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G.R.O.M. software for determining the values of aerodynamic derivatives for isolated wing in supersonic flow

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Software G.R.O.M., presented in this paper, is developed in order to calculate the values of aerodynamic derivatives – 1 the lift force with respect to the angle of attack; 2 drag force with respect to the angle of attack on the square, as static derivatives; 3 roll-dumping moment with respect to the rate of roll; 4 pitch-dumping moment with respect to the rate of pitch, as dynamic derivatives. Linearized theory of supersonic flow is applied for the wing of vanishingly small thickness with the end parallel to free stream, with the condition that leading and trailing edges must be supersonic with respect to free-stream velocity component normal to the edge. It is assumed that the leading edge is sweptback, while two types of the trailing edge were tested (sweptback and unswept).

The results of calculation of aerodynamic derivatives, obtained by G.R.O.M. program, were compared to the corresponding results calculated by Missile Datcom 97 program. High accordance of these two sets of data could be assumed as validation of G.R.O.M. software, signifying its usefulness for engineering practice in the supersonic wing aerodynamic loading calculations.

Key words: **aerodynamic derivates, supersonic flow, wing, aerodynamic load, lift force, drag force, roll-dumping moment, pitch-dumping moment, software.**

Symbols:

A – aspect ratio
$B=\sqrt{M^2-1}$
b – wing span
c_r – root cord
c_t – tip cord
Δc_p – pressure difference coefficient
c_L^{α} – aerodynamic derivative of the lifting force with respect to the angle of attack
$c_l^{w_x}$ – aerodynamic derivative of the roll dumping moment with respect to the rate of roll
$c_m^{w_y}$ – aerodynamic derivative of the pitch dumping moment with respect to the rate of pitch
$c_D^{\alpha^2}$ – aerodynamic derivative of the drag force with respect to the angle of attack on the square
ccl – spanwise load
e – y-wise coordinate of intersection of Mach cone from the root tip of the wing and trailing edge
h – y-wise coordinate of intersection of Mach cone from the tip of the wing and trailing edge
j – y-wise coordinate of intersection of Mach cone emanating from the root tip of the wing, reflected on the end of the wing, and trailing edge

$$K=\frac{\operatorname{ctg}\Lambda_{TE}}{\operatorname{ctg}\Lambda_{LE}}=\frac{AB(1+\lambda)}{AB(1+\lambda)-4mB(1-\lambda)}$$

M – Mach number
S_1 – area of integration
v – velocity
w_x – angular roll velocity
w_y – angular pitch velocity
x,y,z – rectangular coordinates
x_1, x_2 – auxiliary rectangular coordinates
Γ – spanwise distribution of circulation
α – angle of attack
Φ – perturbation potential
μ – Mach line angle
Λ_{TE} – sweep of trailing edge
Λ_{LE} – sweep of leading edge
λ – taper ratio

Introduction

IN carrying out the aerodynamic calculations, knowledge of aerodynamic spanwise loading is of essential importance. Based on the linearized theory of supersonic flow and partly modified functional equations for the circulation of perturbation velocity potential, given in ref. [1], G.R.O.M. software was developed. With respect to the calculated distribution of the spanwise loading depending on the angle of attack, constant rate of roll and constant rate of pitch, values of the following aerodynamic derivatives were calculated:

- aerodynamic derivative of the lift force with respect to the angle of attack
- aerodynamic derivative of the drag force with respect to the angle of attack, as static derivatives,
- aerodynamic derivative of the roll dumping moment with respect to the rate of roll, and
- aerodynamic derivative of the pitch dumping moment with respect to the rate of pitch, as dynamic derivatives.

They were calculated for a series of thin wings with the end of the wing parallel to free-stream, and with supersonic trailing and supersonic or subsonic leading edges.

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Basic equations explaining the connection between the circulation of the perturbation velocity potential of the trailing edge and spanwise loading are given in this work, as well as the final expressions for determining the derivatives mentioned above.

