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Modifications of the equations for distribution of circulation for tapered, sweptback wings with streamwise tips in the supersonic flow presented in NACA TN 2643 paper

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In NACA TN 2643 paper [1] equations for distribution of circulation resulting from a constant angle of attack, constant rate of roll and constant rate of pitch, for series of thin, tapered, sweptback, isolated wings with streamwise tips in supersonic flow, are presented.
Based on this equations for determination of static and dynamic derivatives values, software G.R.O.M. [2], [3], was developed.
In the process of checking, within the presented equations for distribution of circulation, mistakes causing the discontinuities in certain zones of the wings arose, thus violating the basic request that those functions must be continuous. This caused the necessity for equation modifications in order to neutralize discontinuities.
Modified equations are presented in the form of tables, mark of the zone in which the modification has been made included, as well as distribution of circulation in the form of diagrams [2], [3].

Key words: **winds dynamics, supersonic flow, aerodynamic derivatives, velocity potential, software.**

Notification

A – aspect ratio
$B=\sqrt{M^2-1}$
b – wing span
c_r – root cord
d – spanwise coordinate of intersection of trailing edge of wing and Mach line reflected from wing tip
e – spanwise coordinate of intersection of trailing edge of wing and Mach line from leading edge of wing
h – spanwise coordinate of intersection of trailing edge of wing and Mach line from wing tip
$K=\dfrac{\text{ctg}\,\Lambda_{TE}}{\text{ctg}\,\Lambda_{LE}}=\dfrac{AB(1+\lambda)}{AB(1+\lambda)-4mB(1-\lambda)}$
M – Mach number
m – cot Λ
x,y,z – rectangular coordinates (x-axis parallel to free stream direction)
Γ – spanwise distribution of circulation
μ – Mach line angle
Λ – sweep of leading edge
λ – taper ratio

Introduction

BASED on the equations for distribution of circulation presented in NACA TN 2643 [1] paper, software G.R.O.M. was developed. Its used for calculation of the aerodynamic derivatives value - 1. the lift force in respect to angle of attack; 2. drag force in respect to angle of attack on the square, as static derivatives; 3. roll-dumping moment in respect to rate of roll; 4. pitch-dumping moment in respect to rate of pitch, as dynamic derivatives, for two families of wings, with supersonic leading and trailing edges in supersonic flow.

In the frame of this paper two considered families of wings are presented, including numerations and markings of the zones on the wings. These zones are determined by Mach cones positions and intersection points of the cones with the trailing edge.

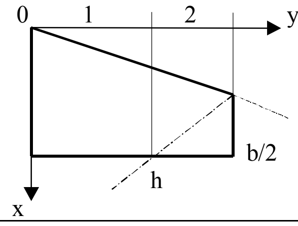
The basic request, while applying the given equations, is the request of function continuity, i.e. diagram presentation of distribution of circulation must be a continuous curve.

However, while checking equations presented in NACA TN 2643 paper discontinuities and interruptions appeared, causing the necessity for modifications.

Reviews of the modified equations are given in the form of tables, mark of the zone in which the modification has been made included along with the distributions of circulation are presented in the form of diagrams.

Method and the procedure of modification

Table reviews of the considered families of wings, with subcase markings and zone numbers and tags:

Subcase	Subcase tag
	PNNI 2

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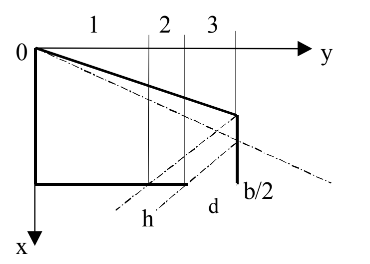
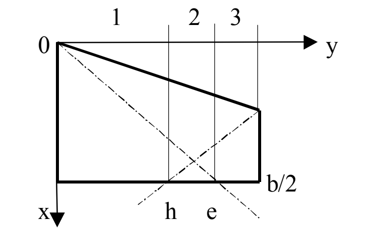
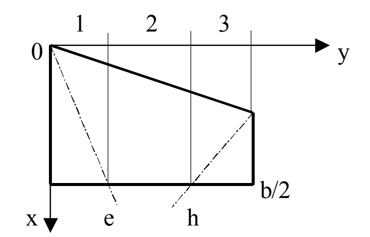
	PNNI 5
	PNNI 4
	PNNI 3

Table 1. Review of subcases for PNNI family wings: thin, tapered wings with sweptback leading and unswept trailing edge

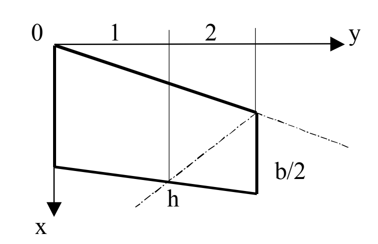
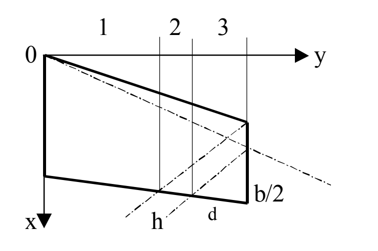
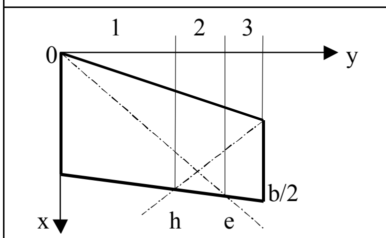
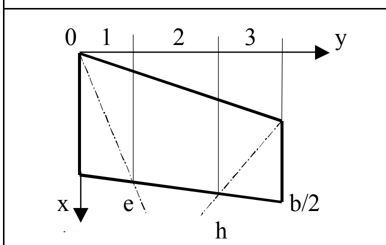
SUBCASE	Subcase tag
	PSNI 2
	PSNI 5
	PSNI 4
	PSNI 3

Table 2. Review of subcases for PSNI family wings: thin, tapered wings with sweptback leading and trailing edge

Mistakes violating the basic principle of functions continuity arose in the following cases:

1. Functions for distribution of circulation resulting from constant rate of roll - subcases PNNI2, as well as PSNI2 and PSNI5.
2. Functions for distribution of circulation resulting from constant rate of roll functions - subcases PNNI2, PNNI5, PNNI4, PNNI3 as well as PSNI2, PSNI5, PSNI4, PSNI3.

