Solving the Problem of General Capacitor Placement in Radial Distribution Systems with Laterals Using Simulated Annealing

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Simulated annealing was introduced as the algorithm for solving general capacitor placement problem by a group of engineers from Cornell University, Ithaca, New York in 1990s. It is the method which searches for a global optimum of the objective function rather than being stuck to the local optima whose number grows as problem dimension increases.

In the developed programs, original methods for solving the problem are introduced. Optimum is reached by initializing an algorithm with more rigorous parameters and for reasonable amount of CPU time.

Key words: capacitor, distribution network, reactive force, algorithm.

Introduction

Capacitors are implemented in radial distribution networks for power factor correction, loss reduction, voltage profile improvement and, in a more limited way, circuit capacity increase.

Capacitors placed in radial distribution systems are fixed or switchable and they are used for minimizing the energy loss thus achieving net dollar savings in comparison with "bare" network i.e. network without capacitors. Because of net dollar savings this problem is of great interest, the figure to which the savings can rise is several tens of thousands of dollars which is not in the least negligible when every cent of loss and capacitor cost ought to be spared.

Optimal capacitor placement has been investigated since the 60s. In the 80s, more rigorous approaches were suggested by Grainger [12], [13]. In the 90s, combinatorial algorithms were introduced as means of solving the capacitor placement problem: simulated annealing was proposed in [2], [3], genetic algorithms in [14], and taboo search algorithms in [15], [6]. This paper proposes a hybrid algorithm based on simulated annealing and greedy search technique which differs from algorithm [7].

Great attention is given to simulated annealing algorithm nowadays because of development of great speed personal computers that can solve complicated problems within one hour computing time which is the case with test circuit analyzed in this paper.

The race for better objective function was one of the main reasons for introducing this paper.

Problem formulation

How to arrange capacitors in radial distribution network in order to minimize energy loss is a very old problem. Recent articles of Baran and Wu, Chiang et al. [2]-[5] give this problem following formulation, minimizing the function:

\[
\sum_{i=1}^{N_c} C_i (U_{i0}^c) + K_e \cdot \sum_{i=1}^{N_t} T_i \cdot P_{loss,i}(X'\cdot U')
\]

(1)

where

- \(N_c\) - is the number of buses where capacitors are installed;
- \(C_i (U_{i0}^c)\) - is the price of capacitor of installed power \(U_{i0}^c\) (kVar), this is step up function (Fig.1);
- \(K_e\) - is the energy cost factor, 0.06$/kWh
- \(N_t\) - is the number of load levels
- \(T_i\) - is the duration of \(i\)-th load level
- \(P_{loss,i}\) - is the active power loss during \(i\)-th load level
- \(X'\) - is the active, reactive power flow and node voltage vector for \(i\)-th load level
- \(U'\) - is the vector of installed capacitors' powers for \(i\)-th load level

Load variation

Consumers are modeled as constant-power-demand customers for each load level given in Table 1:

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Capacitor size and control setting

Capacitor size is given in 300 kVar steps i.e. the least power capacitor has is 300 kVar and its price is 1900 $, 1000 $ for installation (this price does not depend on the capacitor size) and 900 $ for installed power (3 $/kVar). If size is bigger than 1500 kVar the price remains constant, 6400 $.

Objective function

Objective function is given by eq.(1).

It has two terms, capacitor price term and energy loss price term. Peak-power-loss price term is not included for the reason of comparison with the results obtained by Chiang et al. [2], [3].

Objective function is not differentiable as it has steps because of the capacitor price and has many local optima where usual nonlinear optimization techniques get stuck.

Load constraints

The following notations have been adopted:

- $Q_{\text{loss,II}}$ - maximal total power of installed capacitors for a certain load level
- $Q_{\text{dem,II}}$ - reactive power demand for "bear" network for a certain load level
- $Q_{\text{dem,II}}$ - maximal reactive power demand for a certain load level, then $Q_{\text{max,II}} = Q_{\text{loss,II}} + Q_{\text{dem,II}}$ in order for the radial distribution network not overcompensated
- $Q_{\text{min,II}}$ - minimal total power of the installed capacitors for a certain load level
- $P_{00,II}$ - active power demand at supplying node for a certain load level, then

$$Q_{\text{min,II}} = Q_{\text{loss,II}} + Q_{\text{dem,II}} - P_{00,II} \sqrt{\frac{1-0.85^2}{0.85}}$$

to have the power factor at supplying node 0.85 or greater.

