)_1$	...	$(p_n)_1$	$(f_1)_e$
$(p_1)_2$	$(p_2)_2$	$(p_3)_1$	...	$(p_n)_1$	$(f_2)_e$
$(p_1)_2$	$(p_2)_2$	$(p_3)_2$	...	$(p_n)_1$	$(f_3)_e$
...	...	...	...	...	...
$(p_1)_2$	$(p_2)_2$	$(p_3)_2$	...	$(p_n)_1$	$(f_{n-1})_e$
$(p_1)_2$	$(p_2)_2$	$(p_3)_2$	...	$(p_n)_2$	$(f_n)_e$

If K and $\varphi_{1\div3}$ have to be determined for eq, the linear equations are in the form (4), numerator - subdeterminants $D_{i=1,3}$ and denominator - determinant D for $\varphi_{1\div3}$ are given in (5) and solutions for $\varphi_{1\div3}$ (6).

$$\begin{matrix} & |K & \varphi_1 & \varphi_2 & \varphi_3 \,| \end{matrix}$$
$$\begin{bmatrix} {}^*f_0 \\ {}^*f_1 \\ {}^*f_2 \\ {}^*f_3 \end{bmatrix} = \begin{bmatrix} 1 & {}^*(p_1)_1 & {}^*(p_2)_1 & {}^*(p_3)_1 \\ 1 & {}^*(p_1)_2 & {}^*(p_2)_1 & {}^*(p_3)_1 \\ 1 & {}^*(p_1)_2 & {}^*(p_2)_2 & {}^*(p_3)_1 \\ 1 & {}^*(p_1)_2 & {}^*(p_2)_2 & {}^*(p_3)_2 \end{bmatrix} \quad (4)$$

$$\begin{aligned} D &= \left[{}^*(p_1)_2 - {}^*(p_1)_1\right]\left[{}^*(p_2)_2 - {}^*(p_2)_1\right]\left[{}^*(p_3)_2 - {}^*(p_3)_1\right] \\ D_1 &= \left[{}^*f_1 - {}^*f_0\right]\left[{}^*(p_2)_2 - {}^*(p_2)_1\right]\left[{}^*(p_3)_2 - {}^*(p_3)_1\right] \\ D_2 &= \left[{}^*f_2 - {}^*f_1\right]\left[{}^*(p_1)_2 - {}^*(p_1)_1\right]\left[{}^*(p_3)_2 - {}^*(p_3)_1\right] \\ D_3 &= \left[{}^*f_3 - {}^*f_2\right]\left[{}^*(p_1)_2 - {}^*(p_1)_1\right]\left[{}^*(p_2)_2 - {}^*(p_2)_1\right] \end{aligned} \quad (5)$$

$$\begin{aligned} \varphi_1 &= \left[{}^*f_1 - {}^*f_0\right] / \left[{}^*(p_1)_2 - {}^*(p_1)_1\right] \\ \varphi_2 &= \left[{}^*f_2 - {}^*f_1\right] / \left[{}^*(p_2)_2 - {}^*(p_2)_1\right] \\ \varphi_3 &= \left[{}^*f_3 - {}^*f_2\right] / \left[{}^*(p_3)_2 - {}^*(p_3)_1\right] \end{aligned} \quad (6)$$

where in (4-6) * is a sign for logarithms.

These particular solutions may be used for finding general solutions. It can be proven that general solutions for D_i and D have the forms:

$$D = \Delta^* p_1 \Delta^* p_2 \Delta^* p_3 ... \Delta^* p_n$$

$$D_i = \Delta^* f_i \prod_{j}^{n, j\neq i} \Delta^* p_j ; i = 1,...,n; \quad (7)$$

$$\Delta^* p_n = {}^*(p_n)_2 - {}^*(p_n)_1 ; \; \Delta^* f_i = {}^*f_i - {}^*f_{i-1}$$

After finding expressions for D_i and D, solutions for φ_i are as follows:

$$\varphi_i = \frac{\Delta^* f_i}{\Delta^* p_i} ; \; i = 1,...,n \quad (8)$$

From (6, 8) it can be concluded that D_i and D are eliminated from process for determination φ_i, which is the main mathematical result of the given new procedure.

