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Results of computer-experimental performance analysis of a type of fuzzy controllers for the inverted pendulum system

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In memory of Professor Dušan M.Velašević, the PhD Dissertation Supervisor

Fuzzy control techniques are of interest for practical applications. Hence, it is important to understand their features. In this paper some results of computer-experimental analysis of behavior of a fuzzy controller for the cart-ball variant of inverted pendulum system are given. The impacts of changes in uncertainty in the controller are analyzed experimentally: it has been found that the performances of the controller deteriorate mildly towards a certain percentage of the rules that are removed, and that is opposite relative to the process with probabilistic uncertainty model. Also, the robustness of the controller is analyzed experimentally: it has been found that the fuzzy controller is more sensible to wrong, than to missing rules. Further, the results of the experiments with different shapes of primary fuzzy sets membership functions are given: it has been concluded that the fuzzy system is sensitive to the changes of shapes of those functions and that the optimal shapes exists. This result is a new one; it has not been given in the references. To conclude, characteristics of the discussed PD fuzzy controller type are given. The conclusion, based on the results of the computer experiments, is that the behavior of the fuzzy PD controller in the considered task is at least as good as the behavior of the crisp (traditional) PD controller. The subjects for further research activities in the considered paradigm are also pointed out.

Key words: **expert system, fuzzy logic, fuzzy system, fuzzy controllers, controllers, system robustness**

List of notation and symbols

M	– mass of the cart
R	– the radius of the rail arc
y	– length of the rails at the hard support
$\ddot{y}$	– the cart velocity
F	– driving force on the cart
m	– the mass of the ball
I	– the moment of inertia for the ball, (2/5) mr_1^2
r_1	– the radius of the ball
r	– the rolling radius of the ball
ψ	– the rolling angle of the ball
$\varphi \in [-0.22,\ 0.22]$ rad	– the angular position of the ball
$\dot{\varphi} \in [-1,\ 1.5]$ rad/sec	– the angular speed of the ball
V	– the vertical reactive force between the ball and the arc
H	– the horizontal reactive force between the ball and the arc
g	– the gravity, 9.81 m/sec^2
$\times$	– vector (cross) product
PD	– proportional/derivative controller
PID	– proportional/integral/derivative controller
e	– error (ey or $e\varphi$)
$ey = y_{ref} - y$	– error of cart position
ce	– error derivation (cey or $ce\varphi$)
$cey = (ey_k - ey_{k-1})/T_s$	– position error derivation
T_s	– sampling period
u	– controller output, control
$e\varphi = \varphi_{ref} - \varphi$	– error in angular deviation of vertical line of the ball, [rad]
$ce\varphi = (e\varphi_k - e\varphi_{k-1})/T_s$	– angular deviation error derivation
N	– primary fuzzy set “Negative” (in the IF part of the rule)
Z	– primary fuzzy set “Zero”
P	– primary fuzzy set “Positive”
SNO	– output fuzzy set “Strong Negative Output” singleton, (in the THEN part of the rule)
MNO	– output fuzzy set “Moderate Negative Output” singleton
ZO	– output fuzzy set “Zero Output” singleton
MPO	– output fuzzy set “Moderate Positive Output”, singleton
SPO	– output fuzzy set “Strong Positive Output”, singleton

Introduction

THE possibility of using fuzzy sets in modelling gradual properties or softening constraints whose fulfilment is a matter of degree, as well as of information pervaded with imprecision and uncertainty, makes the theory of fuzzy sets useful in many applications. Advances in computer science

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and in computer engineering make wider applications of emerging techniques of intelligent control possible, which requires quite considerable computer resources. The most popular application domain of theory of fuzzy sets is fuzzy control [1]. In many of such applications [2], expert knowledge is coded as fuzzy expert IF-THEN rules, making a fuzzy rule based expert system [3], which describes proposed actions for different classes of situations, represented by fuzzy sets. A fuzzy control element can perform the same task as a proportional-integral-derivative (PID) controller can, because a fuzzy controller implicitly defines numeric function that connects control variable and observed variables. The difference between classical and fuzzy control techniques is in the way in which the control law is obtained [4]. In the context of classic control, a control law is calculated using a mathematical model of the considered process, while fuzzy-logic approach, which is in agreement with artificial intelligence, suggests that the control law should be formed using expertise of a human operator. The tuning of the PID parameters is usually done heuristically, which, in some way is close to the fine-tuning of fuzzy-logic controllers. But, the main difference between a PID controller and a fuzzy controller is that the prior one can attain only linear control law, while the latter can attain not only linear, but non-linear laws also, which can explain some of the better results of the fuzzy controller compared to the PID controller. Any kind of control law can be represented by fuzzy control technique, provided that this control law can be represented by "IF-THEN" expert rules. Expert system approach based on fuzzy logic differs from the standard expert system approach through the existence of an interpolation mechanism from several rules. The control part of a fuzzy-logic controller models linguistic, rule based control. The interpolation mechanism enables the function, simulated by the control part, to remain continual, just like in the case of classic control. In the situation when complex process is considered, it may turn out to be more practical to get the knowledge from a human operator, than to use the mathematical model in order to calculate the control. The reasons for that can be smaller modelling costs in the first case, or the fact that it is impossible to formulate the model.

The base paradigm of the mechanism of fuzzy control and the idea of representing control knowledge using the base of fuzzy rules, are given in [5]. In that paper, the characteristic reasoning mechanism is introduced, so that the class of fuzzy systems is called Mamdani's systems. The basic idea of Mamdani's system consist of decomposition of the input domain by partitioning of input variables in the IF part of an expert IF-THEN rule, and of assigning to each fuzzy domain input a fuzzy value of an input variable.

In [6], it was for the first time pointed to the problem of fuzzy controller stability, the problem which still has research attractiveness.

In [7] the technique of fuzzy controller design consisting of two steps was introduced. In the first step, a controller is designed in such a way that it shows approximately the desired characteristics, while, in the second step, the fine tuning of the system is performed in order to get the system with optimal response.

In [8], the reasoning mechanism, which is an alternative to the mechanism in Mamdani's system, is introduced - the first alternative in the sequence of alternatives which followed. The class of fuzzy systems with this reasoning mechanism is called Larsen's systems. An overview of reasoning mechanisms in fuzzy controllers is given in [4].