Load constraints given by the above mentioned equations limit the configuration space of the possible solutions enormously without losing the global optimum. They make the problem solution feasible, as without them the CPU time is not limited, and problem can not be practically solved.

Power flow equations

In this paper, equations for symmetrical radial distribution system with laterals and one transformer, (supplying one) are given. Method of solution is Newton-Raphson method also called Dist-flow method, explained in full in Baran and Wu's article, [4], [5].

Applying simulated annealing method for more complicated systems is under research and mostly depends on implementing of load flow formula from [1] or [8] into the model of an actual system.

Operational constraints

Operational constraints are voltage constraints, namely voltage of each node must be between $0.9 U_n$ and $1.1 U_n$, where $U_n$ is 12.66 kV. For the reason of obtaining feasible voltage profile, regulating transformer at supplying node which raises supplying voltage for 5% at maximum load level is used.

Simulated annealing

Let $C_i$ be the current solution and $C_j$ a neighboring solution, which is generated exactly before to solution $C_i$. $\Delta C_{ij} = C_i - C_j$ is the difference of these two solutions. $C$ is the control parameter, called temperature which has high value at the beginning of the algorithm and gradually decreases. Sequence of successive solutions is called Markovian chain. There are two types of "simulated annealing" algorithms, homogeneous and inhomogeneous. For the homogeneous one, Markovian chain has constant control parameter for all its solutions and there are several Markovian chains for which the control parameter decreases gradually. For the inhomogeneous one, there is one Markovian chain with different control parameters for the successive pairs of solutions. Simulated annealing can be expressed by the following scheme in Pseudo-Pascal:

1. Start with any initial solution, 
2. Perturb from current $(j)$ solution to the next one $(i)$, 
3. find $\Delta C_{ij}$ ,
4. if $\Delta C_{ij} < 0$ replace the solution $j$ by solution $i$,
5. if not find $\exp(-\Delta C_{ij}/c)$ ,
6. if $\exp(-\Delta C_{ij}/c)$ is greater than random number uniformly distributed in segment $[0,1)$, replace the solution $j$ by solution $i$ (Metropolis criterion),
7. if not retain, current solution $j$ .
8. Stop when system is frozen i.e. there is no noticeable improvement in the solution.

Simulated annealing for constrained problems

There is one more step in "simulated annealing" when there are constraints.

This step is feasibility check, before generating new solutions by random-number-generator perturbation mechanism. If the new solution is feasible, it is compared with the previous one, if it is not, it is is disregarded and a new, feasible one is sought.

For an algorithm with fixed capacitors and algorithm with switchable capacitors, common constraint is the minimal and maximal size of the total installed capacity of capacitors. This size is known in advance and perturbations by random number generator take into consideration this size so that all generated solutions are feasible from this point of view.

In case of switchable capacitors, their size for maximum load level must be greater than the size for nominal load level. Size for nominal load level must be greater than the size for minimal load level (power constraint). This is also taken into consideration by solutions generating the mechanism so that the generated solutions are feasible.
An efficient load flow solution algorithm

The efficient load flow algorithm used in this article is made to suit the analyzed network i.e. the network with one supplying transformer at incoming node and laterals. This algorithm is fast, it converges extremely quickly and suits the simulated annealing algorithm completely because of an enormous number of different network configurations which ought to be analyzed.

Basic elements in simulated annealing

Configuration space

The size of configuration space depends on the number of possible places of capacitor installation i.e. on the number of buses and on possible capacitor size at any location. For the test system, 69-bus, 9-lateral distribution network, and capacitor size in 300 kVar increments, this space is enormous. The question of its reduction is of utmost importance. So constraints which limit configuration space are of ultimate importance under condition that the global optimum is not lost.