Basic assumptions on which this research is based are:

1. Wing is thin.
2. Values of the aerodynamic derivatives are calculated for an isolated wing.
3. Taper ratio is arbitrary.
4. End of the wing is parallel to free-stream.
5. Leading and trailing edge are strait lined, and their angles are constant.
6. Angle of the leading edge is positive i.e. leading edge is sweptback.
7. Angle of the trailing edge can be: zero, positive (sweptback trailing edge), and negative (swept-forward trailing edge), which is taken into account by the sign of factor k. Factor k represents the ratio of cotangent of the angle of the trailing edge with cotangent of the angle of the leading edge, and it can be positive (for both leading and trailing edge being positive-both edges being sweptback), negative (trailing edge being swept-forward) and zero (unswept trailing edge).
8. Theory is valid for the range of Mach numbers for which both leading and trailing edges are supersonic, or leading edge is subsonic and trailing edge is supersonic and free-stream Mach number equal or greater than 1.4.
9. Mach lines emanating from one wing do not influence the other, i.e. there is no wing-to-wing interference

Tested shapes of the wings

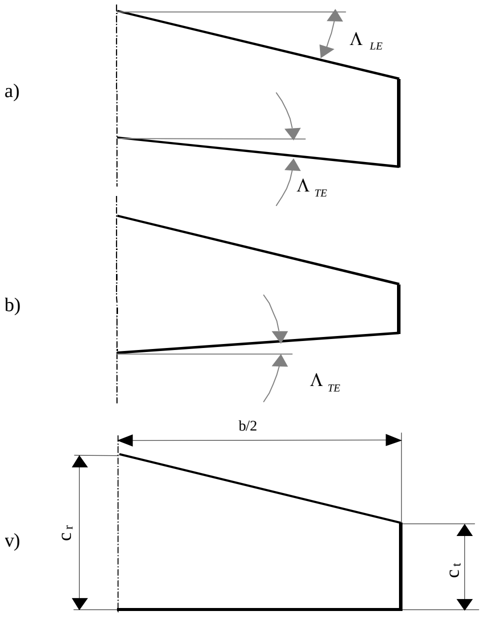


Figure 1. Positions of the trailing edge when leading edge is sweptback: a) sweptback leading and trailing edges – family of PSNI wings ($\Lambda_{LE} > 0$, $\Lambda_{TE} > 0$); b) sweptback leading and sweptforward trailing edge – family of PNKI wings ($\Lambda_{LE} > 0$, $\Lambda_{TE} < 0$); v) sweptback leading and unswept trailing edge – family of PNNI wings: ($\Lambda_{LE} > 0$, $\Lambda_{TE} = 0$)

Three families of the wings are defined with respect to trailing edge sweep angle (Fig.1). Basic requirement is not the sign of the trailing edge sweep angle, but that trailing edge for that value of sweep angle is supersonic.

In Tables 1 and 2 all possible positions of the Mach cones, as well as mutual influence of those cones, are given, with respect to the condition that there is no influence of one wing to another, for tested wing families, along with the review of possible subcases:

Table 1. Wing shape and position of the Mach cones and integration borders for wings with sweptback leading and trailing edges (PSNI family of wings):

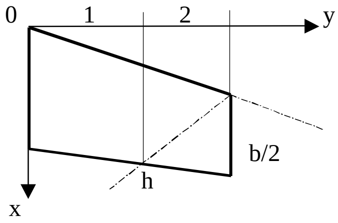
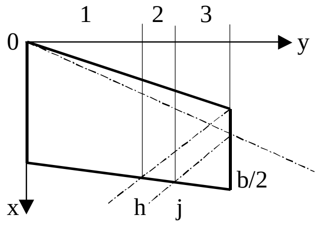
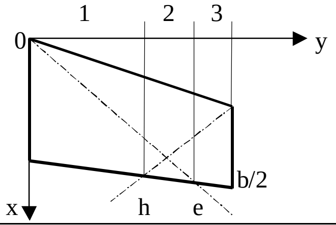
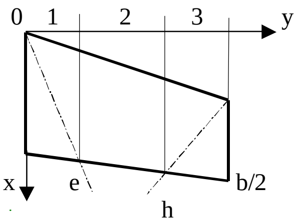
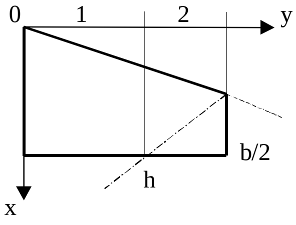
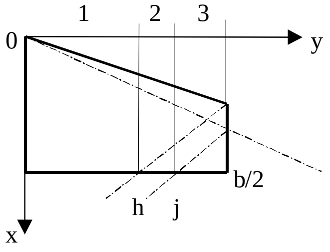
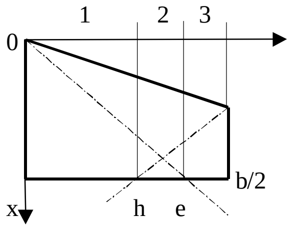
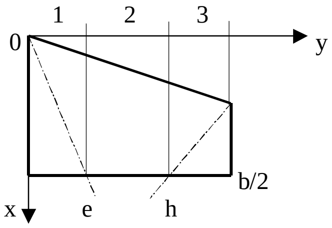
SUBCASE	Subcase tag
	PSNI 2
	PSNI 5
	PSNI 4
	PSNI 3