Organization of the paper is as follows: after the introductory remarks, the paradigmatic cart-ball system is described. This system belongs to the inverted pendulum class of systems. Two kinds of description of the cart-ball system are given: the mathematical model and the fuzzy model. Next, some results of computer-experimental analysis of behaviour of the fuzzy controller of the cart-ball system are given. The influence of uncertainty changes in the fuzzy system is analyzed first, and then the results of an analysis of the changes in robustness are given. The influence of changes in shapes of membership functions of fuzzy sets from input domains has also been analysed. Conclusions, based on the results of the computer experiments, as well as the subjects for further research activities are given.

Description of Cart-Ball System

Computer experiments are performed with the controller of the system for ball balancing–system which belongs to the class of inverted pendulum systems. The software implementations of control systems in MATLAB are used for experiments, [9]. These software implementations emulate an electro-mechanical system, which consists of a cart with a ball on an arc. The ball is rolling across the arc made of the pair of parallel rails, fixed on the cart, Fig.1.

The cart-ball system is controlled with the aim to keep the ball in balance on the top of the arc, and, at the same time, to set the cart in the desired position on its path.

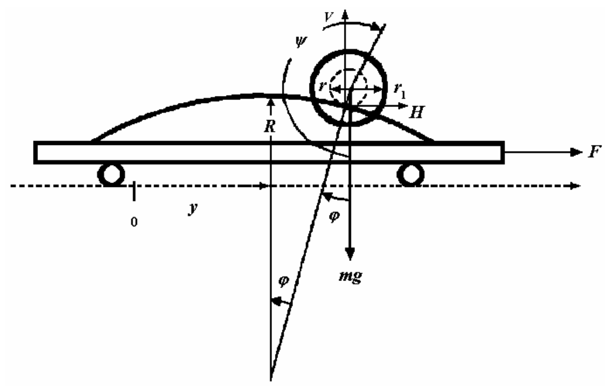

Figure 1. The cart-ball system

A known global general diagram of the considered control system is depicted in Fig.2. A control variable (at Fig.2, $u(t)$, where t is time), is an output of the control element, a controller, and is used as an input of the controlled object.

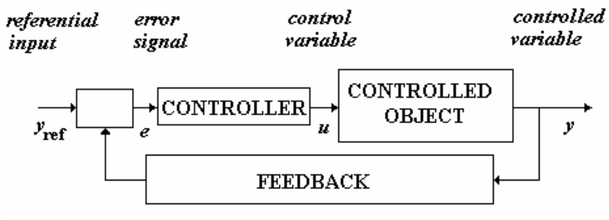

Figure 2. A global general diagram of the control system including a controller

The described problem, which belongs to the class of inverted pendulum problems, is often used as a test problem for different kind of controllers, as sales material for fuzzy design tools, and for teaching. This is due to the fact that the system enables easy observing of the processes in it, even though it is fairly difficult to solve. The inverted pendulum system represents the paradigm of systems in unstable equilibrium, often present in military, industrial, and other systems. Such military systems in unstable equilibrium are control systems present in rockets, airplanes or on ships (autopilots). An example of such a system in industrial processes is a control system used in chemical reactions, where a mixture develops heat itself (exothermal) near the desired temperature; without controller the reaction either stops or it runs away.

Mathematical model

The mathematical model of dynamics of the system depicted in Fig.1 is obtained on the bases of fundamental physical equations describing the physical state of the system. The mathematical model is obtained by linearization, neglecting the members with insignificant influence. The system dynamics is characterized by: y – the cart position relative to the referential initial point, and φ - angular deviation of the ball from the vertical line. The system is in balance when $y=0$, $\varphi=0$, and the driving force on the cart, $F=0$.

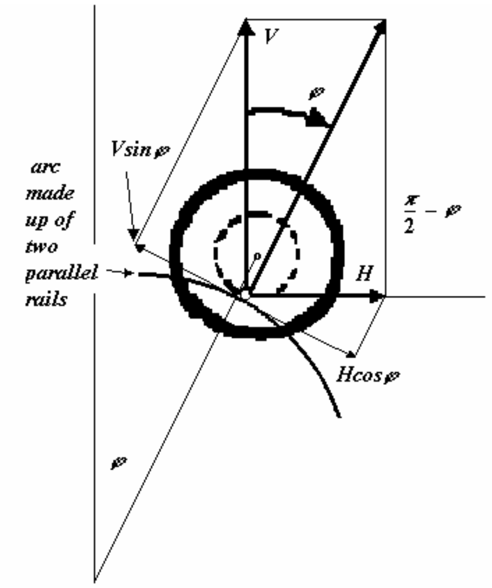

Figure 3. Modelling the rotational ball movement: the arc resistance, reactive force between ball and the arc, is decomposed to two components, V and H

The basic physical equations, which describe the cart-ball system depicted on Figures 1, 3 and 4 and with given cart-ball system features, are:

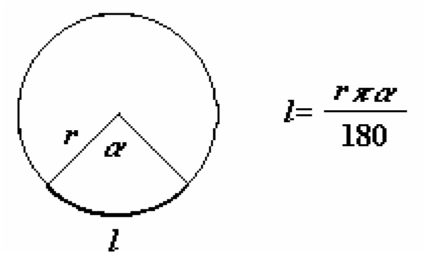

Figure 4. The arc, l

From Figures 1 and 4, due to . $l_\psi = \dfrac{r\pi\psi}{180}$ and $l_\varphi = \dfrac{(r+R)\pi\varphi}{180}$, it holds:

$$\psi = \frac{(R+r)}{r}\varphi . \tag{1}$$

Further more, horizontal reactive force between the ball and the arc, H, is:

$$H = F - M\ddot{y} . \tag{2}$$

At the contact point between the ball and rails, the following momentums annul, Fig.3:

$$\vec{M} = \vec{M}_V + \vec{M}_H ,$$

where

$$\vec{M}_V = \vec{V}\times\vec{r},\ \ \vec{M}_H = \vec{H}\times\vec{r} , \tag{3}$$

so, it holds:

$$I\cdot\ddot{\psi} = (V\sin\varphi - H\cos\varphi)\cdot r . \tag{4}$$

Beside the rotation, the ball also moves translatorily (it falls), so it is:

$$m\cdot(\ddot{y} + (R+r)\cdot(\ddot{\varphi}\cdot\cos\varphi - \dot{\varphi}^2\cdot\sin\varphi)) = H , \tag{5}$$

$$m\cdot((R+r)\cdot(-\ddot{\varphi}\cdot\sin\varphi - \dot{\varphi}^2\cdot\cos\varphi)) = V - mg , \tag{6}$$