Postulates on which modifications are based:

1. Intersection points of Mach cones and trailing edge represent zone boundaries, and, at the same time represent the boundary values – value for distribution of circulation in the previous zone must be, in the boundary (intersection) point, equal to the value for distribution of circulation in the next zone.
2. Equations for distribution of circulation in the first and second zone for the wing with four panels (subcases PNNI4 and PSNI4) are the same as for the wing with five panels (subcases PNNI5 and PSNI5).
3. Equation for distribution of circulation in the first zone for the wing with three panels (subcases PNNI3 and PSNI3) is the same as for the wing with five panels (subcases PNNI5 and PSNI5).
4. Equations for distribution of circulation in the third zone for the wing with three panels (subcases PNNI3 and PSNI3) is the same as for the wing with four panels (subcases PNNI4 and PSNI4).
5. Excluding articles that have factor k in themselves, which takes leading and trailing edge sweeps into account, out of functional expressions for PSNI family of wings, expressions for PNNI family of wings are obtained. For the case of unswept trailing edge $k \to \infty$ (family of the PNNI wings). For any other value of trailing edge sweep angle factor k has a final value.
6. Distribution of circulation values when the leading edge is sonic (Mach line coincident with leading edge – subcases PNNI2 and PSNI2), and when the leading edge is supersonic and there are 5 panels on the wing (subcases PNNI5 and PSNI5) should be very close and the diagram representations of distributions of circulation mutually very similar, provided that Mach numbers for sonic and supersonic leading edge are close to each other (meaning that angles of Mach cones differ in small amount).
7. For the small value of trailing edge sweep (around 1 degree), while the leading edge sweep remains the same, and aspect ratio and area of the wing are slightly changed, distribution of circulation values for PNNI and PSNI family of wings should be very close to each other.

Modification of the equations for distribution of circulation resulting from a constant rate of roll

Since only the diagram for distribution of circulation for subcase PNNI5 is without discontinuities the starting point has to be:

a) method of modification for subcase PNNI2 On the diagram On the diagram 1 distribution of circulations for two subcases of wings (PNNI2 and PNNI5) are presented. Wings are with the root cord length of 1 meter, taper ratio of 0.25 and span of 1 meter. For easier analysis the diagrams were drawn only through characteristic points of the wings (beginning: $y/(b/2)=0$, zones boundary points and end: $y/(b/2)=1$). From the diagrams it is clearly visible that distribution of circulation for subcase PNNI2 has an interruption in the point representing the boundary of the zones 1 and 2. Based on the postulate no.1 and 6 modification for subcase PNNI2 was done, in the zone 2 $\left(h/\left(b/2\right) \le y/\left(b/2\right) \le 1\right)$, in the manner of increasing the value for distribution of circulation at the beginning of the second zone (for $y/\left(b/2\right) = h/\left(b/2\right)$), while maintaining the value for $y/\left(b/2\right) = 1$, thus neutralizing the discontinuity in the boundary point. 1 distribution of circulations for two subcases of wings (PNNI2 and PNNI5) are presented. Wings are with the root cord length of 1 meter, taper ratio of 0.25 and span of 1 meter. For easier analysis the diagrams were drawn only through characteristic points of the wings (beginning: $y/\left(b/2\right) = 0$, zones boundary points and end: $y/\left(b/2\right) = 1$). From the diagrams it is clearly visible that distribution of circulation for subcase PNNI2 has an interruption in the point representing the boundary of the zones 1 and 2. Based on the postulate no.1 and 6 modification for subcase PNNI2 was done, in the zone 2 $\left(h/\left(b/2\right) \le y/\left(b/2\right) \le 1\right)$, in the manner of increasing the value for distribution of circulation at the beginning of the second zone (for $y/\left(b/2\right) = h/\left(b/2\right)$), while maintaining the value for $y/\left(b/2\right) = 1$, thus neutralizing the discontinuity in the boundary point.

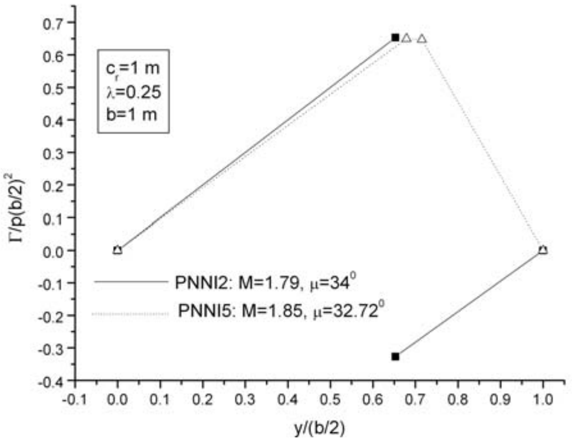


Diagram 1. Distribution of circulation resulting from a constant rate of roll, for subcases marked as PNNI2 and PNNI5, drawn through characteristic points of the zones on the wing; unmodified equations

b) method of modification for subcase PSNI5 – based on the postulates no.1 and 5 modifications in zone 2 $\left(h/\left(b/2\right) \le y/\left(b/2\right) \le d\left(b/2\right)\right)$ and zone 3 $d/\left(b/2\right) \le y/\left(b/2\right) \le 1$) have been conducted in order to neutralize the discontinuities.

c) method of modification for subcase PSNI2 – based on the postulates no.1 and 5 modifications in zone 2 $\left(h/\left(b/2\right) \le y/\left(b/2\right) \le 1\right)$ have been conducted. Modification can also be made in accordance with postulate no.6.

Modification of the equations for distribution of circulation resulting from a constant rate of pitch

Problem with this sets of equations was that for every wing in both families of wings discontinuities of functions in particular zones existed, thus makingit impossible to start modifications from any particular wing. The only solution was to start with the comparison of two wings (using postulates no.1, 5 and 7) and to obtain, through the iteration method, equations with no discontinuities. Since the equations for subcases PNNI2 and PSNI2 are the simplest, comparison and iteration process began with those two subcases.

d) method of modification for subcases PNNI2 and PSNI2

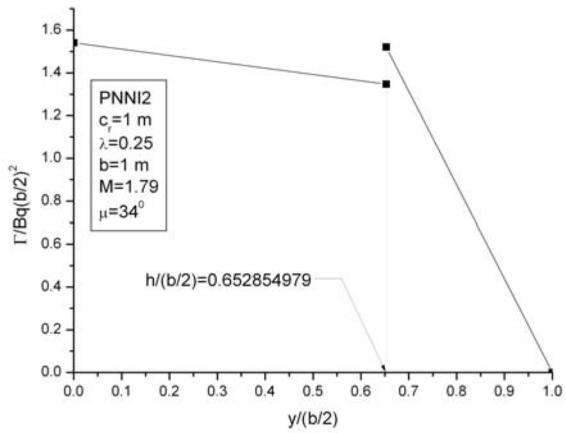


Diagram 2. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PNNI2, drawn through characteristic points of the zones on the wing; unmodified equation**s**

