UDK: 519.87:623.4:629.76
COSATI: 19-01

Mathematical Modeling of Motion of the Clock Safety and Arming Devices

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A theoretical analysis was conducted and the mathematical model of the motion of safety and arming device consisting of the rotor (drive gear), two pairs of involute gears and pallet was developed. Mathematical model, developed on the basis of the equations of the dynamics of rigid material system and impact mechanics, includes differential equations of motion for coupled and free motion, as well as the phase of impact.

Key words: **fuse, arming device, clock mechanism, safety device, mathematical modeling, equations for motion, differential equations, artillery projectile.**

Denotations and abbreviations

A_1 to A_{59} – coefficients depending on geometrical and kinematical parameters of all mechanism components
e_r – restitution coefficient
F_{12} – normal contact force between rotor gear and smaller gear on pinion no.2
F_{23} – normal contact force between gear no.2 and smaller gear on pinion no.3
I_1 – moment of inertia of rotor,
I_2 – moment of inertia of gears and pinion no.2,
I_3 – moment of inertia of escape wheel and pinion no.3
I_p – moment of inertia of pallet
m_1 – mass of rotor
m_2 – mass of gear and pinion no.2
m_3 – mass of escape wheel and pinion no.3
m_p – mass of pallet
$\boldsymbol{n}_n$, $\boldsymbol{n}_t$ – ort-vectors along normal and tangent
P_n – normal contact force between escape wheel and pallet
R_1 – distance of rotor pivot from spin axis
R_4 – distance of pivot of pallet from spin axis
R_{b1} – radius of the basic circle of the rotor
R_{b2} – radius of the basic circle of the gear no.2
r_{b2} – radius of pinion no.2
r_{b3} – radius of escape wheel pinion
r_{c1} – distance from pivot of the rotor to its center of mass
r_{cp} pallet eccentricity
s_1, s_2, s_4 and s_5 – signum of function (to determine the direction of the friction forces)
T_2 – D'Alanbert force on input wheel
T_3 – D'Alambert on escape wheel
$\boldsymbol{V}_{TNf}$, $\boldsymbol{V}_{SNf}$ – normal components of velocity on contact points after impact
a_{en} – inclination angle of entrance pallet working surface
a_{ex} – inclination angle of exit pallet working surface
γ_p' – angle between the positive part of x'-axis and vector $\boldsymbol{n}_4$
μ – coefficient of friction on contact surfaces of all pears of gears (tooth-to-tooth contract) and in bearings of its pinions
μ_1 – coefficient of friction on contact surface of pallet and escape wheel and in bearings of its pinions
ρ_P – radius of pallet pinion (pallet pivot)
ρ_1 – radius of rotor pinion (rotor pivot)
$\phi, \dot{\phi}, \ddot{\phi}$ – rotation angle, angular velocity, and angular acceleration of escape wheel (instantaneous values)
ϕ_{1C} – rotor angle in start position
ϕ_T – cumulative angle of escape wheel rotation
$\psi, \dot{\psi}$ rotation angle and angular velocity of pallet
Ψ_C eccentricity angle of pallet
ω spin rate of projectile, i.e. of fuze
SAD - safety and arming device

System of coordinates

CXYZ - absolute (immobile) coordinate system,
Ox'y'z' relative (mobile) coordinate system,
$O_i n_i t_i z_i$ local (bonded) coordinate system, for all elements of safety and arming device ($i = 1$ to 4)

All others denotations and abbreviations unmentioned in the review given above will be explained in the text.

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Introduction

THE coupled cogged pear consisting of the pallet and escape wheel is the most important part of the clock mechanism that is the core of functioning of all types of mechanical watches. In this case, the elastic force of the compressed spring or gravitation force of the bonded weights support driving (propulsion) and regular functioning of the clock mechanism.

Furthermore, the clock mechanism is being used frequently in the production of the safety and arming devices for all modern types of fuzes for artillery projectiles because of its precision, physical stability, and above all its reliability. In this case, the axial forces of inertia or centrifugal forces of bonded mobile masses drive the mechanism. The optimum values of the construction parameters of this very responsible component of artillery projectiles and relatively complex mechanism is not possible to determine without an appropriate method.

The last given aspect served to define the aim of this theoretical paper, exclusively aimed at corresponding mathematical modeling of the clock mechanism function, that is at the same time the first step to establish the method and tools for computer simulation and optimization of the design and functional parameters of the safety and arming device.

The mathematical modeling of the safety and arming device is based on main dynamics equations of motion for the rigid material system and equations of the impact mechanics. The procedure is it self a rather complicated and long one [1-4]. For that reason only the final equations for motion of the mechanism elements along with certain explorations were shown in this paper.

Description of the safety and arming device

The safety and arming device, i.e. SAD (Fig.1), as a subsystem for safety and arming is a special part of artillery fuze consisting of the mechanism that makes it breaks mechanically the explosive train in the fuze and absolutely safe and secure during the storage, transportation, and handling as well as in the initial phase of the projectile launching. The mechanism makes it possible to reinstate the initial explosive train (arming) only after the projectile has crossed the safety distance from the muzzle. The requirement for muzzle safety zone is a minimum of 400 calibers.

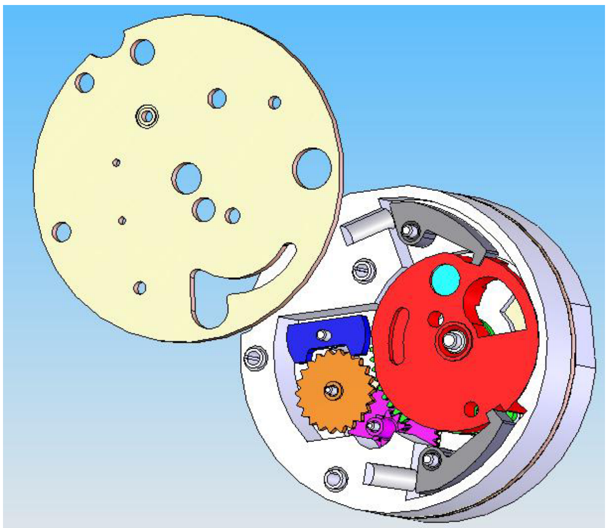

Figure 1. Three-dimensional view of the housing structure with safety and arming mechanism

The fuze subsystem SAD is a special construction intended for performing safety and arming functions, and considering its universal conception, this device is assigned to complete artillery ammunition of hole caliber from 76 mm to 203 mm. It consists of the rotor (driven gear), a two pears of involute gears, and pallet (Fig.2). The mechanism is estimated to function correctly with in a large diapason of loads caused by linear acceleration from 20.000 $\mathrm{m/s^2}$ to 250.000 $\mathrm{m/s^2}$.