Eliminating V, H and ψ the two equations are obtained:

$$\ddot{y}(M+m) = -m\cdot(R+r)\cdot(\ddot{\varphi}\cdot\cos\varphi - \dot{\varphi}^2\cdot\sin\varphi) + F , \tag{7}$$

$$\begin{aligned}&\ddot{\varphi}\cdot I\cdot(R+r)/r = \\ &= m\cdot r\cdot(R+r)\cdot(-\ddot{\varphi}\sin^2\varphi - \dot{\varphi}^2\cdot\sin\varphi\cos\varphi) + \\ &+ mrg\sin\varphi + Mr\ddot{y}\cos\varphi - Fr\cos\varphi\quad .\end{aligned} \tag{8}$$

The model is set in the vicinity of its equilibrium state, (φ small). With a maximum value of φ equal 0.22 rad, it is permissible to set:

$$\sin^2 \approx 0,\ \ \sin\varphi \approx \varphi ,$$

and

$$\cos\varphi \approx 1 \tag{9}$$

The influence of the terms having $\dot{\varphi}^2$ as a factor is less than one percent. Consequently, these terms are neglected, so the two equations now are:

$$\ddot{y}(M+m) = -m\cdot(R+r)\cdot\ddot{\varphi} + F , \tag{10}$$

$$\ddot{\varphi}\cdot I\cdot(R+r)/r = mrg\varphi + Mr\ddot{y} - Fr \tag{11}$$

Substitution of (10) in (11) gives:

$$\begin{aligned}\ddot{\varphi} &= \frac{(M+m)mr^2 g}{(R+r)(Mmr^2+(M+m)I}\varphi - \\ &- \frac{-mr^2}{(R+r)(Mmr^2+(M+m)I}F\end{aligned} \tag{12}$$

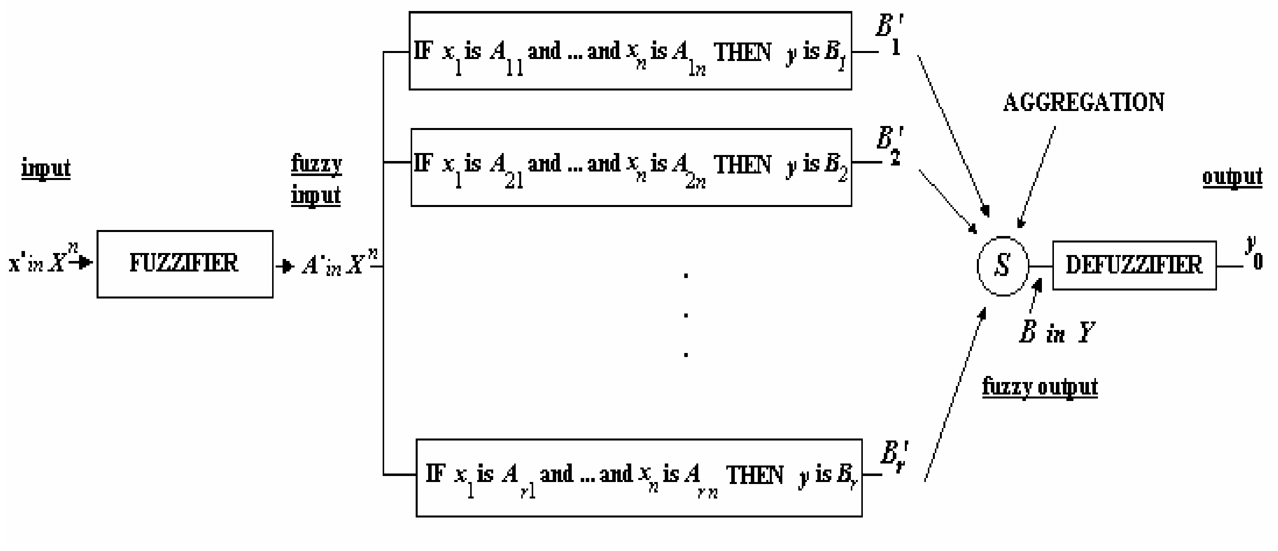

Figure 5. The basic configuration of fuzzy systems

Substitution of (12) in (10) gives:

$$\ddot{y} = \frac{-m^2 r^2 g}{Mmr^2 + (M+m)I}\varphi + \frac{mr^2 + I}{Mmr^2 + (M+m)I}F \qquad (13)$$

Equations (13) and (12) can be written as follows:

$$\ddot{y} = a\varphi + bF \ , \qquad (14)$$

$$\ddot{\varphi} = c\varphi + dF \ , \qquad (15)$$

where:

$$a = \frac{-m^2 r^2 g}{Mmr^2 + (M+m)I}, \quad b = \frac{I + mr^2}{Mmr^2 + (M+m)I}, \qquad (16)$$

$$c = \frac{(M+m)mr^2 g}{(R+r)(Mmr^2 + (M+m)I)},$$

$$d = \frac{-mr^2}{(R+r)(Mmr^2 + (M+m)I)}.$$

Four state variables are introduced: $x_1 = y,\ x_2 = \dot{y}$ - the cart speed, $x_3 = \varphi,\ x_4 = \dot{\varphi}$ - the speed of the angular deviation of the ball. Output variables are $y_1 = y \quad y_2 = \varphi$ and so the mathematical model of the system dynamics is:

$$\dot{\mathbf{x}} = Ax + BF$$

$$\boldsymbol{y} = Cx \qquad (17)$$

$$\boldsymbol{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T, \ \boldsymbol{y} = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T,$$

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & c & 0 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 0 \\ b \\ 0 \\ d \end{bmatrix}, \mathrm{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

where, (please, see *List of notation and symbols* and Fig.1): $M = 3{,}1\,\mathrm{kg}$; $R = 0{,}5\,\mathrm{m}$; $y \in [-0{,}6 \ \ 0{,}6]\,\mathrm{m}$; $\dot{y} \in [-1{,}5 \ \ 1{,}5]\,\mathrm{m/sec}$ $m = 0.675$ kg; $I = (2/5)mr_1^2 = 0{,}204 * 10^{-3}\,\mathrm{kgm^2}$; $r_1 = 0.0275\,\mathrm{m}$; $r = 0.025\,\mathrm{m}$; $\varphi \in [-0.22,\ 0.22]$ rad; $\dot{\varphi} \in [-1,\ 1.5]$ rad/sec and $g = 9.81\,\mathrm{m/sec^2}$. Based on these values, for constants *a*, *b*, *c*, and *d*, the following values are obtained: $a = -1.34$; $b = 0.301$; $c = 14.3$; $d = -0.386$.