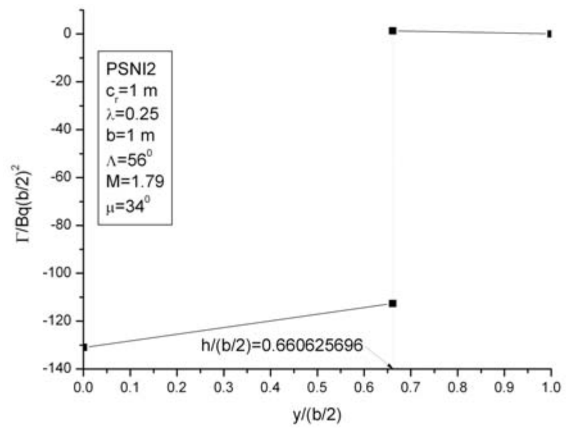


Diagram 3. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI2, drawn through characteristic points of the zones on the wing; unmodified equations

Analyzing these two diagrams (drawn only through characteristic points of the zones on the wing**)** it can be established that values in the points representing boundaries of the zones $\left(h/\left(b/2\right)\right)$ are similar, and that, using iterative method equations should be modified in the way that discontinuities must be neutralized and that new equations must be in accordance with postulates no.1, 5 and 7. With the subcase PNNI2 the discontinuity is by far smaller than the one with the subcase PSNI2, for which the value of distribution of circulation is distinctly negative. The value for distribution of circulation in the first zone (for $y/\left(b/2\right) = h/\left(b/2\right)$), for subcase PNNI2, is around 1.35, and for subcase PSNI2 is around 1.32. Modification was executed in the following way: equation for distribution of circulation in the zone 2 $\left(h/\left(b/2\right) \le y/\left(b/2\right) \le 1\right)$, for subcase PNNI2, was modified so the value for distribution of circulation decreases for $h/(b/2)=y/(b/2)$, while maintaining the value for $y/(b/2)=1$, in accordance with postulates no.1, 5 and 7. After that, in accordance with the same postulates, equation for distribution of circulation in the zone 1 $(0 \le y/(b/2) \le h/(b/2))$, for subcase PSNI2 was modified.

e) method of modification for subcase PSNI5 – After exerting the modifications of equations for the subcase PSNI2, modification of equations or subcase PSNI5 were executed in the similar manner as it was done for the same two wings at a constant rate of roll.

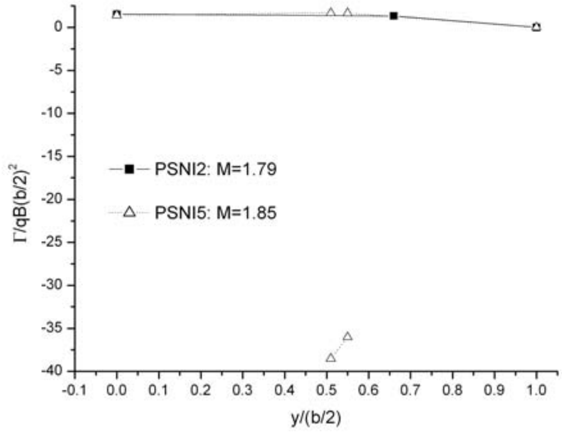


Diagram 4. Distribution of circulation resulting from a constant rate of pitch, for subcases marked as PSNI2 and PSNI5, drawn through characteristic points of the zones on the wing; unmodified equations

On the diagram 4 distribution of circulations for two subcases of wings (PSNI2 and PSNI5) are presented. Wings have the root cord length of 1 meter, taper ratio of 0.5 and span of 1 meter. For easier analysis the diagram were drawn only through characteristic points of the wings (beginning: $y/(b/2)=0$, zones boundary points and end: $y/(b/2)=1$. Function for distribution of circulation for subcase PSNI5 have interruptions in the points representing the boundaries for the zones 1 and 2 and zones 2 and 3, with distinctly negative distribution of circulation in zone 2. In accordance with postulates no.1 and 6 modification for subcase PSNI5, in the zone 2 $(h/(b/2) \le y/(b/2) \le d/(b/2))$, was done.

f) method of modification for subcase PSNI4

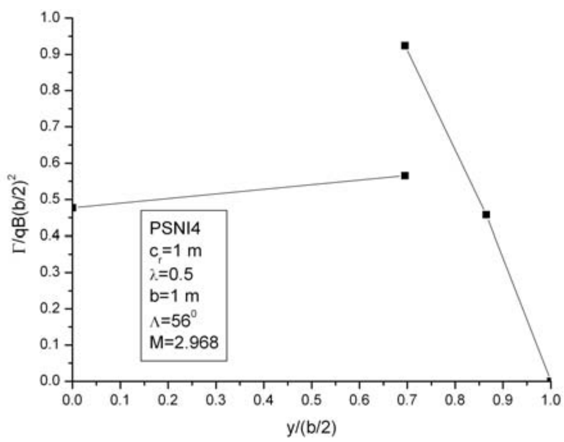


Diagram 5. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI4, drawn through characteristic points of the zones on the wing; unmodified equations

Modification for subcase PSNI4 was executed in zone 2 $(h/(b/2) \le y/(b/2) \le e/(b/2))$, in accordance with postulates no.1 and 2, neutralizing in this way discontinuity existing on the boundary of zones 1 and 2 (for $y/(b/2) = h/(b/2)$), while maintaining the value on the boundary of zones 2 and 3 (for $y/(b/2) = e/(b/2)$).

g) method of modification for subcase PSNI3

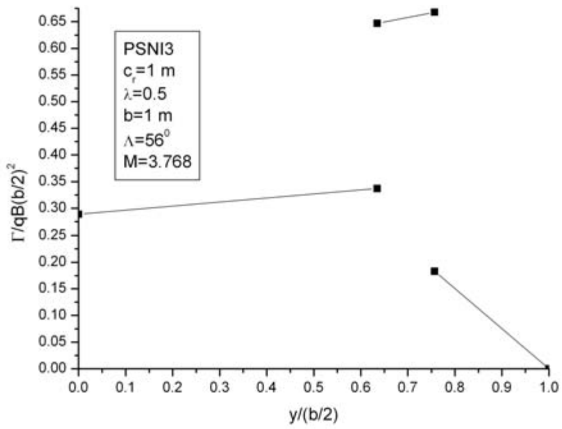


Diagram 6. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI3, drawn through characteristic points of the zones on the wing; unmodified equations