Table 2. Wing shape and position of the Mach cones and integration borders for wings with sweptback leading and unswept trailing edges (PNNI family of wings):

Subcase	Subcase tag
	PNNI 2
	PNNI 5

	PNNI 4
	PNNI 3

It should be emphasized that within the developed software the case of the wings with sweptforward trailing edge (family of PNKI wings) was not considered because the form of functional expressions for circulation Γ is the same as for the case when the trailing edge is sweptback (family of PSNI wings). The newly developed program G.R.O.M. offers the possibility of determining the mentioned aerodynamic derivatives for this family of wings by changing the sign of factor *k* located in the main part of G.R.O.M. software.

Position of the coordinate system is the same as in the NACA TN 2643 rapport – *x*-axis has the direction of freestream, *y*-axis lies along the right half-wing and *z*-axis is pointed upwards.

Method

Generally, determination of the spanwise loading requires knowledge of the pressure distribution on the wing surface, or knowledge of the perturbation velocity potential along the trailing edge. This potential can be calculated by the use of Evvard`s method as:

$$\Phi_{TE}\left(x,y\right) = -\frac{1}{\pi}\iint_{S_1}\frac{\Phi_z}{\sqrt{\left(x-x_1\right)^2 - B^2\left(y-y_1\right)^2}}\,\mathrm{d}x_1\,\mathrm{d}y_1 \qquad (1)$$

Function of the perturbation potential must satisfy the linearized, partial-differential equation of the steady flow and the boundary conditions linked to the movements of the wing referring to the normal components of the velocity for $z = 0$.

Spanwise loading and perturbation velocity potential along the trailing edge are linked by the equation:

$$cc_l = \int_{LE}^{TE}\Delta c_p\,\mathrm{d}x = \frac{4}{v}\Phi_{TE} \qquad (2)$$

Distribution of the spanwise circulation and loading are linked by the following expression:

$$\Gamma = 2\int_{LE}^{TE}\Phi_x\,\mathrm{d}x = 2\Phi_{TE} = \frac{v}{2}cc_l \qquad (3)$$

Boundary conditions, integration area, determination procedure and expressions for perturbation potential are given in the NACA TN 2643 technical note [1] in more detail – equations 3a through 3c, and 5 through 12, and in Figures 2 and 3.

Algorithm description

ENTRY DATA: Concept of G.R.O.M. software is that it consists of two totally separate segments making two separate subprograms for determining the aerodynamic derivative values for the two families of wings: family of PSNI and family of PNNI wings.

First requirement in the use of G.R.O.M. software is to determine the family of wings to which the wing, whose values of derivatives are needed, belongs. That is solved by entering number 1 for PNNI family of wings, and entering number 9 for PSNI (or PNKI) family of wings, i.e. selection of the family of wings depends on the trailing edge sweep.

After determining the family of wings that the particular wing belongs to, the rest of the entry data are being inputted:

- for the family of PNNI wings:

1. Cr – root cord length, in meters
2. LAMBDA – taper ratio
3. BKRILA – wing span, in meters
4. M – Mach number

- for the family of PNNI wings:

1. Cr - root cord length, in meters
2. LAMBDA – taper ratio
3. BKRILA - wing span, in meters
4. LAMLESTEP – leading edge sweep, in degrees
5. M – Mach number

For family of the PNKI wings entry data is the same as for the family of the PSNI wings with the difference that the sign of factor k in the main part of G.R.O.M. program needs to be changed.