The traditional controller can be formed on the basis of values of state variables in the cart-ball system. This controller generates desired control signal, u, i.e. driving force F which drives the cart (proportional-derivative, PD, controller). The controller's operation is based on the influences of the position, y, and of the speed, $\dot{y}$, of the cart, amortized by the influences of the angular position φ and angular speed, $\dot{\varphi}$, of the ball.

Fuzzy model

The cart-ball system dynamics, instead by the mathematical model (17), can be described by a fuzzy system [3], Fig.5. The flow of information in the fuzzy system, Fig.5, is from left to right. The aggregation of the output fuzzy sets B_i', $i = 1, 2, ..., r$ is done, where r is the number of rules.

The input fuzzy set *A*', Fig.5, is mapped to the output fuzzy set *B* by fuzzy inference. An implementation of that mapping is specific for each type of fuzzy system, and is based on fuzzy logic.

Fuzzy systems based on IF – THEN rules, Fig.5, are many-inputs-one-output systems, they map $\mathbf{X} \subset \mathbf{R}^n$, where $\mathbf{R}$ is the set of real numbers, onto $Y \subset \mathbf{R}$. The set $\mathbf{X} = X^n = X_1 \times ... \times X_n$ is an input variable space, a multidimensional universe of discourse, and the set Y is an output variable space. The multidimensional (n dimensional) input fuzzy set A' is, hence, generated in a process of optional fuzzification, in case when there is not A' at the input, but a multidimensional input signal $\mathbf{x}' = [x_1', ..., x_n'] \in \mathbf{X}$. In some systems, an input signal can be given directly by the multidimensional fuzzy set $A' = [A_1, A_2, ..., A_n]$.

Each fuzzy IF-THEN rule from the rule base, Fig.5, defines fuzzy implication $A_{i1} \times ... \times A_{in} \Rightarrow B'_i$, i=1,2, ..., r, given by the fuzzy set defined in the product space **X** × Y, where A_{ij}, j = 1,2, ..., n, are fuzzy sets in IF parts of the rules (primary fuzzy sets). Fuzzy inference engine gives an output fuzzy set B, based on: 1) rules of fuzzy logic, 2) the fuzzy input, A', 3) IF-THEN rules in the rule base, 4) a compositional rule of inference and 5) an aggregation of fuzzy sets B'_i in fuzzy set B. If the Zadeh's inference rule is used for the compositional rule of inference, each rule $\mathbf{R}_i$ specifies a fuzzy set B_i' for input facts A', given by the following membership function, i = 1,2, ..., r:

$$\mu_{B_i'}(y) = \sup_x \left[\min\{ \mu_{A'}, \mu_{A_{i1}\times...\times A_{in}}(x) \Rightarrow \mu_{B_i}(y) \right] \quad (18)$$

Some t-norm [3], usually *min*, can be used in the expression (18) in order to model the connective AND between premises in IF part of the rule. Mamdani's (*min*) or Larsen's (*product*) operator is usually used for the fuzzy implication. Implicitly, the connective *also* is assumed between the rules, and this connective is the most often interpreted as the aggregation operator OR, and modelled by a t-co-norm S, [3] usually by the operator *max*. The output fuzzy sets B_i' are aggregated in the overall fuzzy set B, the output fuzzy set of the fuzzy system, (19):

$$\mu_B(y) = S\left(\mu_{B_n'}(y)\right) \quad (19)$$

That is the way in which a fuzzy reasoning method is implemented.

It is not sufficient, in some applications, to have the fuzzy set B at the output of a fuzzy system. The crisp output value may be required. The optional defuzzification gives the crisp output value y_0 based on the output fuzzy set B. At the widest application of the fuzzy expert systems, at fuzzy-logic controllers, the crisp output value is necessary, because control is done using crisp values.

As it is known, in general, the basic idea of the discrete PID controller is to choose the control law by considering an error $e(kT)$, change-of-error

$$ce(kT) = \left((kT) - e\left((k-1)T\right)\right)/T \quad (20)$$

and the numerically approximated integral of error

$$ie(kT) = ie\left((k-1)T\right) + Te\left((k-1)T\right), \quad (21)$$

where T is the sample period.

The PID control law is

$$u_{PID}(kT) = K_p * e(kT) + K_D * ce(kT) + K_I * ie(kT) \quad (22)$$

where $u_{PID}(kT)$ is the controller output, K_p is proportional constant, K_D is a derivative constant and K_I is an integral constant, defined by characteristics of the process. For a linear process, the parameters K_p, K_D, and K_I are designed in such a way that the closed loop control is stable. In the case of the nonlinear processes, which can be linearized around the operating point, conventional PID controllers also work successfully. However, the PID controller with constant parameters in the whole working area is robust, but not optimal. Hence, tuning of PID parameters has to be performed.

The output of the fuzzy controller $u(kT)$ is given by the following expression:

$$u(kT) = \phi\left(e(kT),\ ce(kT),\ ie(kT)\right) \quad (23)$$

where ϕ is a nonlinear function determined by fuzzy parameters.

The cart-ball system controller may be implemented as a fuzzy controller, a kind of fuzzy systems, which corresponds the traditional PD controller.

The control goal of any controller is to regulate some process output around the setpoint or reference. In the considered case single input - single output control exists. The inputs of the fuzzy PD controller are the error

$$e(kT) = p_{\mathrm{ref}}(kT) - p(kT), \quad (24)$$

and the change_of_error $ce(kT)$, the expression (20), where p_{ref} is setpoint value (y_{ref} or φ_{ref}), and $p(kT)$ is the process output at $t = kT$, T is a sample time. The structure of a fuzzy PD controller is given at Fig.6. A block denoted by f at Fig.6, is a knowledge base, and ge, gce and gu are gains, which correspond K_p, K_D, parameters in (22), and $K_I = 0$.