Numerical example

To illustrate the application of the new procedure for φ_i calculation, projectile flight through atmosphere described in the following set of ordinary differential equations [4] is considered:

$$\frac{dx}{dt} = V_x ; \; \frac{dy}{dt} = V_y ; \; \frac{dz}{dt} = V_z$$

$$\frac{d^2x}{dt^2} = \frac{R_x (V_x - W_x)}{V_R} ;$$

$$\frac{d^2y}{dt^2} = \frac{R_x V_y}{V_R} - 9.81; \quad (9)$$

$$\frac{d^2z}{dt^2} = \frac{R_x (V_z - W_z)}{V_R} ;$$

$$V_R = \sqrt{(V_x - W_x)^2 + V_y^2 + (V_z - W_z)^2}$$

with symbols: x, y, z - mass centre position in space, $V_{x,y,z}$ - corresponding velocities, $W_{x,z}$ - wind velocities, R_x - air resistance along absolute velocity V_R. For numerical solution set of ordinary differential eq.(9) initial values $(x,y,z)_0$, $(V_{x,y,z})_0$, $\alpha_0 \left(\alpha = \frac{V_y}{V}\right)$ and C_x - law of air resistence (taken for numerical example well known Soviet Union $C_{X43}=f\left(M=\frac{V_R}{a}\right)$ defined 1943 must be supplied, where: M - Mach number; a - air speed) and VENCEL-s law of air temperature and pressure variation with height $\tau,\zeta(y)$ (τ_0 - initial value of temperature τ) [4].

For numerical integration set of ordinary differential eq.(9) of the Gill's modification Runge-Kutta metod fourth order [5] of the fourth. Results of the calculated horizontal distance X obtained for initial conditions V_0, C, θ, W_x, $\Delta\tau_0$ are given in Table 2.