Any activation of the booster charge in uncontrolled conditions of accidental inertial loadings that can occur during transportation or handling, for example in case of projectile falling from larger height on to the hard floor, is not possible due to breaking of the explosive initial train of the mechanism. The break is realised physically through removal of one of the initial components, most frequently by placing the most sensitive element in the explosive initial train (high sensitive initial and detonating cap) on the peripheral part of the rotor that is turned round about symmetry axis for some angle, so-called rotor working angle.

The rotor is locked in this safety position with one inertial detent and two centrifugal detents exposed to the action of the springs. During projectile movement through the full bore of the gun-barrel, firstly the inertial force due to axial acceleration compresses the spring of inertial detent, and then, when the projectile achieves some rate of rotation the centrifugal force overtakes a spring resistance of the centrifugal detents, and finally releases, i.e. unlocks the rotor from safety position.

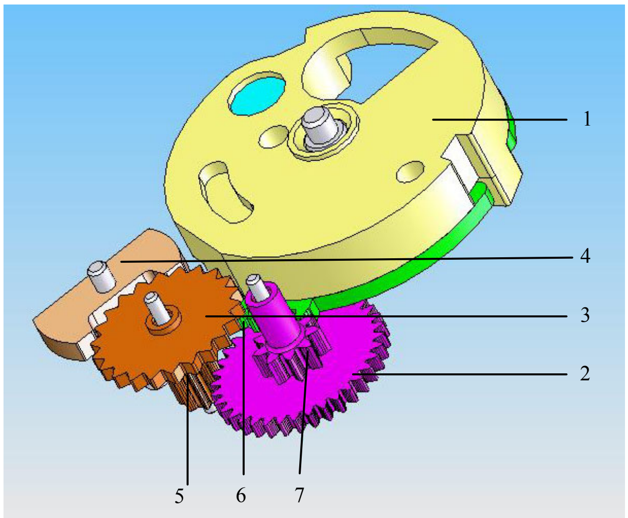

Figure 2. Three-dimensional view and components of the train of coupled gears in the clock mechanism of SAD (1.- Rotor with driving gear no. 1; 2.- Input wheel (pinion no. 2 with gears set no. 2, i.e. pallet); 3.- Escape wheel (pinion no. 3 with gears set no. 3); 4.- Pallet; 5.- Smaller gear in gears set no. 3; 6.- Cogged segment (driven gear); 7.- Smaller gear in gears set no. 2)

The spine axis and center of the mass of the rotor do not coincide with the spine axis of the projectile. Therefore, due to the centrifugal force during rotation of the projectile around symmetry axis, the spin movement of the rotor and the force on the rotor that drives the coupled components of the SAD's clock mechanism are supplied simultaneously.

In other words, when unlocked rotor starts to move, the centrifugal force of its proper mass initializes spin moment about its symmetry axis which transmits to the clock mechanism over the gear segment on the rotor. At the same time, the clock mechanism regulates the spin motion of the rotor, i.e. the arriving delay of initial and detonating cap on the place to reinstate explosive initial train of the fuze.

For the mathematical model of the compound movement of the mechanism has been derived each phase of three main motions:

- coupled motion,
- free motion, and
- impact.

Each phase of motion mentioned above will be discussed individually.

Coupled motion

The coupled motion implies the motion phase when all components of mechanism are connected and it includes time interval from the moment when the tip of the escape wheel (driven by rotor and two pears of involute gears) get in continual contact with either entrance or exit active (working) surface of the pallet (Fig.3) to the moment when the contact is interrupted.

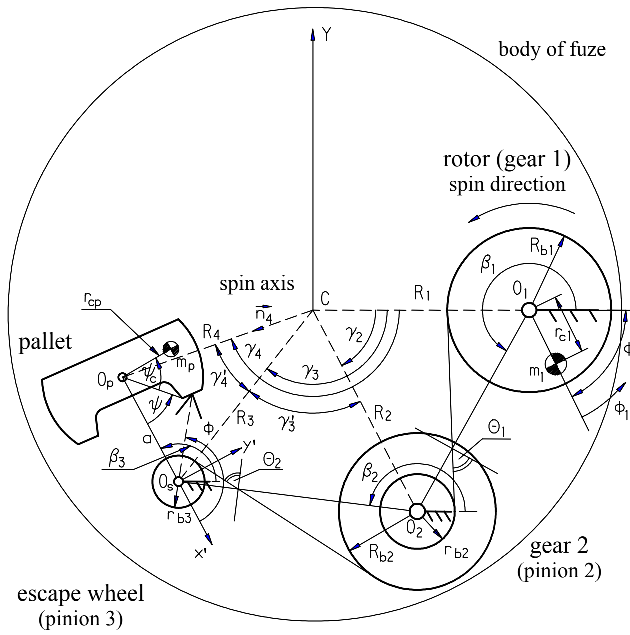

Figure 3. Scheme of safety and arming device with dimensional and main kinematics parameters of mechanism components motion

The differential equations for the entrance as well as the exit coupled motion, which differ from each other to some extent, are derived in reference to the absolute coordinate system and expressed depending on the variable ϕ (rotation angle of escape wheel). The equations are obtained by combining the solutions for the Newtonian force (the Newton's Low) and momentum expressions for each mechanism components particularly [1-4].

Dynamics of pallet and escape wheel in entrance- coupled motion

a) Analysis of forces on pallet

A detailed schematic sketch of the forces that determine the motion of pallet for entrance-coupled motion is shown in Fig.4.