Perturbation mechanism

Generation of new, feasible configuration is done by perturbation mechanism in three steps.

First step: total power of the installed capacitors is chosen by random number generator.

Second step: the power of each installed capacitor is chosen by random number generator.

Third step: the location of each capacitor is chosen by random number generator.

Cooling schedule

The implemented cooling schedule in the applied algorithm is a homogeneous one with the constant value of temperature i.e. control parameter of each Markovian chain. The rule of temperature decrease is a simple one $T_{k+1} = 0.95 T_k$. $T_{k+1}$ is the value of temperature for $(k+1)th$ Markovian chain. By increasing the number of Markovian chains, the value of temperature decreases slowly and the system slowly cools.

The initial value of temperature is very high; for the first: several thousands of degrees is absolutely suitable. For switched capacitors, Markovian chain terminates when the length of Markovian chain reaches one predetermined value given by expression $1.1^* n_0$, where $k$ is the order number of Markovian chain, and $n_0$ is its initial value.

The algorithm in both cases terminates when acceptance ratio becomes small enough. Just how small depends on the operator. The smaller the value, the longer the CPU time, the better the solution. The acceptance ratio is defined as quotient of the number of accepted solutions and number of all solutions generated by one Markovian chain.

A solution algorithm for fixed capacitors placement

Step 1 - Input data:
1. network data and capacitor-price-step-up function,
2. initial temperature $T_0 = 5000$,
3. Markovian chain termination criterion,
4. algorithm stop criterion.

Step 2 - Starting with initial feasible configuration ("bare" network):

it start with "bare" network i.e. network without capacitors for which objective function is calculated, first objective function with which the algorithm is initialized and which is considered as "old" is 1.E30. This value is rejected and "bare" network is the first accepted configuration.

Step 3 - Cooling schedule.

Step 4 - Generate new feasible configuration:
1. random number generator is used,
2. subroutine which represents efficient power flow solution algorithm and gives power flow and voltage profile of the sample network is called.

It is possible to have a slight overcompensation during minimal load level.

Step 5 - Metropolis criterion

By the aforementioned rule the new configuration is rejected or accepted and announced as old, so updating the state. It should be pointed out that in this algorithm more expensive configurations are accepted which is the main difference between simulated annealing and greedy search technique when only cheaper configurations are accepted. In this last case, the local minimum could be the place where the solution gets stuck, which is overcome in simulated annealing by the uphill movement.

Step 6 - Checking of stop criterion:

Algorithm stops in two cases, whichever one occurs first:
1. length of Markovian chain at a certain temperature becomes greater than 5000 counted configurations,
2. acceptance ratio becomes less than 0.001,

Note: When 1 or 2 occurs, the system is considered to be frozen and algorithm terminates.

Step 7 - Output file

The following data are in this file:
1. optimal configuration,
2. load flow solution for each load level,
3. voltage profile for each load level,
4. objective function,
5. objective function elements.

The block diagram algorithm for fixed capacitor placement problem (Fig.2.) is enclosed below.
A solution algorithm for general capacitor placement (combination of fixed and switchable capacitors)

Program developed in Fortran 77 differs greatly from program [2], [3] and gives better results.

1. Main control loop in which simulated annealing algorithm is used with configuration price (1) as the objective function;
2. In the main loop, which can be called peak-load-level loop, there are two separated (they go one after another) control loops, for medium (nominal) and minimal load levels. Objective function for these control loops is active power loss. In these two control loops, as optimization method the greedy search instead of simulated annealing algorithm is used. In this way, less CPU time and exact optima for respective load level loops are obtained.

Step 1 - Input data:
1. network data and capacitor-price-step-up function,
2. initial temperature \( T_0 = 10000 \),
3. initial-Markovian-chain length = 200 generated configurations,
4. increasing-Markovian-chain-length coefficient = 1.1,
5. algorithm stop criterion (maximum of 3000 configurations length or acceptance ratio less than 0.01%).