Table 2. Numerical integration results

K	C	θ	W_x	$\Delta\tau_0$	X	ΔX_{0le}	ΔX_{isq}
NEW 500	0.500	25	10	10	12021.20	-60.10	-55.44
500	0.500	25	10	15	12099.01	-65.87	-60.27
500	0.500	25	10	5	11942.03	2.30	5.36
500	0.500	25	15	10	12189.36	5.79	7.07
500	0.500	25	15	15	12267.09	-0.77	1.43
500	0.500	25	15	5	12044.23	3.42	3.13
500	0.500	25	5	10	11919.49	11.04	21.35
500	0.500	25	5	15	11997.58	6.75	18.02
500	0.500	25	5	5	11776.67	7.76	16.43
500	0.500	26	10	10	12284.69	-59.11	-84.13
500	0.500	26	10	15	12365.58	-63.63	-87.88
500	0.500	26	10	5	12202.49	3.33	-22.97
500	0.500	26	15	10	12458.79	10.49	-18.24
500	0.500	26	15	15	12539.58	5.16	-22.83
500	0.500	26	15	5	12309.29	6.86	-23.13
500	0.500	26	5	10	12178.36	11.16	-7.64
500	0.500	26	5	15	12259.55	8.17	-9.84
500	0.500	26	5	5	12031.23	6.60	-13.53
500	0.500	24	10	10	11745.49	-68.51	-34.37
500	0.500	24	10	15	11820.12	-75.62	-40.35
500	0.500	24	10	5	11669.56	-6.01	26.23
500	0.500	24	15	10	11907.58	-6.43	24.66
500	0.500	24	15	15	11982.18	-14.26	17.95
500	0.500	24	15	5	11767.00	-7.41	21.81
500	0.500	24	5	10	11648.51	3.54	42.76
500	0.500	24	5	15	11723.43	-2.11	38.26
500	0.500	24	5	5	11510.21	1.69	38.97
500	0.505	25	10	10	11979.90	-87.83	-58.65
500	0.505	25	10	15	12057.80	-93.42	-63.12
500	0.505	25	10	5	11900.62	-25.71	1.61
500	0.505	25	15	10	12148.63	-21.26	4.76
500	0.505	25	15	15	12294.42	40.33	67.45
500	0.505	25	15	5	12003.11	-24.18	0.00
500	0.505	25	5	10	11877.82	-17.25	17.21
500	0.505	25	5	15	11956.06	-21.31	14.29
500	0.505	25	5	5	11734.62	-21.07	11.48
500	0.505	26	10	10	12242.22	-87.72	-87.58
500	0.505	26	10	15	12390.95	-24.30	-23.22
500	0.505	26	10	5	12159.83	-25.64	-27.06
500	0.505	26	15	10	12416.89	-17.43	-20.78
500	0.505	26	15	15	12567.07	46.72	44.29
500	0.505	26	15	5	12266.99	-21.63	-26.52
500	0.505	26	5	10	12135.55	-17.98	-12.01
500	0.505	26	5	15	12216.85	-20.77	-13.84
500	0.505	26	5	5	11988.00	-23.12	-18.77
500	0.505	24	10	10	11705.38	-95.35	-37.32
500	0.505	24	10	15	11780.24	-102.14	-42.82
500	0.505	24	10	5	11629.33	-33.13	22.73
500	0.505	24	15	10	11868.12	-32.51	22.68
500	0.505	24	15	15	11942.87	-40.10	16.38
500	0.505	24	15	5	11727.12	-34.06	18.98
500	0.505	24	5	10	11608.06	-23.83	38.92
500	0.505	24	5	15	11683.16	-29.21	34.85
500	0.505	24	5	5	11469.39	-26.21	34.34
500	0.495	25	10	10	12062.84	-32.17	-52.40
500	0.495	25	10	15	12140.53	-38.16	-57.62
500	0.495	25	10	5	11983.91	30.62	9.09
500	0.495	25	15	10	12164.38	-33.02	-56.85
500	0.495	25	15	15	12308.00	26.21	3.14
500	0.495	25	15	5	12085.71	31.23	6.12
500	0.495	25	5	10	11929.76	7.80	-6.41
500	0.495	25	5	15	12039.48	35.03	21.63
500	0.495	25	5	5	11819.10	36.83	21.27
500	0.495	26	10	10	12327.61	-30.21	-80.75
500	0.495	26	10	15	12408.26	-35.06	-85.00
500	0.495	26	10	5	12245.51	32.49	-19.03
500	0.495	26	15	10	12500.99	38.56	-15.92
500	0.495	26	15	15	12581.69	33.04	-20.87
500	0.495	26	15	5	12351.91	35.51	-19.93
500	0.495	26	5	10	12221.62	40.61	-3.33
500	0.495	26	5	15	12302.61	37.32	-6.00
500	0.495	26	5	5	12074.85	36.57	-8.41
500	0.495	24	10	10	11785.95	-41.46	-31.57

500	0.495	24	10	15	11860.44	-48.80	-37.94
500	0.495	24	10	5	11710.15	21.33	29.60
500	0.495	24	15	10	11882.73	-44.81	-38.17
500	0.495	24	15	15	12021.84	11.78	19.36
500	0.495	24	15	5	11807.23	19.46	24.50
500	0.495	24	5	10	11658.20	0.01	15.35
500	0.495	24	5	15	11764.11	25.26	41.59
500	0.495	24	5	5	11551.43	29.84	43.53
510	0.500	25	10	10	12276.27	-35.03	-77.84
510	0.500	25	10	15	12357.03	-39.44	-81.62
510	0.500	25	10	5	12131.31	-35.73	-79.61
510	0.500	25	15	10	12413.04	-2.48	-49.15
510	0.500	25	15	15	12526.54	25.12	-20.93
510	0.500	25	15	5	12298.60	28.56	-19.15
510	0.500	25	5	10	12109.52	-25.64	-61.99
510	0.500	25	5	15	12253.36	34.25	-1.44
510	0.500	25	5	5	11965.88	-27.08	-64.56
510	0.500	26	10	10	12543.32	-35.48	-109.68
510	0.500	26	10	15	12660.42	-5.41	-79.16
510	0.500	26	10	5	12458.00	26.59	-48.35
510	0.500	26	15	10	12718.88	33.59	-44.82
510	0.500	26	15	15	12802.78	29.73	-48.26
510	0.500	26	15	5	12567.16	30.51	-48.60
510	0.500	26	5	10	12370.59	-28.24	-95.39
510	0.500	26	5	15	12518.83	34.21	-32.46
510	0.500	26	5	5	12285.26	31.71	-36.24
510	0.500	24	10	10	11996.93	-41.98	-53.58
510	0.500	24	10	15	12074.43	-47.78	-58.54
510	0.500	24	10	5	11856.57	-41.27	-54.27
510	0.500	24	15	10	12128.19	-12.63	-27.75
510	0.500	24	15	15	12237.80	12.98	-1.32
510	0.500	24	15	5	12017.68	19.11	2.63
510	0.500	24	5	10	11836.31	-30.36	-36.09
510	0.500	24	5	15	11975.56	26.79	21.92
510	0.500	24	5	5	11697.24	-30.38	-37.55
510	0.505	25	10	10	12232.71	-64.76	-82.43
510	0.505	25	10	15	12346.32	-36.24	-53.09
510	0.505	25	10	5	12150.45	-2.93	-21.95
510	0.505	25	15	10	12402.83	1.25	-20.05
510	0.505	25	15	15	12483.77	-3.61	-24.12
510	0.505	25	15	5	12255.18	-1.08	-23.71
510	0.505	25	5	10	12065.34	-56.19	-67.77