In the considered case of the cart-ball system, universes of discourse of variables are the sets of observed values. Each input variable may take values from the set of fuzzy sets associated to it. The output variables take their values from the set of fuzzy singletons (one-element sets). The number of the controller inputs, that is, the number of inputs in the rule base, determines the number of basic premises in the IF part of the rule, i.e. the dimensions of the table. Hence, the controller is implemented as the connection of two fuzzy PD controllers, the cart controller, and the ball controller. For the cart controller the following denotations holds: the inputs are y and $\dot{y}$, that is, the error $e = y_{ref} - y$, and the error derivation $ce = (e_n - e_{n-1})/T_s$, T_s – is the sampling period, and the output is denoted as u. For the ball controller it holds: the inputs are $\dot{\varphi}$, that is, the error $e = \varphi_{ref} - \varphi$, and the error derivation $ce = (e_n - e_{n-1})/T_s$, the output is u. Positional fuzzy PD controllers have identical structures, shown in Fig.6.

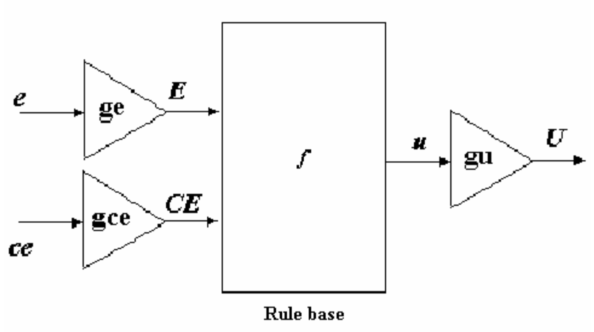

Figure 6. The structure of the considered fuzzy controller

For the fuzzy PD controller from Fig.6, ge, gce, and gu are gains, and the function f represents input-output mapping of fuzzy controller implemented by the following rule base:

R1: IF e is N AND ce is N THEN u is SNO,

R2: IF e is N AND ce is Z THEN u is MNO, R3: IF e is N AND ce is P THEN u is ZO,

R4: IF e is Z AND ce is N THEN u is MNO,

R5: IF e is Z AND ce is Z THEN u is ZO, (25)

R6: IF e is Z AND ce is P THEN u is MPO,

R7: IF e is P AND ce is N THEN u is ZO,

R8: IF e is P AND ce is Z THEN u is MPO,

R9: IF e is P AND ce is P THEN u is SPO,

Having a computer implementation on mind, it is more convenient to represent the upper rule base as a table, [10], Table 1, and so the tabular fuzzy controller is obtained. The main feature of a tabular fuzzy controller is that a rule base is given by a table, for example in the case of (25) where the number of rules is $n = 9$, by the Table 1, with dimensions 3×3:

Table 1. Tabular fuzzy controller′s rule base

e ↓ \ ce →	N	Z	P
N	SNO	MNO	ZO
Z	MNO	ZO	MPO
P	ZO	MPO	SPO

↑ u

Table 1 with 9 rules is interpreted in such a way that, for example, the seventh rule is

$\mathbf{R}_7$: IF e is P AND ce is N THEN u is ZO. (26)

Linguistically expressed, this seventh rule (for example, in the ball controller) is:

$\mathbf{R}_7$: IF the ball is on the right side AND
is moving left THEN do not apply the force on the cart. (27)

In order to have the upper interpretation of the seventh rule (25), (27) clear, the meanings of the denotations from the rule base (25), Table 1, should be given. In the rule base the labels N, Z and P are labels of the primary fuzzy sets in IF part of the rules (N for Negative, Z for Zero, and P for Positive). Fig.7 depicts the membership functions of these fuzzy sets, which map each element of the universe of discourse in the corresponding value of the membership degree.

The labels SNO (Strong Negative Output), MNO (Moderate Negative Output), ZO (Zero Output), MPO (Moderate Positive Output), and SPO (Strong Positive Output) are labels of output fuzzy sets, singletons, Fig.8

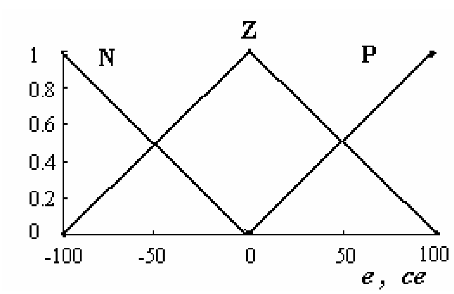

Figure 7. Triangular fuzzy sets.

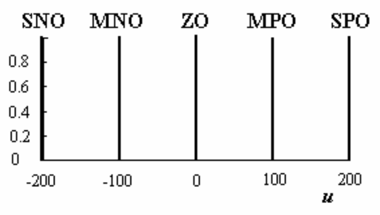

Figure 8. The output sets, singletons.

Table 1 is another way of representing the rule base (25). This representation of a rule base is the basis for improving controller execution speed by its computer implementation as a table based controller. The run-time inference is reduced to a table lookup, which is a lot faster than finding the necessary rules in the rule base in the form like (25), at least when the correct entry can be found without too much searching. An execution speed improvement is realized through realization of the so-called *look-up table*, using a rule base in the form given by table, like Table 1. If the universes of discourse in a fuzzy controller are discrete, it is always possible to pre-calculate all thinkable combinations of inputs and their consequences before putting the controller into operation. The relation between all input combinations and the output are arranged in a look-up table. With two inputs and one output, like in the considered problem, the look-up table is a two-dimensional look-up table. With three inputs, as in the fuzzy PID controller, the table is three-dimensional. Look-up table obtains improvement in execution speed whether the controller is implemented in software or in hardware.

A look-up table is based on a representation of input variables universes of discourse by discrete points. In this case, the trade-off should be made between accuracy and computer resources demand.

In the considered problem, no matter which form of the rule base is used, (25), or given by Table 1, the conjunction AND in "IF" parts of the rules is interpreted as the t-norm *min*. Inference is in the form of sum-multiplication, that is, Larsen's fuzzy system is used. The system input values are the crisp values, and that is the reason for fuzzification of input values in fuzzy singletons. Defuzzification, [12], is done using Center-of-Gravity-with-Singletons method. So, the following expression holds:

$$u = \frac{\sum_i \mu(s_i) s_i}{\sum_i \mu(s_i)}, \tag{28}$$

where: s_1 – is the position of the i-th singleton in the domain, and $\mu(s_1)$ is the activation level of the i-th rule.

As the measure of quality of the system response, determined by the quality of the rule base, usual performance index is used, *Integral Absolute Error*, *IAE*, given by the following expression:

$$IAG = \frac{1}{N} \sum_{k=1}^{N} \left| e(kT) \right|, \tag{29}$$

where N is the number of samples, and time is $t = kT$.