In accordance with postulates no.1, 3 and 4 equations for distribution of circulation in zone 1 $(0 \le y/(b/2) \le e/(b/2))$ and zone 3 $(h/(b/2) \le y/(b/2) \le 1)$ can be treated as correct ones. Based on that, modification for equation in zone 2 $(e/(b/2) \le y/(b/2) \le h(b/2))$ was conducted, neutralizing in that way the discontinuities existing on the boundaries of zones 1 and 2, and zones 2 and 3, changing, in the process, gradient of the function in zone 2.

h) method of modification for subcase PNNI5 – modification was executed in zone 2 $(h/(b/2) \le y/(b/2) \le d/(b/2))$, in accordance with postulates no 1 and 5.

i) method of modification for subcase PNNI4 – modification was executed in zone 2 $(h/(b/2) \le y/(b/2) \le e/(b/2))$ and zone 3 $(e/(b/2) \le y/(b/2) \le 1)$, in accordance with postulates no.1, 2 and 5.

j) method of modification for subcase PNNI3 – modification was executed in zone 3 ($(h/(b/2) \le y/(b/2) \le 1)$, in accordance with postulates no.1, 4 and 5.

Modified equations

Modified equations are given in the form of table review. These tables contain only the equations for the zones in which modification have been conducted. The rest of the equations for a given wing are the same as presented in NACA TN 2643 paper.

Modificatons of the equations for determination of the distribution of circulation resulting from a constant rate of roll:

– subcase marked as PSNI2

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line coincident with leading edge
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le 1$	$\frac{2p(b/2)^2}{3\pi k\sqrt{k}}\left[\frac{y}{b/2}(-1+3k)+2k-\frac{4k}{AB(1+\lambda)}\right]\sqrt{2\left(1-\frac{y}{b/2}\right)\left[\frac{y}{b/2}(1+k)+\frac{4k}{AB(1+\lambda)}\right]}$

– subcase marked as PSNI5

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from the wing apex intersecting tip
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le \frac{d}{b/2}$	$\frac{2p(b/2)^2}{k\pi\left(B^2m^2-1\right)^{3/2}}\left\{-\frac{y}{b/2}\sqrt{\left[\frac{y^2}{(b/2)^2}\left(1-B^2m^2k^2\right)+\frac{8Bmk}{AB(1+\lambda)}\frac{y}{b/2}+\frac{16B^2m^2k^2}{A^2B^2(1+\lambda)^2}\right]\left(B^2m^2-1\right)}-\frac{1}{2k}\left[\left[\frac{y^2}{(b/2)^2}(1+k)\left(-2B^2m^2k+k-1\right)-\frac{8mBk\left(B^2m^2k+1\right)}{AB(1+\lambda)}\frac{y}{b/2}-\frac{16B^2m^2k^2}{A^2B^2(1+\lambda)^2}\right]\bullet\cos^{-1}\left\{\frac{AB(1+\lambda)\left(B^2m^2k+1\right)\frac{y}{b/2}+4mBk}{Bm\left[AB(1+\lambda)(1-k)\frac{y}{b/2}+4mBk\right]}\right\}+\left[\frac{y^2}{(b/2)^2}(1-k)\left(2B^2m^2k-1-k\right)+\frac{8Bmk\left(B^2m^2k-1\right)}{AB(1+\lambda)}\frac{y}{b/2}-\frac{16B^2m^2k^2}{A^2B^2(1+\lambda)^2}\right]\bullet\left(\cos^{-1}\left\{\frac{AB(1+\lambda)\left(B^2m^2k-1\right)\frac{y}{b/2}-4mBk}{Bm\left[AB(1+\lambda)(1-k)\frac{y}{b/2}+4mBk\right]}\right\}-\cos^{-1}\left[\frac{AB(1+\lambda)(1+k+2mBk)\frac{y}{b/2}+4mBk-2kAB(1+\lambda)(Bm+1)}{AB(1+\lambda)(1-k)\frac{y}{b/2}+4mBk}\right]\right)\right]-\frac{1}{3\sqrt{k}}\left[\frac{y}{b/2}\left(3k+1+4mB-2mBk-2m^2B^2k\right)+\frac{4mBk(4mB+1)}{AB(1+\lambda)}+\left(2k-4m^2B^2k-2mBk\right)\right]\bullet\sqrt{(Bm+1)\left\{-\frac{y^2}{(b/2)^2}(1+Bmk)+\frac{y}{b/2}\left[-\frac{4mBk}{AB(1+\lambda)}+k+2mBk+1\right]+\frac{4mBk}{AB(1+\lambda)}-k-Bmk\right\}}\right\}$
$\frac{d}{b/2} \le \frac{y}{b/2} \le 1$	$\frac{p(b/2)^2}{\pi k^2\left(B^2m^2-1\right)^{3/2}}\left(\left[\frac{y^2}{(b/2)^2}(1+k)\left(2B^2m^2k+1-k\right)+\frac{8Bmk\left(B^2m^2k+1\right)}{AB(1+\lambda)}\frac{y}{b/2}+\frac{16B^2m^2k^2}{A^2B^2(1+\lambda)^2}\right]\bullet\cos^{-1}\left[\frac{AB(1+\lambda)(2mBk+1-k)\frac{y}{b/2}-2kAB(1+\lambda)(Bm-1)+4mBk}{AB(1+\lambda)(1+k)\frac{y}{b/2}4mBk}\right]+\frac{2\sqrt{k}}{3}\left[\frac{y}{b/2}\left(-3k+1-4mB-2mBk+2B^2m^2k\right)+\left(-2k+4B^2m^2k-2mBk\right)-\frac{4mBk(4mB-1)}{AB(1+\lambda)}\right]\cdot\sqrt{(Bm-1)\left\{-\frac{y^2}{(b/2)^2}(1+Bmk)+\frac{y}{b/2}\left[-\frac{4mBk}{AB(1+\lambda)}-k+2mBk+1\right]+\frac{4mBk}{AB(1+\lambda)}+k-Bmk\right\}}\right)$

– subcase marked as PNNI2

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line coincident with leading edge; unswept trailing edge
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le 1$	$\frac{2p(b/2)^2}{3\pi}\left[3\frac{y}{b/2}+2-\frac{4}{AB(1+\lambda)}\right]\sqrt{2\left(1-\frac{y}{b/2}\right)\left[\frac{y}{b/2}+\frac{4}{AB(1+\lambda)}\right]}$

Modificatons of the equations for determination of the distribution of circulation resulting from a constant rate of pitch:

– subcase marked as PSNI2

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line coincident with leading edge
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	$\frac{8qB(b/2)^2}{3\pi k}\left[\frac{y}{b/2}+\frac{4k}{AB(1+\lambda)}\right]\sqrt{\frac{y^2}{(b/2)^2}\frac{1-k^2}{k^2}+\frac{8}{kAB(1+\lambda)}\frac{y}{b/2}+\frac{16}{A^2B^2(1+\lambda)^2}}$
$\frac{h}{b/2} \le \frac{y}{b/2} \le 1$	NACA TN 2643

– subcase marked as PSNI5

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from the wing apex intersecting tip
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le \frac{d}{b/2}$	$\frac{2qB(b/2)^2}{\pi kmB(B^2m^2-1)}\left\{\left[\frac{y}{b/2}+\frac{4kmB}{AB(1+\lambda)}\right]\sqrt{\frac{1}{k^2}\left[\frac{y^2}{(b/2)^2}(1-B^2m^2k^2)+\frac{8mBk}{AB(1+\lambda)}\frac{y}{b/2}+\frac{16B^2m^2k^2}{A^2B^2(1+\lambda)^2}\right]}+\frac{1}{2k\sqrt{B^2m^2-1}}\left[\left\{\frac{y^2}{(b/2)^2}(k-1)\left[2-B^2k^2(k+1)\right]+\frac{8Bmk}{AB(1+\lambda)}(k+B^2m^2-2)\frac{y}{b/2}+\frac{16B^2m^2k^2(B^2m^2-2)}{A^2B^2(1+\lambda)^2}\right\}\cdot\left(-\cos^{-1}\left\{\frac{\frac{y}{b/2}(B^2m^2k-1)-\frac{4mBk}{AB(1+\lambda)}}{Bm\left[(1-k)\frac{y}{b/2}+\frac{4mBk}{AB(1+\lambda)}\right]}\right\}+\cos^{-1}\left\{\frac{\frac{y}{b/2}(1+k+2mBk)+\frac{4mBk}{AB(1+\lambda)}-2k(mB+1)}{\frac{y}{b/2}(1-k)+\frac{4mBk}{AB(1+\lambda)}}\right\}\right)+\left\{\frac{y^2}{(b/2)^2}(k+1)\left[-2-B^2m^2(k-1)\right]+\frac{8mBk}{AB(1+\lambda)}(B^2m^2-2-k)\frac{y}{b/2}+\frac{16B^2m^2k^2(B^2m^2-2)}{A^2B^2(1+\lambda)^2}\right\}\cdot\cos^{-1}\left\{\frac{\frac{y}{b/2}(1+B^2m^2k)+\frac{4mBk}{AB(1+\lambda)}}{Bm\left[(1+k)\frac{y}{b/2}+\frac{4mBk}{AB(1+\lambda)}\right]}\right\}\right]+\frac{1}{3}\left[\frac{y}{b/2}(-6-B^2m^2k-4mB+2B^3m^3k+5B^2m^2)+(2B^2m^2k+4mBk-2B^3m^3k)+\frac{4Bkm(-6-4mB+5B^2m^2)}{AB(1+\lambda)}\right]\cdot\sqrt{\frac{1}{k(mB-1)}\left\{-\frac{y^2}{(b/2)^2}(1+Bmk)+\frac{y}{b/2}\left[-\frac{4Bmk}{AB(1+\lambda)}+k+1+2Bmk\right]+\frac{4Bmk}{AB(1+\lambda)}-k(mB+1)\right\}}\right\}$
$\frac{d}{b/2} \le \frac{y}{b/2} \le 1$	NACA TN 2643

– subcase marked as PSNI4

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from wing apex intersecting trailing edge and intersecting Mach line from tip
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le \frac{e}{b/2}$	Based on the postulate no. 2. equation for distribution of circulation in this zone is the same as equation for distribution of circulation in the second zone of the wing marked as subcase PSNI5
$\frac{e}{b/2} \le \frac{y}{b/2} \le 1$	NACA TN 2643

– subcase marked as PSNI3

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from wing apex intersecting trailing edge
$0 \le \frac{y}{b/2} \le \frac{e}{b/2}$	NACA TN 2643
$\frac{e}{b/2} \le \frac{y}{b/2} \le \frac{h}{b/2}$	$\frac{qB(b/2)^2}{k^2mB(B^2m^2-1)^{3/2}}\left\{\frac{y^2}{(b/2)^2}(k-1)\left[2-B^2m^2(k+1)\right]+\frac{8mBk}{AB(1+\lambda)}(k+B^2m^2-2)\frac{y}{b/2}+\frac{16B^2m^2k^2(B^2m^2-2)}{A^2B^2(1+\lambda)^2}\right\}$
$\frac{h}{b/2} \le \frac{y}{b/2} \le 1$	NACA TN 2643

– subcase marked as PNNI2

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line coincident with leading edge; unswept trailing edge
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le 1$	$\frac{2\sqrt{2}q(b/2)^2 B}{3\pi}\left\{\left[\frac{20}{AB(1+\lambda)}+\frac{y}{b/2}-2\right]\sqrt{\left(1-\frac{y}{b/2}\right)\left[\frac{y}{b/2}+\frac{4}{AB(1+\lambda)}\right]}\right\}$

– subcase marked as PNNI5

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from the wing apex intersecting tip; unswept trailing edge
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le \frac{d}{b/2}$	$\frac{-2qB(b/2)^2}{\pi(B^2m^2-1)}\left[\frac{4mB}{AB(1+\lambda)}\sqrt{-\frac{y^2}{(b/2)^2}+\frac{16}{A^2B^2(1+\lambda)^2}}+\frac{1}{2\sqrt{B^2m^2-1}}\cdot\left(\left[-Bm\frac{y^2}{(b/2)^2}-\frac{8}{AB(1+\lambda)}\frac{y}{b/2}+\frac{16Bm(B^2m^2-2)}{A^2B^2(1+\lambda)^2}\right]\cdot\cos^{-1}\left[\frac{ABmB(1+\lambda)\frac{y}{b/2}+4}{\frac{y}{b/2}AB(1+\lambda)+4mB}\right]+\left[-Bm\frac{y^2}{(b/2)^2}+\frac{8}{AB(1+\lambda)}\frac{y}{b/2}+\frac{16Bm(B^2m^2-2)}{A^2B^2(1+\lambda)^2}\right]\cdot\left\{\cos^{-1}\left[\frac{mBAB(1+\lambda)\frac{y}{b/2}-4}{4mB-\frac{y}{b/2}AB(1+\lambda)}\right]+\cos^{-1}\left[\frac{\frac{y}{b/2}(2Bm+1)+\frac{4mB}{AB(1+\lambda)}-2(mB+1)}{-\frac{y}{b/2}+\frac{4mB}{AB(1+\lambda)}}\right]\right\}\right)+\frac{1}{3Bm}\left[\frac{y}{b/2}(-B^2m^2+2B^3m^3)+\frac{4mB}{AB(1+\lambda)}(-6-4mB+5B^2m^2)+4mB+2B^2m^2-2B^3m^3\right]\cdot\sqrt{\frac{1}{(mB-1)}\left\{-Bm\frac{y^2}{(b/2)^2}+\frac{y}{b/2}\left[-\frac{4mB}{AB(1+\lambda)}+1+2Bm\right]+\frac{4mB}{AB(1+\lambda)}-(mB+1)\right\}}\right]$
$\frac{d}{b/2} \le \frac{y}{b/2} \le 1$	NACA TN 2643