510	0.505	25	5	15	12209.53	4.14	-6.61
510	0.505	25	5	5	11983.13	3.63	-9.36
510	0.505	26	10	10	12498.51	-66.17	-114.58
510	0.505	26	10	15	12615.93	-35.68	-83.46
510	0.505	26	10	5	12413.07	-4.38	-53.82
510	0.505	26	15	10	12674.68	3.64	-48.75
VI 510	0.505	26	15	15	12758.70	-0.01	-51.80
V 510	0.505	26	15	5	12522.57	0.00	-53.39
510	0.505	26	5	10	12325.20	-59.71	-101.46
510	0.505	26	5	15	12473.82	3.22	-37.87
IV 510	0.505	26	5	5	12239.78	-0.01	-42.85
510	0.505	24	10	10	11954.61	-70.78	-57.89
510	0.505	24	10	15	12064.27	-44.32	-30.43
510	0.505	24	10	5	11875.66	-8.83	2.39
510	0.505	24	15	10	12118.60	-8.59	1.01
510	0.505	24	15	15	12196.25	-14.85	-4.26
510	0.505	24	15	5	11975.61	-9.48	-1.53
510	0.505	24	5	10	11793.43	-59.91	-41.52
510	0.505	24	5	15	11933.05	-2.30	17.12
III 510	0.505	24	5	5	11714.45	0.00	16.68
510	0.495	25	10	10	12320.26	-5.02	-73.34
510	0.495	25	10	15	12400.86	-9.69	-77.54
510	0.495	25	10	5	12175.66	-5.20	-74.29
510	0.495	25	15	10	12456.54	26.92	-45.48
510	0.495	25	15	15	12569.78	54.17	-17.79
510	0.495	25	15	5	12342.36	58.39	-14.76
510	0.495	25	5	10	12154.12	5.19	-56.29
510	0.495	25	5	15	12297.56	64.57	3.59
510	0.495	25	5	5	12010.83	4.25	-58.06
510	0.495	26	10	10	12588.52	-4.57	-104.92
510	0.495	26	10	15	12672.30	-7.91	-108.00
510	0.495	26	10	5	12439.48	-6.05	-106.84
510	0.495	26	15	10	12730.16	30.47	-74.33
510	0.495	26	15	15	12847.23	59.67	-44.88
510	0.495	26	15	5	12612.11	61.23	-43.97
510	0.495	26	5	10	12416.36	3.45	-89.47
510	0.495	26	5	15	12564.24	65.45	-27.16
510	0.495	26	5	5	12268.67	1.21	-92.21
510	0.495	24	10	10	12039.62	-12.96	-49.41
510	0.495	24	10	15	12116.99	-18.98	-54.76
510	0.495	24	10	5	11899.65	-11.71	-49.26