Step 2 - Starting with initial feasible configuration:
1. by random number generator, complying with load constraints, generate peak load level configuration and call subroutine to find node voltage values for power loss calculation;
2. by random number generator, complying with load constraints and power constraint, generate nominal-load-level configuration and call subroutine to calculate the loss;
3. same step as above but for minimum load level;
4. calculate initial objective function;

Step 3 - Make global cooling mechanism for outer loop:
1. on previous algorithm basis (for fixed capacitors) and the fact that rejections are now more frequent, it can be concluded that initial temperature satisfies completely;
2. cooling schedule rule, same simple cooling mechanism as for fixed capacitors, is adopted:
   \[ T_{k+1} = 0.95 T_k \]

Step 4 - By random number generator generate new peak-load-level configuration. Call subroutine to determine power flow. In this way second peak-load-level configuration is obtained. This configuration is a limiting configuration for lower load levels.

Step 5 - First inner loop is "middle", nominal-load-level loop. Greedy search is applied here in order to find the cheapest configuration for nominal load level regarding active power loss criterion.

NOTE: to calculate power flow for certain configuration subroutine is always called.

Step 6 - Second inner loop (minimal-load-level loop) goes after the first inner loop. Again greedy search is used in order to find the cheapest (regarding active power loss) configuration for current-old-nominal-load-level configuration. The later is the optimal configuration for current-peak-load-level configuration.

Step 7 - A complete, current and new configuration is obtained.

Step 8 - At this point everything is ready for applying Metropolis criterion only for outer loop in which new maximum load level configuration is generated.

Step 9 - From this point on, the procedure is the same as for fixed capacitors.

The block diagram algorithm for general-capacitor-placement problem (Fig.3.) is enclosed below.
Numerical results

Adopted data

Two programs were developed, one simpler for fixed capacitor placement problem, another more sophisticated for general capacitor placement problem, both of them written in Fortran 77 computer language.

As the test system 69-branch is used, 9-lateral test system derived from a portion of the PG&E distribution system. The network data of this system are given in [4], [5].

The substation voltage without a tap is taken as the base kV and the lower and upper voltage limits are assumed to be 0.9 and 1.1 p.u. respectively.

In every node there are active and reactive load demands. Supplying voltage at supplying network node is 12.66kV during minimal and nominal load level. During peak load level it is 1.05*12.66 kV.

"Bare" network (network without capacitors)

a) Objective function of a "bare" network amounts to 139,642 $.
b) Power factor at supplying node is less than 0.85 and power factor correction is necessary.
c) Voltage goes below the permitted value (0.9*12.66 kV) at node 50th, so better voltage profile is also desirable.

This program’s generated configuration for fixed capacitors placement

In node 17th, 300 kVar and in node 50th, 1200 kVar capacitors are placed.

a) Objective function amounts to: 101,201 $, with the following components:
   - capacitors price: 6.500 $,
   - active energy loss price: 91.644 $.
b) Power factor at the supplying node is greater than 0.85. The network is slightly overcompensated for minimal load level.
c) Voltage profile complies with voltage constraints.
d) $\Delta C/C = 0.01 \%$, maximal-final-Markovian-chain length $n_{\text{max}} = 5000$ counted network configurations.
e) Simple cooling schedule is used $\alpha(Tk) = 0.95$.
f) Acceptance ratio is 0.001.
g) CPU execution time is 1min on PC Pentium II, 533 Mhz.

Solution given by authors [2], [3]

In node 20th, 300 kVar fixed capacitor is placed.

In node 51th, switchable capacitor of the following rates is placed: 1800kVar for maximum load level, 1200 kVar for medium load level and 600 kVar for minimal load level.
a) Objective function amounts to: 99.944 $, with In node 51th, switchable capacitor of the following rates is placed: following components:
   - capacitor price: 8.300 $,
   - active energy loss price: 91.644 $.
b) Power factor at In node 51th, switchable capacitor of the following rates is placed: supplying node is greater than 0.85. The network is not overcompensated.
c) Voltage profile complies with In node 51th, switchable capacitor of the following rates is placed: voltage constraints.

Way of capacitors placement in the network

In general, capacitor placement problem let the following four different configurations be analyzed:

1. configuration

solution given by authors [2],[3], objective function is 99.944 $.