Based on the leading edge sweep (for PNNI wing family), or the trailing edge sweep (for families of PSNI or PNKI wings) entry data computations, as well as the angles of the Mach cones, and their position, thus determining the subcase to which the given wing belongs to, by the use of corresponding IF conditions, these are being executed.

METHOD OF NUMERICAL INTEGRATION: Depending on the positions of the Mach cones and their interaction on the wing surface, corresponding zones with corresponding flow field are defined. Boundaries of integration, for the given expressions for distribution of circulation which determine the given flow field, are determined by intersection points of the Mach cones and trailing edge. The values of boundaries of integration are calculated in normalized form – *y*/(*b*/2), where *y* represents y-wise coordinate of the intersection point of the Mach cone with the trailing edge.

As a method of numerical integration for the integration of expressions for velocity circulation (Γ), in order to obtain the needed values of aerodynamic derivatives, Simpson's method was chosen due to its dependability and accuracy.

All of the functional expressions for determining the values of aerodynamic derivatives are written in the body of the corresponding subroutine. Which function is to be integrated and the value called back in the corresponding subprogram, program decides based on the numerical value of the indicator (ind) of each function, thus allowing one integration subroutine to integrate more than one external function.

METHOD FOR DETERMINATION OF THE AERODYNAMIC DERIVATIVES VALUES: In the NACA TN 2643 paper [1] the expressions for velocity circulation obtained by integration along the cord are presented, determining, in that way, functional expressions for the distribution of the circulation Γ in normalized form, depending on the wing geometry characteristics and characteristics of the flow field for constant values of the angle of attack, rate of roll and rate of pitch. Based on this circulation, calculation of the needed aerodynamic derivatives is done, by the process of its integration along the wing span and multiplying and dividing with corresponding variables in accordance with the boundary conditions.

Expressions for circulation for constant angle of attack are used for determining the value of aerodynamic derivative of the lift force with respect to the angle of attack, and aerodynamic derivative of the drag force with respect to the angle of attack on the square; expressions for circulation for constant rate of roll are used for determining the value of aerodynamic derivative of the roll-dumping moment with respect to the rate of roll; expressions for circulation for constant rate of pitch are used for determining the value of aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch, for isolated wing of vanishingly small thickness.

Taking into account the way of normalization of the expressions for circulation and the definitions of aerodynamic derivatives for forces and moments, final expressions for determining the needed aerodynamic derivatives are given in the following forms:

- aerodynamic derivative of the lift force with respect to the angle of attack

$$c_L^{\alpha} = \sum_{i=1}^{n} 2A \int_{dg}^{gg} \frac{\Gamma_{zone}}{v\cdot\alpha\cdot(b/2)}\, \mathrm{d}\left(\frac{y}{b/2}\right) \tag{4}$$

$$c_l^{w_x} = \sum_{i=1}^{n} -\frac{1}{2} \int_{dg}^{gg} \frac{\Gamma_{zone}}{w_x\cdot(b/2)^2}\, \mathrm{d}\left(\frac{y}{b/2}\right)^2 \tag{5}$$

- aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch

$$c_m^{w_y} = \sum_{i=1}^{n} -\frac{1}{2} \int_{dg}^{gg} \frac{\Gamma_{zone}}{w_y\cdot B\cdot(b/2)^2}\, \mathrm{d}\left(\frac{y}{b/2}\right)^2 \tag{6}$$

dg – value of upper integration boundary
gg – value of lower integration boundary

- aerodynamic derivative of the drag force with respect to the angle of attack on the square

$$\frac{D}{S_K q} = \frac{L^{\alpha}}{S_K q}\alpha^2 \Rightarrow c_D^{\alpha^2} = c_L^{\alpha} \tag{7}$$

From eq. (7) it is clear that the value of the aerodynamic derivative of the drag force with respect to the angle of attack on the square for wings of vanishingly small thickness is equal to the value of the aerodynamic derivative of the lift force with respect to the angle of attack, for such flow conditions for which there is no separation of the stream of the contour and there are no vortexes, i.e. for small angles of attack.