The final form of the equation for acceleration of the center of mass of pallet in the absolute coordinate system obtained on the basis of the Newton's Law is

$$\begin{aligned} & P_n\boldsymbol{n}_n + \mu_1 s_4 P_n \boldsymbol{n}_t + F'_{xp}\boldsymbol{i}' - \mu_1 s_5 F'_{yp}\boldsymbol{i}' + F'_{yp} j' + \mu_1 s_5 F'_{xp}\boldsymbol{j}' = \\ & = m_p\left\{-\omega^2 R_4 \boldsymbol{n}_4 - (\omega+\dot{\psi})^2 r_{cp}\left[\cos(\psi+\psi_c)\boldsymbol{i}' + \right.\right. \\ & \left.\left. + \sin(\psi+\psi_c)\boldsymbol{j}'\right] + \ddot{\psi}\, r_{cp}\left[-\sin(\psi+\psi_c)\boldsymbol{i}' + \cos(\psi+\psi_c)\boldsymbol{j}'\right]\right\} \end{aligned} \qquad (1)$$

Left side of eq.(1) represents a sum of all external forces that affects the pallet, when the complex factor next to the mass of pallet, on its right side, represents absolute acceleration consisting of vector sum of traveled and relative acceleration.

The final moment equation of the pallet for pivot point O_p is given regarding its acceleration

$$\boldsymbol{M}_{Op} = -\ddot{\boldsymbol{r}}_{Op} \times m_p \boldsymbol{r}_{cp} + \dot{\boldsymbol{H}}_{Op} \qquad (2)$$

where:

$\boldsymbol{M}_{Op}$ - sum of all external moments in reference to the pivot point O_p on pallet pinion,

$\ddot{\boldsymbol{r}}_{Op}$ - absolute acceleration of pivot point O_p,

$\dot{\boldsymbol{H}}_{Op}$ - time derivation of angular kinetics momentum of the pallet in reference to the pivot point O_p.

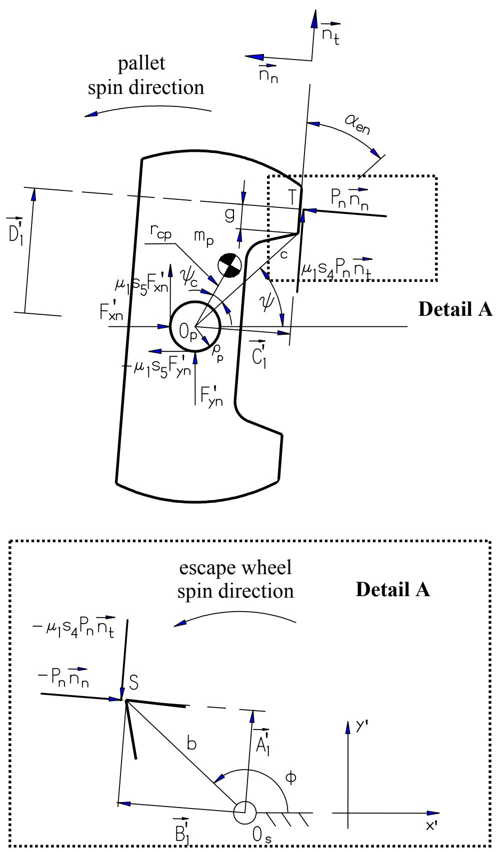

Figure 4. Schematic sketch of the forces that determine the motion of pallet for entrance-coupled motion

The momentum eq. (2) expressed in scalar form is

$$M_{Op} = D_1' P_n - C_1' \mu_1 s_4 P_n - \rho_p \mu_1 s_5 \left(F_{xp} + F_{yp}\right) \qquad (3)$$

Transferring the vector equation (1) to $Ox'y'$ -coordinate system and resolving the obtained system of equations terms for components of the force on pivot of the pallet are determined. Substituting the terms for components of the force on pivot of the pallet in eq.(2), the equation for contact force P_n becomes

$$P_n = \frac{I_{PR}\ddot{\psi} + A_{21}\dot{\psi}^2 + 2\dfrac{\omega^2}{|\omega|}A_{20}\dot{\psi} + \omega^2 A_{19}}{A_{18}} + \\ + \frac{-m_p r_{cp} R_4 \omega^2 \sin\left(\gamma_p' - \psi - \psi_c\right)}{A_{18}} \tag{4}$$

Coefficients A_{18} to A_{21} in eq.(4) depend exclusively on the geometrical parameters of the mechanism, and the effective momentum of inertia I_{PR} would have the following values

$$I_{PR} = I_P + A_{22} \tag{5}$$

where the same signs precede $\dot{\psi}$ and $\ddot{\psi}$, and

$$I_{PR} = I_P - A_{22} \tag{6}$$

where the opposite signs precede $\dot{\psi}$ and $\ddot{\psi}$.

Eq.(4), which will be used later to establish a differential equation of pallet motion in the phase of coupled motion, must be written as a function depending on the angular velocity and angular acceleration of escape wheel: $P_n = f\left(\dot{\phi},\ddot{\phi}\right)$, since this form is more favorable for performing a kinematics analysis and determining the required parameters of motion of gear mechanism components.

On the basis of equations that represent the relation between angular velocities and angular acceleration of pallet and escape wheel, as well as eq.(4), the final form of expression for contact force becomes

$$P_n = \frac{1}{A_{18}}\left[I_{PR}U\ddot{\phi} + \left(A_{21}U^2 + I_{PR}V\right)\dot{\phi}^2 + 2\frac{\omega^2}{|\omega|}A_{20}U\dot{\phi} + \omega^2 A_{19}\right] + \\ + \frac{1}{A_{18}}\left[-m_p r_{cp} R_4 \omega^2 \sin\left(\gamma_p' - \psi + \psi_c\right)\right] \tag{7}$$

b) Analysis of forces on the escape wheel

A detailed schematic sketch of the forces that determine the motion of the escape wheel and pinion no.3 for entrance-coupled motion is shown in Fig.5.