The smaller *IAE*, the better the system performance. The value *IAE* is used to compare transient responses, obtained under different conditions, or for different values of parameters, which have been changed, for the same stop time.

On the bases of observation of the response quality measure of the system *IAE*, through the experiments, the behaviour of the fuzzy controller was observed.

Results of computer experiments

For computer experiments the MATLAB for Windows software implementation consisting of two parts was used: 1) the implementation of the traditional PD controller of the cart-ball system based on the model (17) and, 2) the implementation of the fuzzy controller in the rule version (25), and in tabular version, (Table 1). The performances of these controllers have been compared. The fuzzy PD controller can have slower response time and smaller overshoot, than its traditional counterpart. The control signal coming from the fuzzy controller is smoother than the control signal coming from the traditional controller. As the traditional PD controller is, the fuzzy controller is also sensitive to noise and to sudden changes in the referent value y_{ef}. In general, the fuzzy PD controller is expected to behave as least as good as the traditional (the crisp) PD controller. This is theoretically based on the fact that some fuzzy systems are universal approximators, [11]. The described fuzzy PD controller is among such fuzzy systems. Inputs to the fuzzy PD controller, Fig.6, are the error e_n and the derivative of the error de/dt, usually called the change in the error ce_n, expression (20). The controller output is the control signal u_n, which is a nonlinear function of e_n and ce_n

$$u_n = f\left(e_n, ce_n\right) \tag{30}$$

The function f is the set of implications in the rule base of the fuzzy controller, Fig.6. A linear approximation of a fuzzy PD controller is the controller where the control surface is replaced by a linear plane. This is equivalent to replacing the rule base f with an ordinary summation, see also Fig.6:

$$U_n = \left(ge * e_n + gce * ce_n\right) * gu \,. \tag{31}$$

The expression (31) is similar to the expression for a PD controller (expression (22) for $K_I = 0$, and adequate values of ge, gce, gu, K_p, K_D). So, for a linear fuzzy PD controller it should be possible to have the same response as its crisp counterpart. Generally, having in mind that the range for φ in fuzzy PD controller is not restricted to [-0.22 rad, 0.22 rad] (the expressions (9), for the crisp PD controller), it is expected that a fuzzy PD controller performs at least as well as a crisp PD controller.

The fact that some fuzzy systems are universal approximators may also be the basis for generation of a new programming paradigm, a fuzzy rule based programming paradigm.

If the behaviour of the fuzzy controller implemented with the rule base (25) is compared with the behaviour of the tabular fuzzy controller with same rules, Table 1, [10], for this relatively small rule base, it has been noticed that the tabular controller is faster for 0.01 sec relative to the first one, using software implementations on PC at 300 MHz. It is expected that this effect should be stronger if the larger rule base is considered.

The fine-tuning of the fuzzy controller has been done. The fine-tuning is done heuristically, [1], and the gains in the fuzzy ball controller, Fig.6, are obtained: $ge_b = 230$, $gce_b = 45$, $gu_b = 0.5$. For optimal fuzzy PD controller the value of the performance index is obtained: *IAE*=4.0855, for the starting conditions: $y = 0\,\text{m}$, $\dot{y} = 0\ \text{m/s}$, $\varphi = 12^\circ$, $\dot{\varphi} = 0^\circ/\text{sec}$.

Analysis of changes in uncertainty

In order to analyze the influence of uncertainty changes on the behaviour of the considered fuzzy system, some rules have been removed from the rule base given by Table 1. That corresponds the enlarging of the uncertainty in the system. The fuzzy controller behaviour has been changed with the increasing of the uncertainty. The performance index *IAE* has been observed as the indicator of system performances. If the rule base (25) (Table 1) has been replaced with the rule base with 7 rules, given by Table 2,

Table 2. Look-up table with removed rules

e ↓ / ce →	N	Z	P
N	/	MNO	ZO
Z	MNO	ZO	MPO
P	ZO	MPO	/

↑ u

then *IAE*=4.1392. So, the performance index *IAE* is increasing. The growth is mild until the percentages of the removed rules reach 60% of the number from the beginning.

In the case of the probabilistic uncertainty model, [13], which contains the member with the noise ω_n, the value of variance of the noise should be assumed. If $Var\left(\omega_n\right)$ increases, the state equations of the system expresses a greater uncertainty. In the probabilistic model of uncertainty, quadratic root of the mean quadratic error of the output represents the integral index similar to the index *IAE*, used in this paper. The growth of the quadratic root of the mean quadratic error, with the growth of $Var\left(\omega_n\right)$, for small values of $Var\left(\omega_n\right)$ is large, and than this growth is mild, hence, an opposite process relative to the process with fuzziness.

Robustness analysis of fuzzy control rules

The robustness of the considered fuzzy system, fuzzy controller, has been investigated by inserting the wrong rules in the rule base, which corresponds to the situation when there is a lack of correct expertise, or when the problem is highly non-structural. For the rule base with two wrong rules (3rd and 7th.), given by Table 3, instead of the rule base given by Table 1, the value of the performance

Table 3. Look-up table with wrong rules

e ↓ / ce →	N	Z	P
N	SNO	MNO	MPO
Z	MNO	ZO	MPO
P	MNO	MPO	SPO

↑ u

index *IAE*=4.6 is obtained. The fuzzy controller is more sensitive to wrong, than to missed rules. In case of wrong rules, the fuzzy system adjusts relatively quickly, and it is partly stable.

Results of some other analyses

In the software simulation of the fuzzy PD controller, the experiments were organized with the different shapes of primary fuzzy sets membership functions, [14].

For the described controller parameters, and primary fuzzy sets membership functions given by Fig.7, the error e is obtained as given by Fig.9. It has a small overshoot, and the performance index is *IAE* = 4.1392.

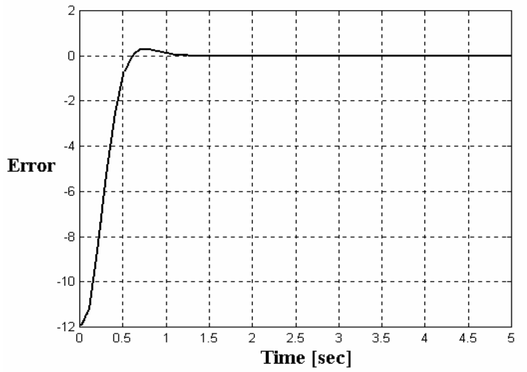

Figure 9. Error of the output relative to the setpoint value, for triangular fuzzy sets.