Validation of G.R.O.M. software – test results

In this work test results for two families of wings are presented:

1. PNNI – family of wings with sweptback leading and trailing edges
2. PSNI – family of wings with sweptback leading and unswept trailing edge

In order to validate newly developed software, sets of testing were executed to compare the results obtained by the use of this new computer program with data presented in references [1], and values obtained by the use of other programs (Missile Datcom 97).

"Internal" testing

While executing "internal" testing wing geometry and Mach number were varied.

Variation of the geometry – Fig. 4:

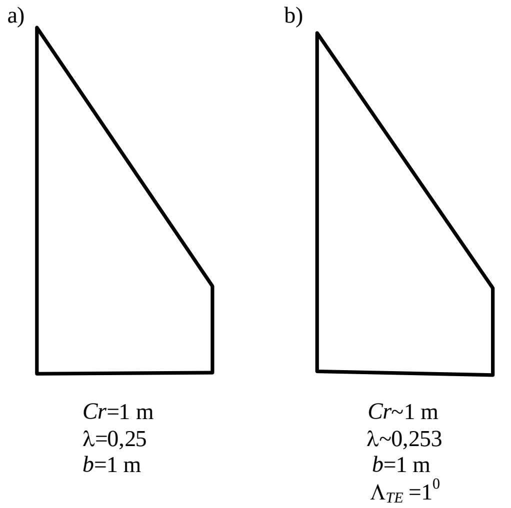


Figure 2. "Internal" testing – change of geometry: a) family of the PNNI wings; b) family of the PSNI wings

Excluding articles that contain factor k, which takes leading and trailing edge sweeps into account, out of functional expressions for PSNI family of wings, expressions for PNNI family of wings are obtained. For the case of unswept trailing edge $k\to\infty$ (family of the PNNI wings). For any other value of trailing edge sweep angle factor k has a final value.

That means that for a small value of the trailing edge sweep angle (1 degree) while sweep angle of the leading edge rests unchanged and aspect ratio and wing surface are slightly changed, results obtained for the PNNI family of wings and PSNI family of wings should be very close. This testing method shows the accuracy of formulas for these two families of wings and this is the reason for which this testing was conducted.

Review of the "internal" testing results will be presented only as data. There will be no graphical presentation of the results since graphs match almost 100%.

Indexes v and p occurring in Tables 5 - 12, along with the subcase tag, are used for marking the values of aerodynamic derivatives of roll-dumping and pitch-dumping moments.

Aerodynamic derivative of the lift force with respect to the angle of attack and aerodynamic derivative of the drag force with respect to the angle of attack on the square

Table 3. Review of comparison for PNNI2 and PSNI2 subcases:

M=1.6	PNNI2	PSNI2	deviation [%]
zone 1	2.574	2.581	0.26
zone 2	2.679	2.664	0.53
total	5.253	5.245	0.14

Table 4. Review of comparison for PNNI5 and PSNI5 subcases:

M=2.236	PNNI5	PSNI5	deviation [%]
zone 1	3.108	3.081	0.90
zone 2	0.624	0.628	0.59
zone 3	1.27E-06	1.272 E-06	0.12
total	3.733	3.747	0.36

Table 5. Review of comparison for PNNI4 and PSNI4 subcases:

M=2.5702	PNNI4	PSNI4	deviation [%]
zone 1	2.829	2.812	0.59
zone 2	0.148	0.150	1.12
zone 3	0.255	0.252	1.20
total	3.231	3.247	0.47

Table 6. Review of comparison for PNNI3 and PSNI3 subcases:

M=3.09	PNNI3	PSNI3	deviation [%]
zone 1	2.112	2.117	0.24
zone 2	0.319	0.325	1.60
zone 3	0.225	0.229	1.78
total	2.657	2.671	0.53

Aerodynamic derivative of the roll-dumping moment with respect to the rate of roll

Table 7. Review of comparison for PNNI2v and PSNI2v subcases:

M=1.6	PNNI2v	PSNI2v	deviation [%]
zone 1	-0.053	-0.052	0.88
zone 2	-0.163	-0.164	0.22
total	-0.216	-0.216	0.07

Table 8. Review of comparison for PNNI5v and PSNI5v subcases:

M=2.236	PNNI5v	PSNI5v	deviation [%]
zone 1	-0.127	-0.128	0.79
zone 2	-0.060	-0.059	0.35
zone 3	-1.32E-07	-1.324E-07	0.014
total	-0.186	-0.187	0.65

Table 9. Review of comparison for PNNI4v and PSNI4v subcases:

M=2.5702	PNNI4v	PSNI4v	deviation [%]
zone 1	-0.126	-0.127	0.83
zone 2	-0.015	-0.015	0.96
zone 3	-0.029	-0.029	0.69
total	-0.169	-0.171	0.82

Table 10. Review of comparison for PNNI3v and PSNI3v subcases:

M=3.09	PNNI3v	PSNI3v	deviation [%]
zone 1	-0.086	-0.086	0.64
zone 2	-0.032	-0.033	1.70
zone 3	-0.028	-0.029	1.39
total	-0.147	-0.148	1.00

Aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch

Table 11. Review of comparison for PNNI2 and PSNI2 subcases:

M=1.6	PNNI2p	PSNI2p	deviation [%]
zone 1	-0.429	-0.429	0.03
zone 2	-0.464	-0.461	0.46
total	-0.893	-0.891	0.25

Table 12. Review of comparison for PNNI5p and PSNI5p subcases:

M=2.236	PNNI5p	PSNI5p	deviation [%]
zone 1	-0.314	-0.314	0.07
zone 2	-0.071	-0.072	1.90
zone 3	-1.344E-07	-1.332E-07	0.12
total	-0.403	-0.403	0.17

Table 13. Review of comparison for PNNI4p and PSNI4p subcases:

M=2.5702	PNNI4p	PSNI4p	deviation [%]
zone 1	-0.238	-0.238	0.11
zone 2	-1.46E-02	-1.358E-02	7.40
zone 3	-2.443E-02	-2.257E-02	8.00
total	-0.276	-0.274	0.70

Table 14. Review of comparison for PNNI3p and PSNI3p subcases:

M=3.09	PNNI3p	PSNI3p	deviation [%]
zone 1	-0.141	-0.141	0.055
zone 2	-2.409E-02	-2.226E-02	8.23
zone 3	-1.87E-02	-1.777E-02	5.30
total	-0.184	-0.183	0.43

Change of Mach number – Fig.5

This set of testing refers to the leading edge. In the first case, the Mach cone angle is such that the leading edge is subsonic in that way that the Mach cone lies just in front of the leading edge (0.85 degrees). Second case is such that the leading edge is supersonic in the way that the Mach cone lies just behind (1.28 degrees) the leading edge. Small difference in Mach number value, in the cases of subsonic and supersonic leading edge, makes the aerodynamic derivative values mutually very similar.

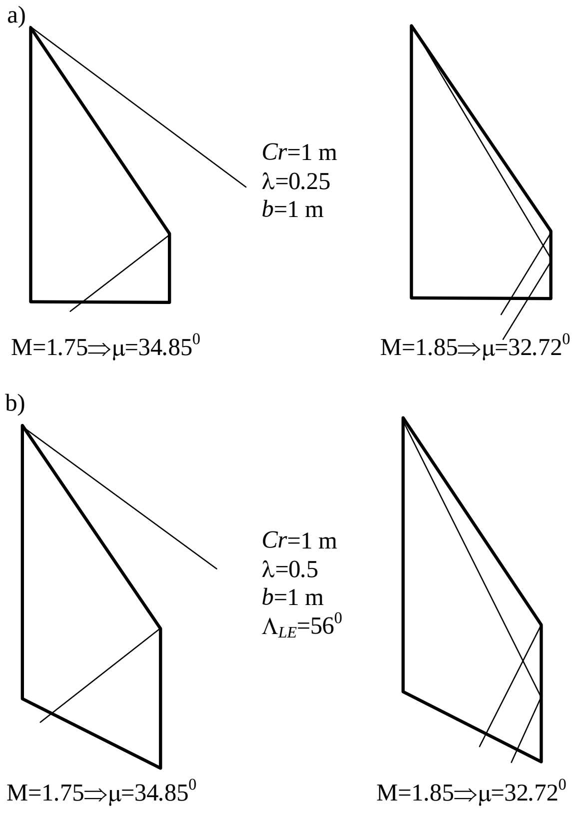


Figure 3. “Internal” testing by changing the Mach number: a) family of the PNNI wings; b) family of the PSNI wings