2. configuration

$n_1 = 300$ kVar fixed capacitor,

$n_{a1} = 2100, 1200, 600$ kVar switchable capacitor, objective function is 99.116 $.

3. configuration

$n_4 = 300$ kVar fixed capacitor,

$n_{a4} = 2700, 1200, 600$ kVar switchable capacitor, objective function is 98.909 $.

4. configuration

cheapest solution (this program’s generated configuration),

- The worst solution is solution given by authors [2], [3] where the objective function is 28.4% less than "bare" network objective function.
- Second configuration is generated in the process of applying simulated annealing algorithm and its objective function is 29% less than "bare" network objective function.
- Third configuration is generated in the process of applying simulated annealing algorithm and its objective func-
tion is 29.2% less than “bare” network objective function.
Fourth configuration gives the cheapest solution obtained in the process of applying simulated annealing algorithm and its objective function is 29.4% less than for “bare” network.
All these solutions have something in common, two capacitors are placed, one fixed on the main feeder and one switchable on the longest lateral on which voltage goes below the permitted limits in case of “bare” network.

Conclusions
In this article, solving general capacitor placement problem in radial distribution system with laterals by simulated annealing is presented.

Short resume of the obtained results and economical advantages of applying capacitors in radial distribution networks
Program aspects of simulated annealing algorithm in this case are very complicated but the analysis of the obtained results is possible to summarize as follows.
The obtained results show advantages of the developed method and give firm basis for its application in real distribution networks.
However, comparing these with the results that can be obtained using other analytical and heuristic methods is not done in this paper. The mere fact that heuristic is in question says that those results are bound to be worse.

Way of capacitor placement in radial distribution network
There is no general answer to this question. The answer can be obtained after program application in the actual case (actual radial distribution network).
It can be seen that in each case it was necessary to place a switchable capacitor on the longest lateral where voltage goes below the permitted limits in case of “bare” network.
There is one more thing worth mentioning: if there is more than one capacitor, some are placed on the main feeder and some on the longest lateral and nowhere else.

Achieving good power factor
Real solutions obtained in this paper, show that poor power factor of a network can always be improved by capacitor placement.
Slight overcompensation appears in fixed-capacitor-placement problem for minimal load level. The reason for overcompensation is that the upper limit for placing capacitors does not allow overcompensation in the maximum load level but not in lower load levels.

CPU elapsed time theoretical analyses
Theoretical conclusions are only confirmed with practical results obtained after several repetitions of the same algorithm (actual programs) for different values of the incoming parameters.
The longer the Markovian chain, less ΔC/C, cooling schedule closer to 1, the less the acceptance ratio, execution of program is longer, elapsed CPU time is longer and the obtained solution is closer to optimum.

Global optimum closeness
With the more restrictive input parameters, the global optimum is captured more closely, but computing time is drastically longer.

Algorithm accepts worse solutions (enhancement of objective function value) and can terminate with a worse solution. This is avoided by monitoring the program during its actual execution.
Program monitoring is actually noting all minimal objective function values up to the actual program termination.

Applicability
Capacitor placement problem in a radial distribution network is a combinatorial optimization problem with many independent variables (placed capacitor prices) of discrete values and non-differentiable objective function. For this reason there is no better optimization algorithm for this actual problem than simulated annealing. However, even for simulated annealing application there are some limitations.
In simple radial distribution networks, with a small number of possible network configurations simulated annealing algorithm is enormously slower than simple greedy search technique and it should not be applied.
Reasons for this are:
1. enormous elapsed time in the generation of Markovian chain although there are a few different network configurations,
2. even when Markovian chain is small there is cooling schedule which additionally prolongs the elapsed CPU time.
It is useful to be reminded of the fact that simulated annealing as an optimization method is applied in cases where the configuration space is huge, eternal or countable space.