In experiments, the shapes of membership functions of primary fuzzy sets were changed. For the trapezoidal primary fuzzy set, Fig.10, the error diagram is very similar to one given by Fig.9, with relatively smaller overshoot and better performance index, *IAE*= 4.086.

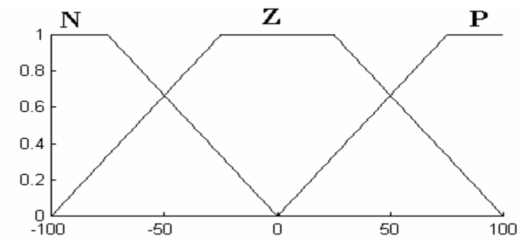

Figure 10. Trapezoidal input fuzzy sets

For the nonlinear (bell shaped) primary fuzzy set, Fig.11, the error diagram is given by Fig.12, with much worse characteristics and *IAE*= 14.4621.

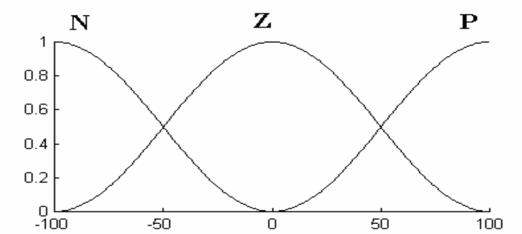

Figure 11. Nonlinear input fuzzy sets

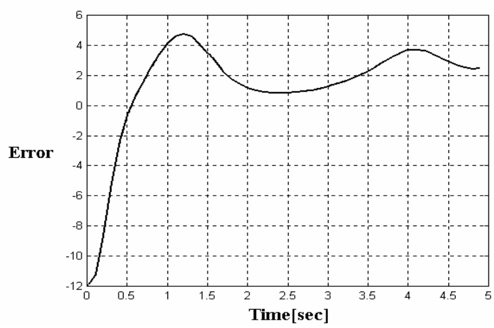

Figure 12. Error of the output relative to the set point value, for the bell shaped input fuzzy sets.

For the nonlinear (flatten bell shaped) input fuzzy set, Fig.13, the error diagram is given by Fig.14, with *IAE*= 5.9692.

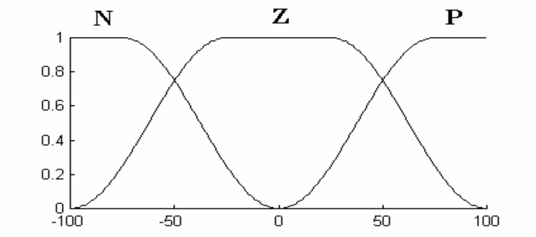

Figure 13. Flatten bell shaped input fuzzy sets.

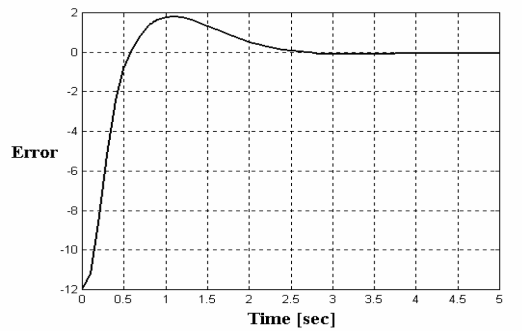

Figure 14. Error of the output relative to the setpoint value, for flatten bell shaped input fuzzy sets.

For combined linear and nonlinear input fuzzy sets, Fig.15, the error diagram is similar to the one given by Fig.9, *IAE* = 4.2018.

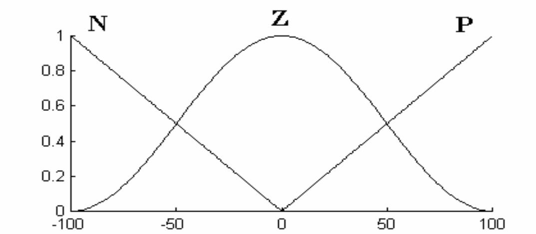

Figure 15. Combined linear and nonlinear input fuzzy sets

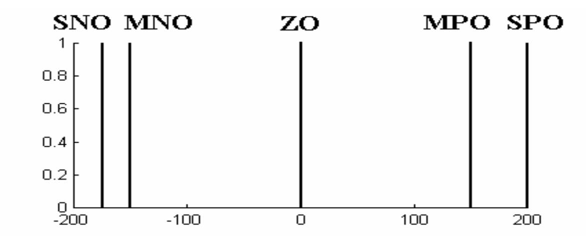

Figure 16. Different singletons as output fuzzy sets

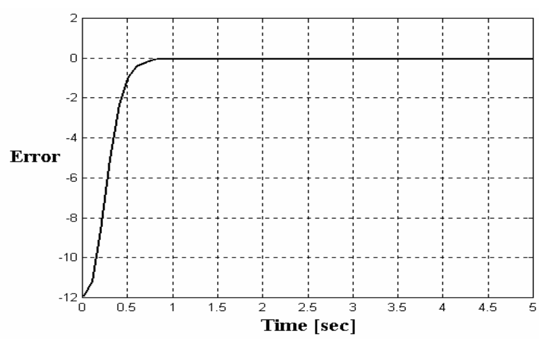

Figure 17. Error of the output relative to the set point value, for the combined input fuzzy sets and changed output singletons.

For combined linear and nonlinear input fuzzy sets, Fig.15, and changed output fuzzy sets, singletons, Fig.16, the error diagram is smooth, nice, with no overshoot, Fig.17, and that controller results with the best *IAE*=4.0855, among the cases experimented with.

From those experiments, it is obvious that the fuzzy system is sensitive to changes of shapes of the membership functions of primary fuzzy sets, and that optimal choice of the shapes for the considered fuzzy system exists. This result is a new one; it has not been given in the references.

Conclusions

The computer model of the kind of inverted pendulum system, implemented as the cart-ball system has been considered. As result of computer experiments with fuzzy PD controller used to control the cart-ball system, it can be said that when uncertainty increases in the fuzzy system, the fuzzy controller shows first mild, and than more intensive performance aggravation. Also, the robustness of the controller is analyzed experimentally by inserting the wrong rules in the rule base: it has been found that fuzzy PD controllers are robust; they adjust to the disturbances generated from incorrect rule base and that the fuzzy controller is more sensitive to wrong, than to missed rules. Further, the results of the experiments with different shapes of primary fuzzy sets membership functions are given: it has been concluded that the fuzzy system is sensitive to changes of shapes of those functions and that the optimal shapes exists. This result is a new one, it has not been considered in references till now.