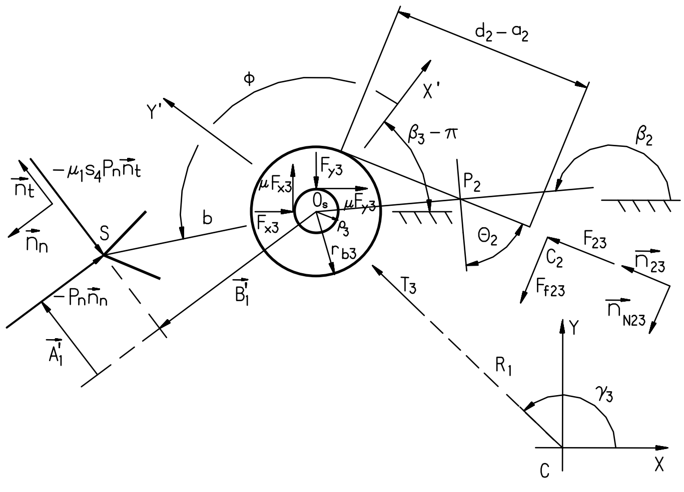

Figure 5. Schematic sketch of the forces that determine the motion of the escape wheel and pinion no.3 for entrance-coupled motion

The final form of the main equation of dynamics for the escape wheel derived from balance condition of forces is

$$-P_n\boldsymbol{n}_n - \mu_1 s_4 P_n \boldsymbol{n}_t + F_{23}\boldsymbol{n}_{23} + \mu s_2 F_{23}\boldsymbol{n}_{N23} + \boldsymbol{T}_3 + F_{X3}\boldsymbol{i} + \\ + \mu F_{Y3}\boldsymbol{i} + \mu F_{X3}\boldsymbol{j} - F_{y3}\boldsymbol{j} = 0 \tag{8}$$

Eq.(8) includes expression for D'Alembert force in the form

$$\boldsymbol{T}_3 = m_3 R_3 \omega^2 \left(\cos\gamma_3 \boldsymbol{i} + \sin\gamma_3 \boldsymbol{j}\right) \tag{9}$$

The momentum equation for the escape wheel is

$$A_1'\boldsymbol{n}_t \times \left(-P_n\right)\boldsymbol{n}_n + B_1'\boldsymbol{n}_n \times \left(-\mu_1 s_4 P_n\right)\boldsymbol{n}_t - \\ -\mu\rho_3\left(F_{x3} + F_{y3}\right)\boldsymbol{k} + r_{b3}F_{23}\boldsymbol{k} - \mu s_2\left(d_2 - a_2\right)F_{23}\boldsymbol{k} = I_3\ddot{\phi}\boldsymbol{k} \tag{10}$$

which in scalar form becomes:

$$-P_n\left(A_1' - B_1'\mu_1 s_4\right) - \mu\rho_3\left(F_{x3} + F_{y3}\right) + \\ + r_{b3}F_{23} - \mu s_2\left(d_2 - a_2\right)F_{23} = I_3\ddot{\phi} \tag{11}$$

Transferring the vector equation (8) to *CXY*-coordinates system and resolving the given system of equations the terms for components of the force on pivot of the escape wheel are obtained. Substituting the terms for components of the force on pivot in momentum eq.(11), the equation for contact force P_n as a function of the angular velocity and angular acceleration of escape wheel, $P_n = f\left(\dot{\phi},\ddot{\phi}\right)$, becomes

$$P_n = \frac{-I_3\ddot{\phi} + F_{23}A_{15} - T_3 A_{16}}{A_{17}} \tag{12}$$

Equalizing the expressions (7) and (12), given as functions of the same parameters, a unique differential equation of entrance-coupled motion of the escape wheel and pallet is obtained:

$$\left[I_3 A_{18} + I_{PR}A_{17}U\right]\ddot{\phi} + \left[I_{PR}A_{17}V + A_{17}A_{21}U^2\right]\dot{\phi}^2 + \\ + 2\frac{\omega^2}{|\omega|}A_{17}A_{20}U\dot{\phi} = A_{15}A_{18}F_{23} - A_{16}A_{18}T_3 - A_{17}\omega^2 \cdot \\ \cdot\left[A_{19} - m_p r_{cp} R_4 \sin\left(\gamma_p' - \psi - \psi_c\right)\right] \tag{13}$$

Coupled motion of the pallet and escape wheel

The schematic sketch of the forces that determine the motion of pallet for exit-coupled motion is shown in Fig.6.

Comparing the diagrams of forces on Figures 4 and 5 with the relevant diagram on Fig.6, it can be concluded that the intensity of the contact force remains unchanged and only takes an opposite sign

$$\boldsymbol{P}_n = -P_n\boldsymbol{n}_n \tag{14}$$

The change of the sign reflects itself on the equation of forces as well as on the momentum equation. From both equations mentioned, (force and momentum), the expression for contact force P_n is obtained by analogy from the preliminary consideration

$$P_n = \frac{I_{PR}\ddot{\psi} + A_{21}\dot{\psi}^2 + 2\dfrac{\omega^2}{|\omega|}A_{20}\dot{\psi} + \omega^2 A_{19}}{AA_{18}} + \\ + \frac{-m_p r_{cp} R_4 \omega^2 \sin\left(\gamma_p' - \psi - \psi_c\right)}{AA_{18}} \tag{15}$$

The final form of eq.(15) for contact force P_n depending on the angular velocity and angular acceleration of the escape wheel, $P_n = f\left(\dot{\phi},\ddot{\phi}\right)$ is

$$P_n = \frac{1}{AA_{18}}\left[I_{PR}U\ddot{\phi} + \left(A_{21}U^2 + I_{PR}V\right)\dot{\phi}^2 + 2\frac{\omega^2}{|\omega|}A_{20}U\dot{\phi} \right] + \frac{1}{AA_{18}}\left[\omega^2 A_{19} - m_p r_{cp} R_4 \omega^2 \sin\left(\gamma_p^{'} - \psi + \psi_c\right)\right] \tag{16}$$

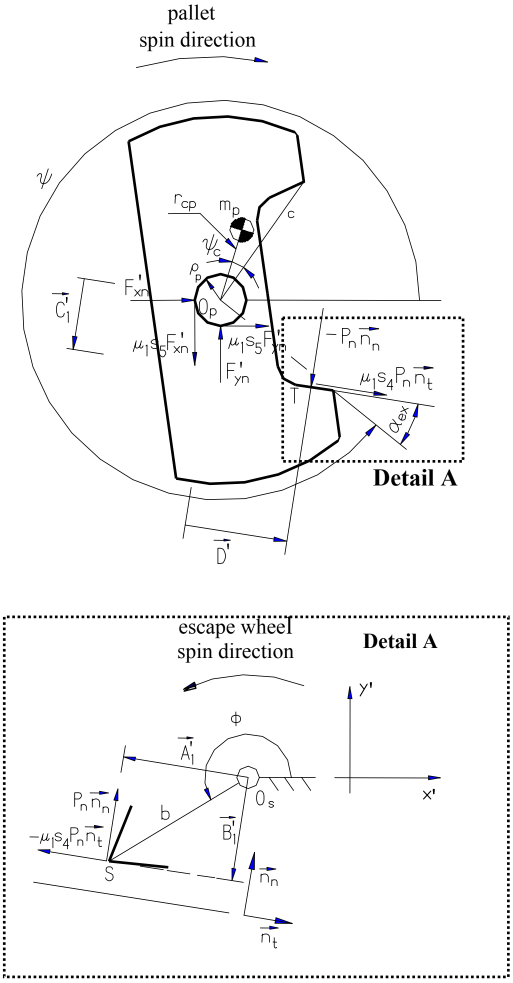

Figure 6. Schematic sketch of the forces that determine the motion of pallet for exit-coupled motion