– subcase marked as PNNI4

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from wing apex intersecting trailing edge and intersecting Mach line from tip; unswept trailing edge
$0 \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le \frac{e}{b/2}$	Based on the postulate no. 2. equation for distribution of circulation in this zone is the same as equation for distribution of circulation in the second zone of the wing marked as subcase PNNI5
$\frac{e}{b/2} \le \frac{y}{b/2} \le 1$	$\frac{qB(b/2)^2}{\pi mB(m^2B^2-1)^{3/2}}\left(\left[-B^2m^2\frac{y^2}{(b/2)^2}+\frac{8Bm}{AB(1+\lambda)}\frac{y}{b/2}+\frac{16B^2m^2(B^2m^2-2)}{A^2B^2(1+\lambda)^2}\right]\cdot\cos^{-1}\left[\frac{\frac{y}{b/2}(2Bm+1)+\frac{4mB}{AB(1+\lambda)}-2(mB+1)}{-\frac{y}{b/2}+\frac{4mB}{AB(1+\lambda)}}\right]+\frac{2\sqrt{B^2m^2-1}}{3}\cdot\left[\frac{y}{b/2}(-B^2m^2+2B^3m^3)+\frac{4mB}{AB(1+\lambda)}(-6-4mB+5m^2B^2)+4mB+2B^2m^2-2B^3m^3\right]\cdot\sqrt{\frac{1}{mB-1}\left\{-Bm\frac{y^2}{(b/2)^2}+\frac{y}{b/2}\left[-\frac{4mB}{AB(1+\lambda)}+1+2Bm\right]+\frac{4mB}{AB(1+\lambda)}-(mB+1)\right\}}\right)$

– subcase marked as PNNI3

Range of $\frac{y}{b/2}$	Expression for circulation along the span; Mach line from wing apex intersecting trailing edge; unswept trailing edge
$0 \le \frac{y}{b/2} \le \frac{e}{b/2}$	NACA TN 2643
$\frac{e}{b/2} \le \frac{y}{b/2} \le \frac{h}{b/2}$	NACA TN 2643
$\frac{h}{b/2} \le \frac{y}{b/2} \le 1$	Based on the postulate no. 4. equation for distribution of circulation in this zone is the same as equation for distribution of circulation in the third zone of the wing marked as subcase PNNI4

Diagrams for the distribution of circulation

In this section diagrams for the distribution of circulation are given, for the wings for which modification of the equations have been conducted.

Distribution of circulation resulting from a constant rate of roll

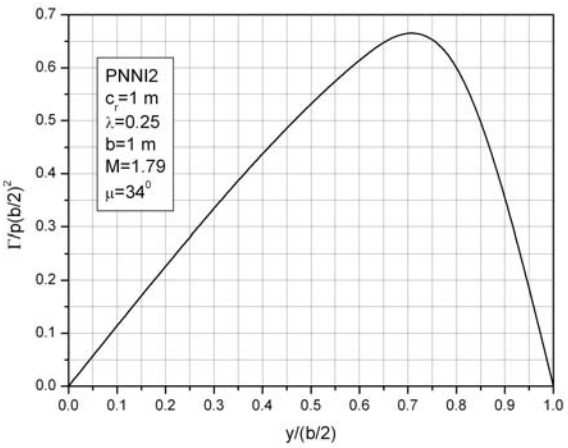


Diagram 7. Distribution of circulation resulting from a constant rate of roll, for subcase marked as PNNI2, modified equations

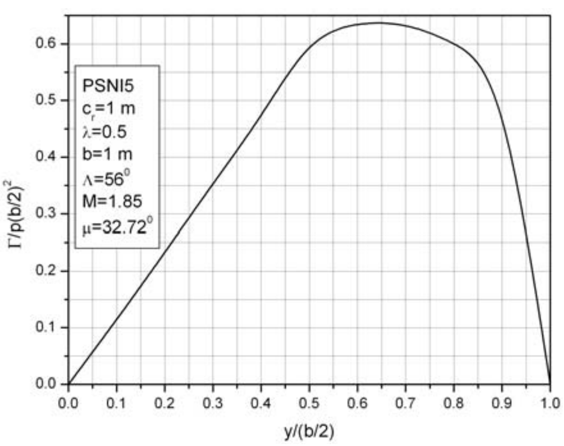


Diagram 8. Distribution of circulation resulting from a constant rate of roll, for subcase marked as PSNI5, modified equations

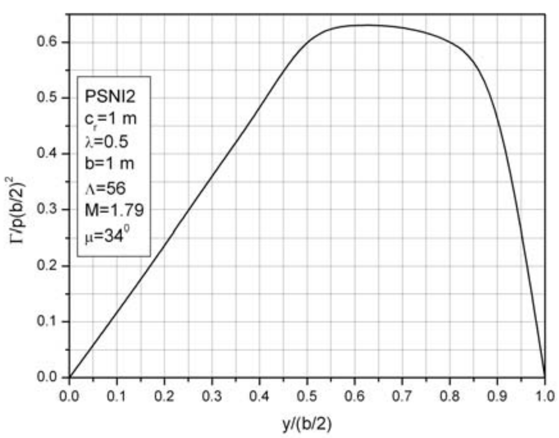


Diagram 9. Distribution of circulation resulting from a constant rate of roll, for subcase marked as PSNI2, modified equations