It can be said that, in general, it is expected that the fuzzy PD controller is at least as good as the traditional PD controller, because linear fuzzy PD controller should have the same response as its crisp PD counterpart.

Some interesting research problems have been noticed. One of them is the problem of performance study of other types of fuzzy controllers, for example, of fuzzy PID controllers. Performances of fuzzy controllers which consist of several fuzzy controllers are also an interesting research problem. In the problem considered in this paper, the ball is rolling across the arc made of a pair of parallel rails, fixed on the cart, that is in the plane. One of the directions of the generalization is to consider the ball rolling across the half ball that is in the three-dimensional space. From the application point of view, it is interesting to consider the problem of inverted pendulum implemented as the system cart-pole, where the flexible pole is hinged on a movable cart. Flexibility of the pole introduces additional complexity into the system. Results obtained in such research tasks may be of interest for realizations of different kinds of electromechanical systems.

In the paper, a possible research direction in the domain of computer engineering has been indicated. The conclusion based on the experiments, where, generally speaking it is expected that the fuzzy PD controller performances are at least as good as those of the traditional PD controller, is theoretically based on the fact that some fuzzy systems are universal approximators. This fact may also be the basis of generating a new programming paradigm, a fuzzy rule based programming paradigm.

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Rezultati računarsko-eksperimentalne analize performansi jedne vrste rasplinutih regulatora sistema inverznog klatna

Metodi rasplinuto-logičkog upravljanja su od interesa za praktične primene. Stoga, važno je znati njihova svojstva. U ovom radu dati su neki rezultati računarsko-eksperimentalnih analiza ponašanja rasplinutih-logičkih regulatora sistema inverznog klatna u varijanti kolica-loptica. Eksperimentalno je analiziran uticaj promena u neizvesnosti u regulatoru: uočeno je da se performanse regulatora neznatno pogoršavaju do izvesnog procenta uklonjenih pravila, što je suprotno u odnosu na proces sa probabilističkim modelom neizvesnosti. Takođe, eksperimentalno je analizirana robusnost regulatora: uočeno je da je rasplinuti regulator osetljiviji na pogrešno, nego na nedostajuće pravilo. Dalje,

dati su rezultati eksperimenata sa različitim oblicima funkcija pripadanja primarnih rasplinutih skupova: zaključeno je da je rasplinuti sistem osetljiv na promene oblika ovih funkcija i da postoje optimalni oblici tih funkcija. Ovo je novi rezultat: taj rezultat nije dat u literaturi. Kao zaključak rada date su karakteristike diskutovanog rasplinutog PD regulatora. Zaključak, zasnovan na rezultatima računarskih eksperimenata je da je ponašanje rasplinutog PD regulatora, primenjenog u razmatranom problemu, bar tako dobro kao ponašanje jasnog (tradicionalnog) PD regulatora. Ukazano je na moguće pravce daljih istraživanja u razmatranoj paradigmi.

Ključne reči: **ekspertni sistem, rasplinuta logika, rasplinuti sistem, rasplinuti regulatori, robusnost sistema.**

Результаты вычислительно-экспериментального анализа характеристик одного сорта размытых контроллеров системы инвертирующего маятника

Методы размыто-логического управления от большого интереса для практических применений, и из-за того очень важно знать их особенности. В этой работе приведены некоторые результаты вычислительно-экспериментального анализа поведения размытых - логических контроллеров системы инвертирующего маятника в варианте тележка-шарик. Экспериментально анализировано влияние изменений в неизвестности в контроллере: обнаружено, что характеристики контроллера незначительно ухудшаются до определенного процента устранимых правил, что противоположно по отношению к процессу с пробабаллистической моделью неизвестности. Также, экспериментально анализирована живучесть контроллера: обнаружено, что размытый контроллер является более чувствительным на ошибочное, чем на нехватающее правило. Дальше, здесь приведены результаты экспериментов со различными формами функций принадлежности примарных размытых комплектов: сделан вывод, что размытая система более чувствительна на изменения форм этих функций и что существуют оптимальные формы этих функций. Это новый результат: этот результат не приведен в литературе. Как вывод этой работы приведены характистики обсужденного размытого ПД контроллера. Вывод, обоснован на результатах вычислительных экспериментов, такой - поведение размытого ПД контроллера, применяемого в рассматриваемой проблеме, такое хорошее по крайней мере как поведение ясного (традиционного) ПД контроллера. Здесь указано на возможные пути (курсы) дальнейших (будущих) исследований в рассматриваемой парадигме.

Ключевые слова: система специалистов, размытая логика, размытая система, размытые контроллеры, живучесть системы.

Les résultats de l'analyse informatique expérimentale des performances d'une sorte de régulateurs diffus chez le système de la pendule intervertie

Les méhodes du contrôle logique diffuse peuvent être de l'intérêt dans l'usage pratique.C'est pourquoi il est important de connaître leurs propriétés. Dans ce travail sont donnés des résultats des analyses numériques du comportement des régulateurs logiques diffus chez le système de la pendule intervertie en variante chariot-balle. L'analyse est basée sur les essais expérimentaux concernant l'influence des changements de l'incertitude du régulateur : on a constaté que les performances du régulateur s'empirent un peu jusqu'à un pourcentage des règles supprimées, ce qui est opposé au procès avec le modèle de l'incertitude probable. On a également analysé par voie expérimentale la robustesse du régulateur ; on a constaté que le régulateur diffus est plus sensible à la règle inexacte qu'à la règle qui manque. Ensuite on a donné les résultats d'essais des différentes formes de fonctions faisant partie des assembles diffus primaires ; on a conclu que le système est sensible aux changements des formes de ces fonctions et que les formes optimales de ces fonctions existent. Ce résultat est nouveau : il ne figure pas dans les livres de spécialités. Pour finir, on a donné les caractéristiques du régulateur PD diffus. La conclusion, basée sur les résultats des essais numériques, est que le comportement du régulateur PD diffus est aussi bon que le comportement du régulateur concis (traditionnel). On a aussi indiqué les possibilités des futures recherches dans le paradigme considéré.

Mots clés: **système d'expert, logique diffuse, système diffus, régulateurs diffus, robustesse de système.**