a) Analysis of the escape wheel motion

The value of contact force P_n on the escape wheel for exit-coupled motion carried out on the basis of the new conditions and analogically to the previous analysis of the exit-coupled motion, is given through equation

$$P_n = \frac{-I_3\ddot{\phi} + F_{23}A_{15} - T_3 A_{16}}{AA_{17}} \tag{17}$$

Equalizing expressions (16) and (17), given as functions of the same parameters, a unique differential equation for exit-coupled motion of the escape wheel depending on the angular velocity and angular acceleration is obtained

$$\begin{aligned}&\left[I_3 AA_{18} + I_{PR}AA_{17}U\right]\ddot{\phi} + \left[I_{PR}AA_{17}V + AA_{17}A_{21}U^2\right]\dot{\phi}^2 + \\ &+2\frac{\omega^2}{|\omega|}AA_{17}A_{20}U\dot{\phi} = A_{15}AA_{18}F_{23} - A_{16}AA_{18}T_3 - \\ &-AA_{17}\omega^2\left[A_{19} - m_p r_{cp} R_4 \sin\left(\gamma_p' - \psi - \psi_c\right)\right]\end{aligned} \tag{18}$$

b) Dynamics of rotor (driven gear)

A detailed schematic sketch of the forces that determine the motion of rotor that moves in a counterclockwise direction is shown in Fig.7.

To express the motion of rotor depending on the rotation angle of the escape wheel ϕ, the following relations will be used

$$\dot{\phi}_1 = N_{31}\dot{\phi} \quad \text{and} \quad \ddot{\phi}_1 = N_{31}\ddot{\phi}$$

where :

$$N_{31} = \frac{N_{P3} \times N_{P2}}{N_{G2} \times N_{G1}} \tag{19}$$

- N_{G1} number of teeth of rotor (along full perimeter),
- N_{G2} number of teeth of the big gear on pinion no.2,
- N_{P2} number of teeth of the small gear on pinion no.2,
- N_{P3} number of teeth of the big gear on pinion no.3.

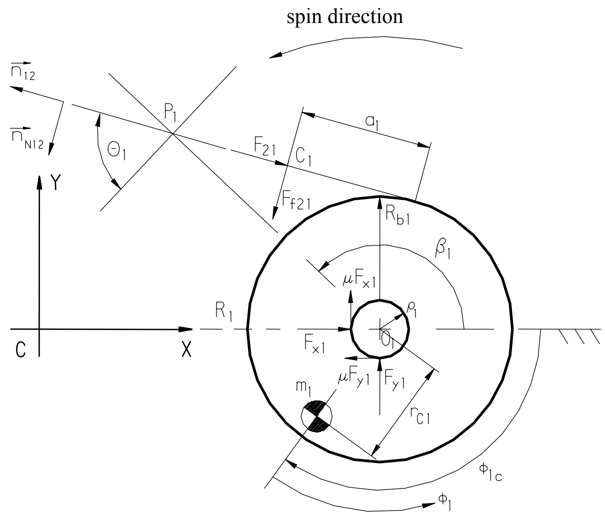

Figure 7. Schematic sketch of the forces that determine the motion of rotor

The rotation angle of rotor $\phi_{1c} + \phi_1$ given as function of the total rotation angle of the escape wheel ϕ_T can be written in the form

$$\phi_{1C} + \phi_1 = \phi_{1C} + N_{31}\phi_T \tag{20}$$

In this case, the final form of the balance equation is

$$-F_{21}\boldsymbol{n}_{12} + \mu s_1 F_{12}\boldsymbol{n}_{N12} + F_{x1}\boldsymbol{i} - \mu F_{y1}\boldsymbol{i} + F_{y1}\boldsymbol{j} + \mu F_{x1}\boldsymbol{j} = \\ = m_1\left\{\begin{aligned}&-\omega^2 R_1\boldsymbol{i} - \left(\omega + N_{31}\dot{\phi}\right)^2 r_{c1}\left[\cos\left(\phi_{1c} + N_{31}\phi_T\right)\boldsymbol{i} + \right.\\ &\left.+\sin\left(\phi_{1c} + N_{31}\phi_T\right)\boldsymbol{j}\right] + \\ &+N_{31}\ddot{\phi} r_{c1}\left[-\sin\left(\phi_{1c} + N_{31}\phi_T\right)\boldsymbol{i} + \right.\\ &\left.+\cos\left(\phi_{1c} + N_{31}\phi_T\right)\boldsymbol{j}\right]\end{aligned}\right\} \tag{21}$$

Finally, the momentum equation in the scalar form is

$$\begin{aligned}&-F_{12}R_{b1} + \mu s_1 a_1 F_{12} - \mu\rho_1\left(F_{x1} + F_{y1}\right) = \\ &= m_1\omega^2 R_1 r_{c1}\sin\left(\phi_{1c} + N_{31}\phi_T\right) + I_1 N_{31}\ddot{\phi}\end{aligned} \tag{22}$$

Transferring the vector equation (21) to CXY -coordinate system and resolving the obtained system of equations produce the terms for components of the force on the pivot of rotor. On the basis of the determined value of the force on pivot and eq.(22), the force F_{12} depending on the angular velocity and angular acceleration of escape wheel, $F_{12} = f\left(\dot{\phi}, \ddot{\phi}\right)$ is derived

$$F_{12} = \frac{-A_{32} - A_{33}N_{31}\dot{\phi} - A_{34}\left(N_{31}\dot{\phi}\right)^2}{A_{31}} + \\ + \frac{-A_{35}\sin\left(\phi_{1c} + N_{31}\phi_T\right) - I_{1R}N_{31}\ddot{\phi}}{A_{31}} \tag{23}$$

The eq.(23) includes the term for effective moment of inertia I_{1R}, that takes the values as follows

$$I_{1R} = I_1 + |\mu|\rho_1\left(A_{26} + A_{30}\right) \tag{24}$$

where the same sign precedes $\dot{\phi}$ and $\ddot{\phi}$, and

$$I_{1R} = I_1 - |\mu|\rho_1\left(A_{26} + A_{30}\right) \tag{25}$$

where the opposite signs precede $\dot{\phi}$ and $\ddot{\phi}$.

c) Dynamics of gear and pinion (gears set no.2)

A detailed schematic sketch of the forces that determine the motion of gear and pinion (gears set no. 2) is shown in Fig.8.