Distribution of circulation resulting from a constant rate of pitch

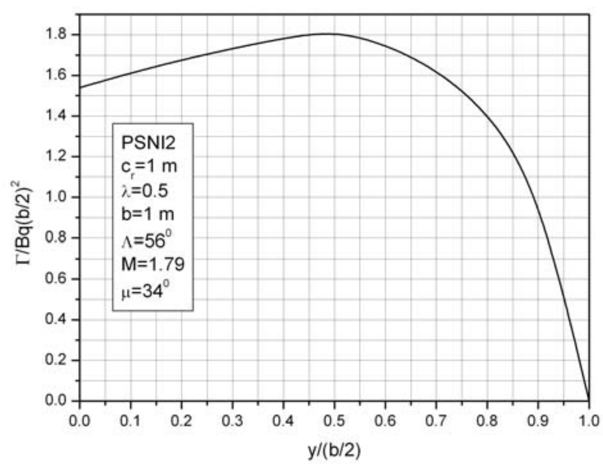


Diagram 10. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI2, modified equations

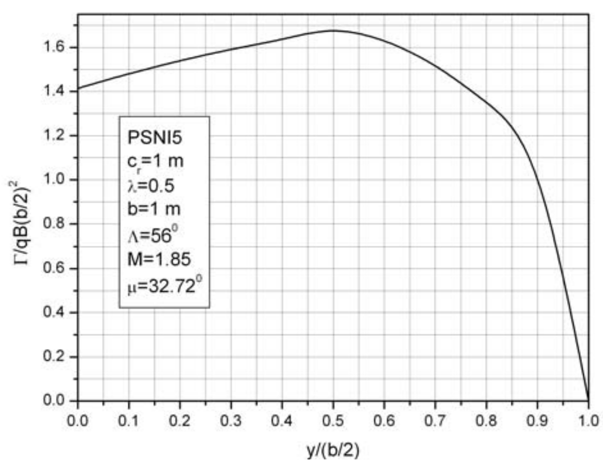


Diagram 11. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI5, modified equations

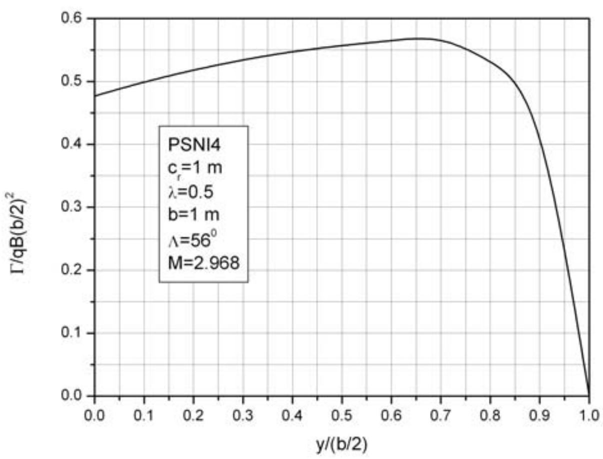


Diagram 12. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI4, modified equations

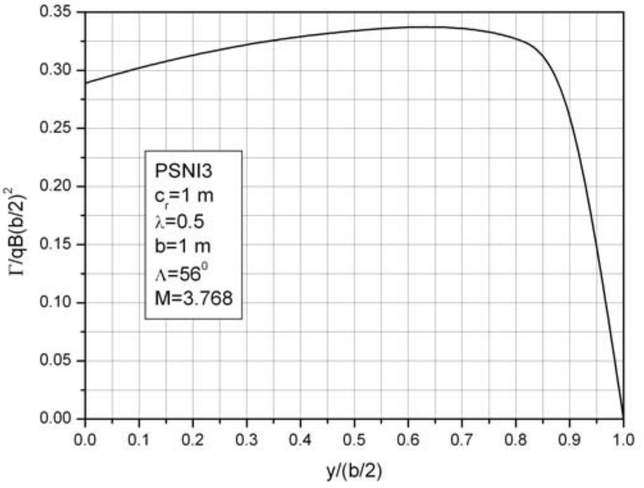


Diagram 13. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PSNI3, modified equations

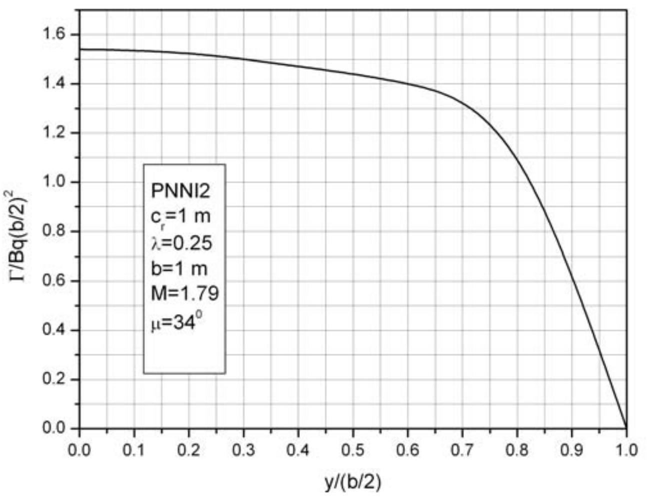


Diagram 14. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PNNI2, modified equations

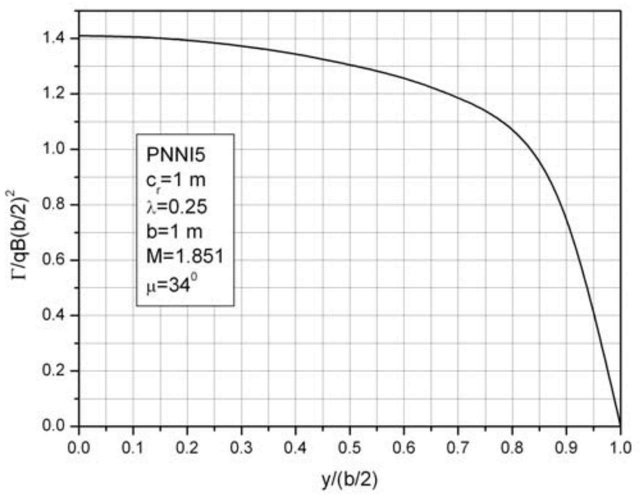


Diagram 15. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PNNI5, modified equations

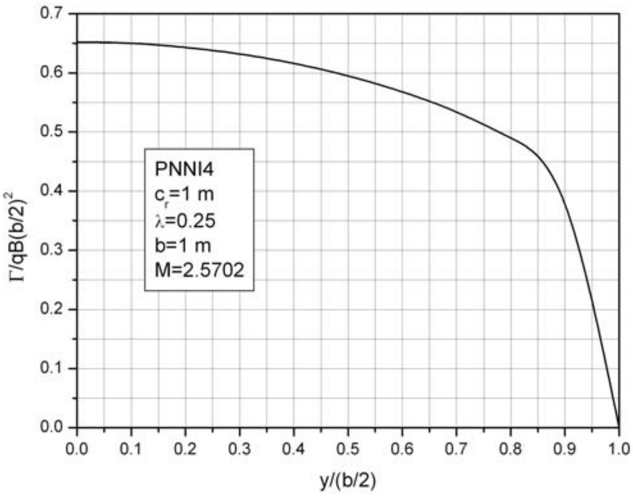


Diagram 16. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PNNI4, modified equations