510	0.495	24	15	10	12170.42	15.81	-24.37
510	0.495	24	15	15	12279.70	41.00	1.46
510	0.495	24	15	5	12060.20	48.01	6.75
510	0.495	24	5	10	11879.64	-0.50	-30.71
510	0.495	24	5	15	12018.50	56.17	26.65
II 510	0.495	24	5	5	11740.93	-0.01	-31.37
490	0.500	25	10	10	11831.91	-19.17	31.92
490	0.500	25	10	15	11907.00	-26.07	26.26
490	0.500	25	10	5	11689.88	-22.33	26.65
490	0.500	25	15	10	11931.78	-19.62	28.55
490	0.500	25	15	15	12075.05	40.96	90.37
490	0.500	25	15	5	11855.74	44.38	90.48
490	0.500	25	5	10	11666.65	-14.87	41.07
490	0.500	25	5	15	11807.59	45.25	102.46
490	0.500	25	5	5	11525.97	-18.67	35.13
490	0.500	26	10	10	12093.16	-15.42	7.68
490	0.500	26	10	15	12171.12	-21.23	2.95
490	0.500	26	10	5	11946.85	-19.85	1.45
490	0.500	26	15	10	12197.52	-13.56	6.31
490	0.500	26	15	15	12345.05	49.48	70.42
490	0.500	26	15	5	12118.43	50.43	68.52
490	0.500	26	5	10	11922.06	-13.28	15.21
490	0.500	26	5	15	12067.24	49.33	78.92
490	0.500	26	5	5	11777.14	-18.35	8.30
490	0.500	24	10	10	11558.62	-30.25	48.61
490	0.500	24	10	15	11630.61	-38.44	41.84
490	0.500	24	10	5	11421.03	-32.05	44.42
490	0.500	24	15	10	11653.85	-33.13	43.14
490	0.500	24	15	15	11792.69	24.86	102.53
490	0.500	24	15	5	11580.86	30.83	104.72
490	0.500	24	5	10	11399.27	-23.79	59.38
490	0.500	24	5	15	11471.40	-30.70	53.92
490	0.500	24	5	5	11263.00	-26.21	54.55
490	0.505	25	10	10	11793.14	-44.63	30.37
490	0.505	25	10	15	11868.35	-51.32	25.09
490	0.505	25	10	5	11650.68	-48.38	24.25
490	0.505	25	15	10	11927.50	-10.48	61.82
490	0.505	25	15	15	12037.02	16.45	90.15
490	0.505	25	15	5	11817.14	19.04	88.99
490	0.505	25	5	10	11627.20	-41.20	38.29
490	0.505	25	5	15	11768.56	19.43	100.35
490	0.505	25	5	5	11486.10	-45.58	31.51

490	0.505	26	10	10	12053.17	-41.81	5.82
490	0.505	26	10	15	12131.35	-47.31	1.56
490	0.505	26	10	5	11906.41	-46.85	-1.31
490	0.505	26	15	10	12192.75	-4.62	40.00
490	0.505	26	15	15	12305.85	24.09	69.94
490	0.505	26	15	5	12078.72	24.27	66.82
490	0.505	26	5	10	11881.48	-40.46	12.19
490	0.505	26	5	15	12027.10	22.68	76.59
490	0.505	26	5	5	11736.16	-46.08	4.44
490	0.505	24	10	10	11521.01	-54.85	47.31
490	0.505	24	10	15	11593.15	-62.80	40.93
490	0.505	24	10	5	11382.98	-57.24	42.26
490	0.505	24	15	10	11616.58	-57.27	42.50
490	0.505	24	15	15	11755.85	1.23	102.57
490	0.505	24	15	5	11543.48	6.42	103.55
490	0.505	24	5	10	11361.07	-49.17	56.95
490	0.505	24	5	15	11498.02	8.84	116.55
490	0.505	24	5	5	11224.40	-52.14	51.30
490	0.495	25	10	10	11871.08	6.55	33.37
490	0.495	25	10	15	11945.97	-0.65	27.26
490	0.495	25	10	5	11729.48	3.97	28.98
490	0.495	25	15	10	11970.58	5.61	29.31
490	0.495	25	15	15	12113.38	65.63	90.39
490	0.495	25	15	5	11894.70	69.93	91.83
490	0.495	25	5	10	11706.43	11.65	43.69
490	0.495	25	5	15	11781.42	5.73	38.87
490	0.495	25	5	5	11566.16	8.41	38.59
490	0.495	26	10	10	12133.46	11.13	9.35
490	0.495	26	10	15	12211.33	5.13	4.26
490	0.495	26	10	5	11987.58	7.29	4.00
490	0.495	26	15	10	12237.45	12.50	7.27
490	0.495	26	15	15	12384.50	74.97	70.63
490	0.495	26	15	5	12158.60	76.90	70.18
490	0.495	26	5	10	11962.97	14.08	18.06
490	0.495	26	5	15	12107.74	76.18	81.10
490	0.495	26	5	5	11818.43	9.55	11.97
490	0.495	24	10	10	11596.56	-5.47	49.76
490	0.495	24	10	15	11668.39	-13.91	42.58
490	0.495	24	10	5	11459.41	-6.67	46.44
490	0.495	24	15	10	11691.38	-8.87	43.56
490	0.495	24	15	15	11829.77	48.57	102.25
490	0.495	24	15	5	11618.65	55.50	105.83