The equation of forces that includes the term of D'Alembert force has the form

$$T_2 = m_2 R_2 \omega^2$$

The balance condition of the forces is give in the equation

$$-F_{23}\boldsymbol{n}_{23} - s_2\mu F_{23}\boldsymbol{n}_{N23} + F_{12}\boldsymbol{n}_{12} - \mu s_1 F_{12}\boldsymbol{n}_{N12} + F_{x2}\boldsymbol{i} + \\ + \mu F_{y2}\boldsymbol{i} + F_{y2}\boldsymbol{j} - \mu F_{x2}\boldsymbol{j} + T_2\left(\cos\gamma_2\boldsymbol{i} + \sin\gamma_2\boldsymbol{j}\right) = 0 \tag{26}$$

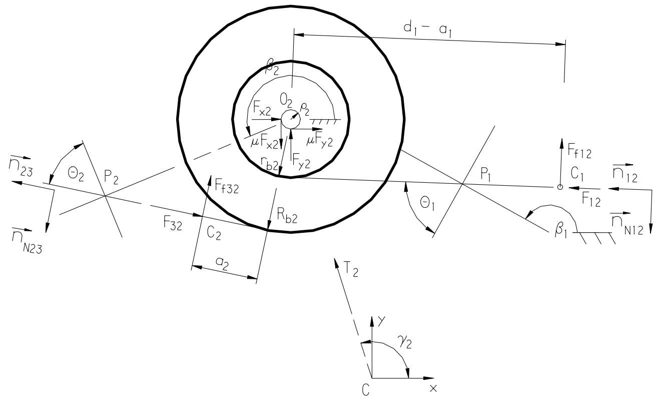

Figure 8. Schematic sketch of the forces that determine the motion of gear and pinion (gears set no.2)

The momentum equation in the scalar form is

$$F_{23}R_{b2} - \mu s_2 F_{23}a_2 - F_{12}r_{b2} + \mu s_1 F_{12}\left(d_1 - a_1\right) + \\ + \mu\rho_2\left(F_{x2} + F_{y2}\right) = I_2 N_{32}\ddot{\phi} \tag{27}$$

Solving the system eqs. (26) and (27) the expression for force F_{23} is obtained

$$F_{23} = \frac{F_{12}A_{42} - T_2 A_{43} + I_2 N_{32}\ddot{\phi}}{A_{44}} \tag{28}$$

Dynamics of system in the phase of coupled motion

To obtain a unique differential equation of the whole system in the phase of coupled motion depending on the rotation angle of the escape wheel ϕ, it is necessary to define an appropriate term for force F_{23} that includes rotor influence, and substitute this-one in eq.(13) for entrance-coupled motion and eq.(18) for exit-coupled motion.

Therefore, it is necessary firstly to substitute the term for force F_{12} from eq.(23) in eq.(28) that gives

$$F_{23} = \frac{1}{A_{44}}\left[\frac{-A_{42}A_{32}}{A_{31}} - T_2 A_{43} - \frac{A_{42}A_{33}}{A_{31}}N_{31}\dot{\phi} - \right. \\ - \frac{A_{42}A_{34}}{A_{31}}N_{31}{}^2\dot{\phi}^2 - \frac{A_{42}A_{35}}{A_{31}}\sin\left(\phi_{1c} + N_{31}\phi_T\right) + \\ \left. + \left(I_2 N_{32} - \frac{A_{42}}{A_{31}}I_{1R}N_{31}\right)\ddot{\phi}\right] \tag{29}$$

Secondly, to obtain the differential equation of system in the phase of coupled entrance-motion depending on the rotation angle of the escape wheel it is necessary to substitute eq.(29) in eq.(13) as follows

$$A_{45}\ddot{\phi} + A_{46}\dot{\phi}^2 + A_{47}\dot{\phi} = \\ = A_{48} - A_{49}\sin\left(\phi_{1c} + N_{31}\phi_T\right) + A_{50}\sin\left(\gamma_p^{'} - \psi - \psi_c\right) \tag{30}$$

Non-homogenous differential eq.(30) has a universal character. In other words, this equation can also be used to calculate the parameters for exit-coupled motion, with the difference that in this case coefficients AA_{17} and AA_8 are used instead of coefficients A_{17} and A_{18}.

Contact forces during coupled motion

The contact force F_{23}, expressed as a function of rotation angle of the escape wheel, is derived on the basis of eq.(29):

$$F_{23} = \frac{1}{A_{44}}\left[\frac{-A_{42}A_{32}}{A_{31}} - T_2 A_{43} - \frac{A_{42}A_{33}}{A_{31}}N_{31}\dot{\phi} - \frac{A_{42}A_{34}}{A_{31}}N_{31}{}^2\dot{\phi}^2 - \right. \\ \left. - \frac{A_{42}A_{35}}{A_{31}}\sin\left(\phi_{1c} + N_{31}\phi_T\right) + \left(I_2 N_{32} - \frac{A_{42}}{A_{31}}I_{1R}N_{31}\right)\ddot{\phi}\right]$$

The contact force F_{12} can be calculated similarly on the basis of eq.(28)

$$F_{12} = \frac{F_{23}A_{44} - T_2 A_{43} - I_2 N_{32}\ddot{\phi}}{A_{42}}$$

The contact force P_n is defined by Eqs. (12) and (17) depending on the actual motion phase for entrance-coupled motion or exit-coupled motion, respectively.