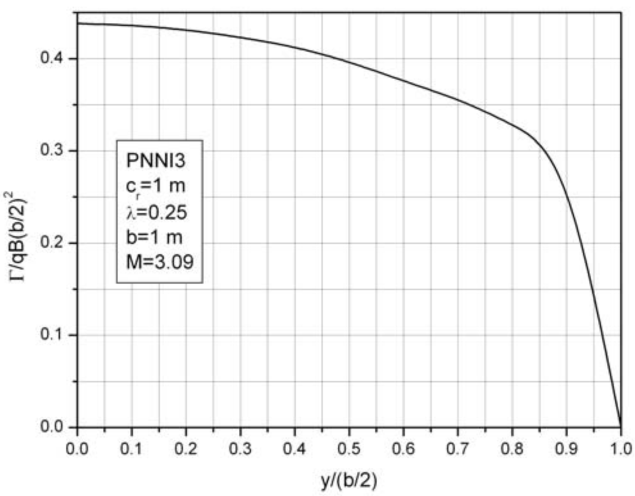


Diagram 17. Distribution of circulation resulting from a constant rate of pitch, for subcase marked as PNNI3, modified equations

Conclusion

In the frame of this work review of modified equations for distribution of circulation have been presented. Basic equations, some of which have been modified, are the equations presented in NACA TN 2643 [1] paper, based on which software G.R.O.M. [2], [3] was developed. This software is used for calculation of the aerodynamic derivatives values - 1. the lift force in respect to angle of attack; 2. drag force in respect to angle of attack on the square, as static derivatives; 3. roll-dumping moment in respect to rate of roll; 4. pitch-dumping moment in respect to rate of pitch, as dynamic derivatives. While checking presented equations it was established that certain equations have discontinuities, thus violating the basic principle of functions continuity.

This caused the necessity for equations modifications in order to neutralize discontinuities.

Methods and ways of corrections are given in this paper, along with table reviews of modified equations, and diagrams for the distribution of circulation obtained through the use of modified equations.

The quality of corrections is checked through the sets of conducted testing [2], [3].

References

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[2] NINKOVIĆ,D.: *Mathematical model of the wing in supersonic flow,* Faculty of Mechanical Engineering, MSc thesis, Belgrade, 2004

[3] NINKOVIĆ,D.: P: *G.R.O.M. software for determination of the aerodynamic derivatives values for isolated wing in supersonic flow*, Scientific technical review Belgrade, 2004, Vol.LIV, No.2.

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Modifikacija jednačina za cirkulaciju poremećajnog potencijala brzine za krilo sa supersoničnom napadnom i izlaznom ivicom u supersoničnoj struji datih u radu NACA TN 2643

U okviru rada NACA TN 2643 [1] date su jednačine za određivanje poremećajnog potencijala brzine pri konstantnom napadnom uglu, konstantnoj brzini valjanja i konstantnoj brzini propinjanja za izolovano krilo u supersoničnoj struji.
Na osnovu ovih jednačina razvijen je softverski program G.R.O.M. [2], [3] za određivanje vrednosti statičkih i dinamičkih aerodinamičkih derivativa
Prilikom provere datih jednačina pojavile su se greške koje su uzrokovale prekidnost funkcija za cirkulaciju poremećajnog potencijala brzine u nekim zonama odnosnog krila, čime je narušen osnovni zahtev da ove funkcije budu neprekidne, što je uslovilo potrebu korekcije jednačina.
Korigovane jednačine date su u tabelarnom prikazu, sa oznakom zone u kojoj je korekcija izvršena, kao i raspodela cirkulacije u obliku dijagrama [2], [3].

Ključne reči: **aerodinamika krila, supersonično strujanje, aerodinamički derivativi, potencijal brzine, softver**

Modification des équations de circulation du potentiel perturbant de vitesse pour l'aile à bord de sortie d'attaque supersonique présenté dans le travail NACA TN 2643

Dans le cadre du travail NACA TN 2643 [1] sont présentées les équations servant pour la détermination du potentiel perturbant de vitesse avec l'angle d'attaque constant où la vites-se de roulage et la vitesse de tangage sont constantes pour une aile isolée dans le courant d'air supersonique. Sur la base de ces équations un programme logiciel G.R.O.M. [2], [3] a été développé pour déterminer les valeurs dérivatives aérodynamiques et statiques.Pendant la vérification des équations données certaines erreurs se sont manifestées. Ces erreurs ont provoqué la discontinuité des fonctions de circulation du potentiel perturbant de vitesse dans certaines zones de l'aile traitée, ce qui a bouleversé la condition principale de la conti-nuité de ces fonctions. Cela a exigé une correction des équations. Les équations corrigées sont présentées dans les tableaux, avec les zones de corrections effectuées ainsi que la distribution de circulation en forme de diagrammes.

Mots clés: **ailes aérodynamiques, courant supersonique, dérivés aérodynamiques, potentiel de vitesse, logiciel**

Модификация уравнений для циркуляции поврежденого потенциала скорости для сверхзвуковой передней и задней кромки крыла в сверхзвуковом потоке, которые даны в работе NACA TN 2643

В рамках работы NACA TN 2643 [1] назначены уравнения для определения поврежденного потенциала скорости при постоянном угле атаки, при постоянной скорости крена и при постоянной скорости тангажа для изолированного крыла в сверхзвуковом потоке.
На основании этих уравнений развилось программное обеспечение Г.Р.О.М. [2], [3] для определения величины статических и динамических аэродинамических дериватив.
При проверке даных уравнений явились ошибки, причиной которых появилось прерывание функций для циркуляции поврежденного потенциала скорости в некоторых зонах относительного крыла, чем нарушено основное требование чтобы эти функции были беспрерывными, а это обусловило надобность коррекции уравнений.
Корректированные уравнения даны в таблицах, с обозначением зоны, в которой сделана коррекция, как и распределение циркуляции в виде диаграммы [2], [3].

Ключевые слова: аэродинамика крыла, сверхзвуковой поток, аэродинамические дериативы, потенциал скорости, программное обеспечение