Comparing the developed algorithm with other solving algorithms
Fixed capacitor placement problem is solved exclusively by applying simulated annealing algorithm implemented in one program loop.
General capacitor placement problem is solved by the algorithm which differs very much from the algorithm in [2], [3]. It combines simulated annealing in the main program loop of the maximum load level, with greedy search technique in two lower program loops of nominal and minimal load levels. Such hybrid algorithm of simulated annealing and greedy search technique (which differs from the algorithm in [7]) finds solution in a very short time. The algorithm in [2], [3] where simulated annealing is applied in all program loops is very long, and does not terminate in a reasonable (acceptable) time.

Voltage constraint
For the network in question [4], [5] not a single configuration complies with the constraint of 1.00 p.u. voltage at the supplying node (all load levels). Namely, during maximum load level in each configuration voltage at 46th and 47th nodes falls below the permitted limit which is 0.9*12.66kV.
Therefore, the question is: "How to minimize network objective function and comply with voltage constraint?"
With capacitors only it is impossible to achieve voltage control which is the case in this example. It is necessary to place voltage transformers along with capacitors which is done in [4], [5]. It is important to stress once again, that only when voltage at the supplying node becomes 1.05 p.u. at the maximum load level, the problem can be solved by capacitor placement.
Further perspectives in applying simulated annealing algorithm in radial distribution networks

Because of the interdependence between feeder voltages and outcome of the capacitors, a simultaneous analysis of voltage regulators and capacitors is very complicated. Furthermore, there is a great interest in the acceptable load demand modelling. There has not been a generally accepted model up to now. As consequence, the constant power model has been accepted. To apply simulated annealing in the simultaneous analysis of voltage regulators and capacitors in radial distribution networks the following problems ought to be solved in the first place:

1. efficient load flow algorithm with voltage regulators ([1], [8]);
2. how to include voltage regulator's price in the network objective function [11].

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Rešavanje problema kompenzacije reaktivne snage u radijalnim distributivnim mrežama sa bočnim granama primenom metode simulacije kaljenja

"Simulacija kaljenja" je prvi put primenjena za rešavanje problema kompenzacije reaktivne snage od strane grupe inženjera sa Kornel Univerziteta, Itaka, Nju Jork 1990-ih godina. To je metoda koja traga za globalnim optimumom funkcije cilja a ne zaglavljuje kod lokalnih optimuma čiji broj raste sa povećanjem dimenzije problema. Originalne metode rešavanja problema kroz razvijene programe su prikazane. Bolja konvergencija se postiže inicializacijom algoritma sa rigoroznijim vrednostima parametara i to još uvek za prihvatljivo računarsko (CPU) vreme.

Ključne reči: fiksni i regulacioni kondenzatori, globalni optimum , simulacija kaljenja , naponski profil.
Продесс решения задач размещения конденсаторов реактивной мощности в радиальных распределительных сетях со боковыми ветвями применением метода "Имитирование закалки"

"Имитирование закалки" впервые применено для решения задач размещения конденсаторов реактивной мощности от стороны группы инженеров Университета Корнелл, Итака, в Нью-Йорке в 1990-чых годах. Этот метод ищет глобальный оптимум функции цели, а не пропадает у местных оптимумов, число которых возрастёт с увеличением размера задачи. Здесь показаны оригинальные методы решения задач через усовершенствованные программы. Лучшую конвергенцию возможно достичь инициализацией алгоритма со схожими точными значениями параметров, а именно ещё за приемлемое машинное время (CPU).

Ключевые слова: конденсатор, распределительная сеть, реактивная мощность, алгоритм.

Solution du problème de la compensation de la force réactive dans le réseau radial distributif aux branches latérales par la méthode simulation de la trempe

La simulation de la trempe pour la solution de problème de compensation de la force réactive a été appliquée pour la première fois par un groupe d'ingénieurs de l'Université Corneille, Ithaca, à New York, en 1990. C'est la méthode qui cherche l'optimum global de la fonction du but et ne se bloque pas chez les optimums locaux dont le nombre augmente avec l'augmentation de la grandeur du problème. Les méthodes originales de la solutions du problème par les programmes développés sont exposés. Une meilleure convergence s'obtient en utilisant l'algorithme avec les valeurs rigoureuses des paramètres et cela pour un temps informatique raisonable.

Mots clés: condensateur, réseau de distribution, force de réaction, algorithme.