490	0.495	24	5	10	11437.90	1.87	61.77
490	0.495	24	5	15	11509.77	-5.39	55.79
I 490	0.495	24	5	5	11302.03	0.00	57.75

The results in Table 2 are obtained for parameters arrangement given in Table 3. Rows in Table 2 correspond eq.(2) andits sequences are denominated from I-VI. These sequences are used for φ_i calculation using NEW method and results are given in Table 4. Coefficient K is calculated from data given in the first row of the Table2.

Table 3. Example experiment plan of least square method

p_1	p_2	p_3	p_4	…	p_n	f_i
$(p_1)_1$	$(p_2)_1$	$(p_2)_1$	$(p_4)_1$	…	$(p_n)_1$	$(f_1)_1$
					$(p_n)_2$	$(f_1)_2$
					$(p_n)_3$	$(f_1)_3$
				…		
				…		
			$(p_4)_2$			
			$(p_4)_3$			
		$(p_3)_2$				
		$(p_3)_3$				
	$(p_2)_2$					
	$(p_2)_3$					
$(p_1)_2$						
$(p_1)_3$						

The same sequences of ordinary linear equations OLE are used for obtaining coefficients as numerical solution and data in Table 2 for the same solutions by least square method LSQ. On the basis of the calculated φ_i and K differences ΔX_{ole}, ΔX_{isq} given in Table 2 are derived. Mean square root differences from differences ΔX_{ole} ΔX_{isq} are also given in Table 4.

Table 4. Calculated values for $(\varphi_i,\, K,\, m_{srd})$

φ_i, K, m_{srd}	**LSQ**	**OLE**	**NEW**
φ_{V0}	1.1470986	9.5233440E-01	9.5233440E-01
φ_c	-3.1752747E-01	-1.1291144E-01	-1.1291145E-01
φ_θ	6.0951495E-01	5.4807090E-01	5.4807090E-01
φ_{Wx}	2.1482959E-02	2.0790317E-02	2.0790317E-02
$\varphi_{\Delta\tau}$	1.6819049E-02	1.7004655E-02	1.7004658E-02
K	1.0000201	4.718855	4.695382
m_{srd}	50.60136	35.36713	64.77618

For eq.(8) and Table 4, it is easy to see the advantages of the new procedure for obtaining φ_i and K. It can be stated as follows:

- number of experiments is significantly reduced compared to the least square method for the same results;
- simplicity of the expression (8) eliminates any need for using numerical methods for solving ordinary linear equations;
- new method is predominantly compared to the existing, least square and other methods, due to the established correspodent parameter sequences in eq.(2) and experiment plan given with Table 1.

Conclusion

Dimensional analysis procedure used in all problems where there are $n > 3$ variables to be included on the right-hand side of equations like (2) is inadequate.

In that case, the final form of the relation will include determinate powers of three of these variables, along with the arbitrary functions of $(n-3)$ non-dimensional parameters, each of which is a group of some or all of the n variables.

The result of author's investigation concerning determining the exponents in equations like (2) without restrictions as for number of parameters and associated problems is given.