It is important to note that for the entrance-coupled motion given by eq.(12), the coefficient A_{17} becomes AA_{17}. In this case, it is necessary to express the contact force depending on the rotation angle of pallet ψ, then either eq.(4) or eq.(15) will be used in accordance to the actual motion phase

$$P_n = \frac{I_{PR}\ddot{\psi} + A_{21}\dot{\psi}^2 + 2\frac{\omega^2}{|\omega|}A_{20}\dot{\psi} + \omega^2 A_{19}}{A_{18}} + \\ + \frac{-m_p r_{cp} R_4 \omega^2 \sin\left(\gamma_p^{'} - \psi - \psi_c\right)}{A_{18}} \tag{31}$$

In the case of the exit-coupled motion the coefficient A_{18} becomes AA_{18}.

Free motion

In this phase of motion the pallet on one side, and the system consisting of the rotor, gear train, and escape wheel on the other, are moving independently from each other. In this phase the motion of the pallet is given throuth a differential equation depending on the rotation angle of the pallet ψ , and for the whole system depending on the rotation angle of the escape wheel ϕ .

Pallet movement in the phase of free motion

In the phase of free motion there is no contact between the teeth of the escape wheel neither with the entrance nor with the exit active (working) surfac; so the contact force P_n equals zero. By substituting the zero value $P_n = 0$ in eq.(31) a differential equation of the pallet movement in the phase of free motion is obtained

$$A_{51}\ddot{\psi} + A_{21}\dot{\psi}^2 + A_{52}\dot{\psi} = -A_{53} + A_{54}\sin\left(\gamma_p^{'} - \psi - \psi_c\right) \quad (32)$$

where constants from A_{51} to A_{54} are coefficients depending on the kinematics and geometrical parameters of the system components.

Movement of the escape wheel, gears train, and rotor in the phase of free motion

By substituting the zero value $P_n = 0$ in eq.(12) or (17) a differential equation of free phase for the whole system of the clock mechanism without the pallet, is obtained

$$I_3\ddot{\phi} = A_{15}F_{23} - T_3A_{16} \quad (33)$$

The substitution of the terms for contact force F_{23} (eq.(29)) and D'Alambert force T_3 (eq.(9)) in eq.(33) a differential equation has the following form:

$$A_{55}\ddot{\phi} + A_{56}\dot{\phi}^2 + A_{57}\dot{\phi} = A_{58} - A_{59}\sin\left(\phi_{1c} + N_{31}\phi_T\right) \quad (34)$$

If $\ddot{\phi}$ is known, the contact force F_{F23} in the phase of free motion can be solved from eq.(33) in the form

$$F_{F23} = \frac{I_3\ddot{\phi} + T_3A_{16}}{A_{15}} \quad (35)$$

Eq.(27), which was derived from the equations of force and momentum equation for gear and pinion (set no.2), can be modified to enable determining the contact force F_{F12} in the phase of free motion

$$F_{F12} = \frac{F_{F23}A_{44} + T_2A_{43} - I_2N_{32}\ddot{\phi}}{A_{42}} \quad (36)$$

Impact

The mathematical model of impact is based on the classical equations of the impact mechanics and derived in the form of the momentum equation, including the coefficient of restitution. It should be emphasized that the equation of impact is identical for the phase of entrance contact (beginning of entrance coupled motion) and the phase of the exit contact (beginning of exit coupled motion). At the same time the moment arms, i.e. the moment distances A_1' , B_1' , C_1' , and D_1' depending on the angle α_{en} for the phase of entrance contact, i.e. depending on the angle α_{ex} for the phase of exit contact are included into the impact analysis.

Impact at the beginning of entrance coupled motion (entrance impact)

The schematic sketches of the external active forces that determine the motion of pallet, i.e. the escape wheel in the phase of entrance impact, are shown in Fig.9. Mutual normal impulse force P_n between these components of the system appears as only one active impulse on the pallet and escape wheel. Considering its very small intensity the frictional impulse force μP_n will be neglected in the further analysis.

Under the given conditions the final form of the momentum equation of impulse for the pallet is

$$\boldsymbol{J}_p = D_1^{'}\boldsymbol{n}_t \times P_n\boldsymbol{n}_n = P_nD_1^{'}\boldsymbol{k} \quad (37)$$

Analogically, the equation for escape wheel is

$$\boldsymbol{J}_3 = A_1^{'}\boldsymbol{n}_t \times \left(-P_n\right)\boldsymbol{n}_n = -P_nA_1^{'}\boldsymbol{k} \quad (38)$$

Momentum equations in the scalar form, derived from Eqs.(37) and (38) are

$$I_p\left(\dot{\psi}_f - \dot{\psi}_i\right) = P_nD_1^{'} \quad (39)$$

and

$$I_3\left(\dot{\phi}_f - \dot{\phi}_i\right) = -P_nA_1^{'} \quad (40)$$

where I_p and I_3 represent the polar moments of inertia of the pallet and escape wheel. The subscript i (*initial*) indicates the angular velocities before impact, while the subscript f (*final*) indicates the same quantities after the impact. If P_n is eliminated from Eqs.(39) and (40), impulse equation that define the motion of the pallet and escape wheel depending on its property angular velocity is obtained

$$I_pA_1^{'}\dot{\psi}_f + I_3D_1^{'}\dot{\phi}_f = I_pA_1^{'}\dot{\psi}_i + I_3D_1^{'}\dot{\phi}_i \quad (41)$$

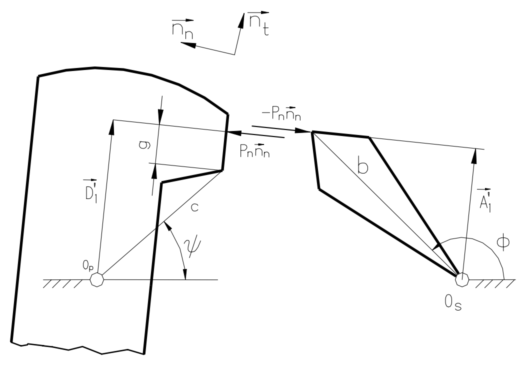

Figure 9. Schematic sketches of the external active forces on the pallet and escape wheel in the phase of entrance impact

Impact at the beginning of exit coupled motion (exit impact)

Schematic sketches of the external active forces that determine the motion of the pallet, i.e. the escape wheel in the phase of exit impact, are shown in Fig.10.