The advantage of the new procedure for obtaining φ_i and K can be stated as follows:

- number of experiments is significantly reduced compared to the least square method for the same results;
- simplicity expression (8) eliminates the necessity to use any numerical method for solving ordinary linear equations;
- -new method is predominantly compared to the existing, least square and other methods, due to the corresponding parameter sequences established in eq.(2) and experiment plan given in Table 1.

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Novo rešenje posebno formirane jednačine linearne regresije

Prikazan je nov način dobijanja koeficijenata φ_i, i korektivnog faktora K u izrazu oblika $f_1 = K\, p_1^{\varphi_1}\, p_2^{\varphi_2}\, p_3^{\varphi_3} ... p_n^{\varphi_n} = K \prod_{i=1}^{n} p_i^{\varphi_i}, (i = 1, ..., n)$ gde su p_1 parametri mehaničkog sistema. Novim načinom je eliminisana metoda najmanjih jednačina za koju treba obaviti daleko veći broj eksperimenata nego što ima koeficijenata φ_i plus jedan za korektivni faktor K. Po datom postupku, dovoljno je izvršiti tačno onoliko eksperimenata koliko ima koeficijenata φ_i i jedan više zbog K.

U isto vreme, φ_i se dobijaju elementarno, bez korišćenja numeričke ili bilo koje druge analitičke metode za rešavanje sistema običnih linearnih jednačina. Ovo je omogućeno zahvaljujući posebno definisanoj strukturi sistema običnih linearnih jednačina koja prati raspored parametara p_1 u planu eksperimenta.

Ključne reči: **linearna regresija, sistem linearnih jednačina, planiranje eksperimenta**

Une nouvelle solution d'une équation de regression linéare spécialement formée

On a démonré une nouvelle manière d'obtenir le coefficient φ_i et le facteur correctif K dans l'expression en forme $f_1 = K\, p_1^{\varphi_1}\, p_2^{\varphi_2}\, p_3^{\varphi_3} ... p_n^{\varphi_n} = K \prod_{i=1}^{n} p_i^{\varphi_i}, (i = 1, ..., n)$ ou le p_1 représente les paramètres du système mécanique. Par ce nouveau procédé on a éliminé la méthode des plus petites équations pour lesquelles il faut effectuer le nombre d'essais plus grand que chez le coefficient φ_i plus un pour le facteur correctif K. Selon le procédé donné il suffit juste d'effectuer un nombre d'essais correspondant au nombre du coefficient φ_i plus un, toujours à cause du K. Simultanément on obtient φ_i de façon élementaire, sans appliquer la méthode numérique ou d'autres méthodes analytiques afin de résoudre le système d'équations linéaires ordinaires. Ceci est possible grace à une structure du système d'équations linéaires ordinaires spécialement définie qui suit la répartiton des paramètres p_1 dans la planification d'un essai.

Mots clés: **regression linéaire, système d'équations linéaires, planification d'un essai**

Новое решение специально установленного уравнения линейной регрессии в плане экспериментов

В этой работе представлен новый способ φ_i получения коэффициента и корректировочного коэффициента K в уравнении: $f_1 = K\, p_1^{\varphi_1}\, p_2^{\varphi_2}\, p_3^{\varphi_3} ... p_n^{\varphi_n} = K \prod_{i=1}^{n} p_i^{\varphi_i}, (i = 1, ..., n)$, где p_1 являются параметрамы механической системы. Новым способом элиминирован метод наименьших квадратов, чтобы было возможно сложить систему обыкновеных линейных уровнений, для которой нужно сделать гораздо больше экспериментов от существующего числа коэффициентов φ_i с добавлением одного для корректировочного коэффициента K. По даном поступке, довольно сделать точно столько экспериментов, сколько существует коэффициентов φ_i и за один больше из-за K.

В то же время, φ_i получаются элементарно, без пользования численного метода или какого-нибудь другого аналитического метода для решения системы обыкновеных линейных уровнений. Это сделано возможным благодаря специально определенной структуре системы обыкновеных линейных уровнений, следящей распределение параметров p_1 в плане экспериментов.

Клычевюе слова: линейнаь регрессиь, система линейнюх уровнений, план &кспериментов