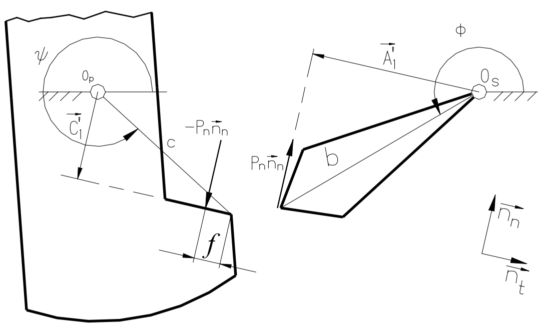

Figure 10. Schematic sketches of the external active forces on the pallet and escape wheel in the phase of exit impact

Using analogy from the above given procedure, the momentum equation of impulse for the pallet in the phase of exit impact was derived

$$\boldsymbol{J}_p = C_1'\boldsymbol{n}_t \times \left(-P_n\right)\boldsymbol{n}_n = -P_n C_1'\boldsymbol{k} \tag{42}$$

The relevant impulse equation for the escape wheel is

$$\boldsymbol{J}_3 = A_1'\boldsymbol{n}_t \times \left(P_n\right)\boldsymbol{n}_n = P_n A_1'\boldsymbol{k} \tag{43}$$

The momentum equations in the scalar form, derived from (42) and (43) are

$$I_p\left(\dot{\psi}_f - \dot{\psi}_i\right) = -P_n C_1' \tag{44}$$

and

$$I_3\left(\dot{\phi}_f - \dot{\phi}_i\right) = P_n A_1' \tag{45}$$

If P_n is eliminated from Eqs.(44) and (45), an impulse equation identical to (41) is obtained; it means that the equations of entrance motion and the exit motion of the pallet and escape wheel in the phase of impact are absolutely similar.

Impact equations for pallet

In order to obtain equations for calculating the post impact angular velocities $\dot{\phi}_f$ and $\dot{\psi}_f$, it is necessary to include the term for coefficient of restitution e_r in eq.(41)

$$e_r = \frac{V_{TNf} - V_{SNf}}{V_{TNi} - V_{SNi}} = \frac{\dot{\psi}_f D_1' - \dot{\phi}_f A_1'}{\dot{\psi}_i D_1' - \dot{\phi}_i A_1'} \tag{46}$$

The velocities V_{TNf} and V_{SNf} represent the normal component of velocities for contact points after the impact.

From the previous the following equations were obtained

$$\dot{\phi}_f = \frac{\dot{\phi}_i\left(I_3 D_1' + e_r I_p A_1'\right) + \dot{\psi}_i I_p A_1'\left(1 - e_r\right)}{I_p A_1'^2 + I_3 D_1'^2} \tag{47}$$

and

$$\dot{\psi}_f = \frac{\dot{\phi}_f A_1' + e_r\left(\dot{\psi}_i D_1' - \dot{\phi}_i A_1'\right)}{D_1'} \tag{48}$$

Thay define angular velocities of the pallet and escape wheel.

Finally recapitulating of the results obtained on the basis of the performed dynamics analysis of the clock mechanism function the following conclusions can be drown:

- it is evident, the procedure of mathematical modeling of the clock mechanism is relatively complex and extensive and therefore only the main statements of interpretation and final results of the performed analysis were given in this paper,
- mathematical model of the clock mechanism function in function of the rotation angle of the pallet and escape wheel was derived, since these two parameters enable determining indirectly the muzzle safety zone, using the arming time (rotation delay time of the cap pivot), and
- performed theoretical analysis has been used to verify the procedures given in [3] as well as to constitute a proper software solution for the computer simulation and optimization of the function of safety arming device.

Conclusion

Modern constructions of fuzes for artillery projectiles are based on the use of mechanical safety and arming devices that successfully integrate and reliably perform the functions of safety and arming.

The mathematical model of the mechanism motion for all phases (coupled motion, free motion, and impact) was carried out as the first step to establish the tools for computer simulation and optimization of the function of safety and arming device. The mathematical model consists of differential equations of motion that are expressed depending on kinematics, geometrical, and physical parameters of each component in the train of the mentioned mechanism.

It is necessary to emphasize that due to the established mathematical model as and the use of computer, the next step will be creating software for solving the system of differential equations of mechanism motion. Finally, using the software solutions with variation of the significant parameters will make it possibile to theoretically define the optimum parameters of the safety arming device for the assigned design conditions. And the main design condition would be an arming time that provides the required muzzle safety zone.

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Received: 21.01.2005.

Matematičko modelovanje kretanja satnih osiguravajuće-armirajućih mehanizama

Izvršena je teorijska analiza i izveden je matematički model kretanja satnog osiguravajuće-armirajućeg mehanizma koji se sastoji od rotora (pogonski zupčanik), dva para evolventnih zupčanika i nemirnice. Matematički model, izveden na osnovu jednačina dinamike krutog materijalnog sistema i mehanike sudara, obuhvata diferencijalne jednačine kretanja za zajedničko i slobodno kretanje i za fazu sudara.

Ključne reči: **upaljač, armiranje upaljača, satni mehanizam, sigurnosni mehanizam, matematičko modelovanje, jednačine kretanja, diferencijalne jednačine, artiljerijski projekti**

Modélisation mathématique du mouvement des mécanismes de sureté d'horlogerie armants

Une analyse théorique a été faite et on a réalisé un modèle mathématique du mouvement d'un mécanisme de sureté d'horlogerie. Ce mécanisme se compose d'un rotor (la roue des minutes), deux paires de roues évolventes et d'un balancier. Le modèle mathématique dé-rivé à partir des équations de dynamique d'un système matériel rigide et de mécanique de choc, englobe les équations différencielles du mouvement d'ensamble et pour le mouve-ment libre et pour la phase du choc.

Mots clés: **fusée, armement de fusée, mécanisme d'horlogerie, mécanisme de sureté; équ-ation de mouvement, équation différentielle, projectile d'artillerie**

Математическое моделирование движения часовых предохранительно-подготавливающих механизмов

Здесь проведен теоретический анализ и выведена математическая модель движения часового предохранительно-подготавливающего механизма, который состоит из якоря (зубчатый привод), две пары возведенных в степень зубчатых колец и опытного поддона (для грузов).
Математическая модель, выведенная на основании уровнений динамики жесткой материальной системы и механики столкновения, охватывает дифференциальные уравнения движения для совместного и свободного движения и для фазы столкновения.

Ключевые слова: взрыватель, подготовка взрывателя, часовой механизм, предохранительный механизм, математическое моделирование, уравнения движения, дифференциальные уравнения, артиллерийский снаряд