Aerodynamic derivative of the lift force with respect to the angle of attack and aerodynamic derivative of the drag force with respect to the angle of attack on the square

Table 15. Comparative review of derivative value for PNNI family of wings:

TAG	M=1.75	value
PNNI2	zone 1	3.334
	zone 2	-0.105
	total	-0.209

TAG	M=1.85	value
PNNI5	zone 1	3.431
	zone 2	0.162
	zone 3	0.851
	total	4.443

Deviation: 6.25%

Table 16. Comparative review of derivative value for PSNI family of wings:

TAG	M=1.75	value
PSNI2	zone 1	2.246
	zone 2	1.929
	total	4.175

TAG	M=1.85	value
PSNI5	zone 1	2.346
	zone 2	0.168
	zone 3	1.147
	total	3.984

Deviation: 4.79%

Aerodynamic derivative of the roll-dumping moment with respect to the rate of roll

Table 17. Comparative review of derivative value for PNNI family of wings:

TAG	M=1.75	value
PNNI2v	zone 1	-0.104
	zone 2	-0.105
	total	-0.209

TAG	M=1.85	value
PNNI5v	zone 1	-0.120
	zone 2	-0.012
	zone 3	-0.071
	total	-0.202

Deviation: 3.23%

Table 18. Comparative review of derivative value for PSNI family of wings:

TAG	M=1.75	value
PSNI2v	zone 1	-0.065
	zone 2	-0.146
	total	-0.211

TAG	M=1.85	value
PSNI5v	zone 1	-0.076
	zone 2	-0.011
	zone 3	-0.123
	total	-0.210

Deviation: 0.57%

Aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch

Table 19. Comparative review of derivative value for PNNI family of wings:

TAG	M=1.75	value
PNNI2p	zone 1	-0.484
	zone 2	-0.207
	total	-0.691

TAG	M=1.85	value
PNNI5p	zone 1	-0.457
	zone 2	-0.020
	zone 3	-0.128
	total	-0.606

Deviation: 14%

Table 20. Comparative review of derivative value for PSNI family of wings:

TAG	M=1.75	value
PSNI2p	zone 1	-0.412
	zone 2	-0.407
	total	-0.819

TAG	M=1.85	value
PSNI5p	zone 1	-0.398
	zone 2	-0.030
	zone 3	-0.288
	total	-0.716

Deviation: 14.43%

Comparative testing:

This testing was performed in order to determine the quality of the obtained results. For comparison of values for the aerodynamic derivatives of: the lift force with respect to the angle of attack, roll-dumping moment with respect to the rate of roll and for the drag force with respect to the angle of attack on the squire software Missile Datcom 97 was used.

A problem that occurred concerning comparison of the value of the aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch was the impossibility for Missile Datcom 97 to provide data for this dumping moment for an isolated wing. For the reason of completeness of this work values for aerodynamic derivatives of the pitch-dumping moment with respect to the rate of pitch, for PNNI and PSNI families of wings, will be presented without comparative data. As orientation control, a diagram of the value change of this derivative with respect to the Mach number change presented in ref. [2] can be used.

Comparative test results are given in graphical form (diagrams).

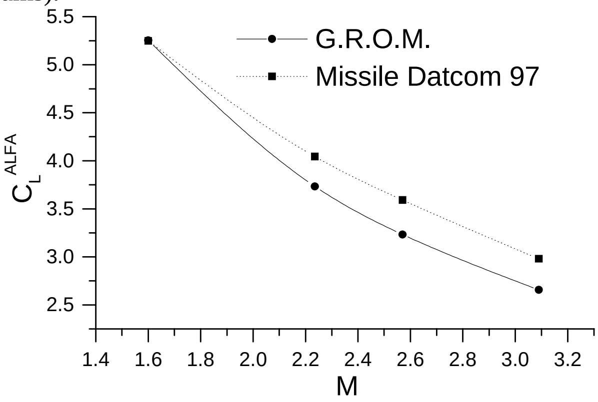


Diagram 1. Comparative review of the change of the aerodynamic derivative of the lift force with respect to the angle of attack for the PNNI family of wings

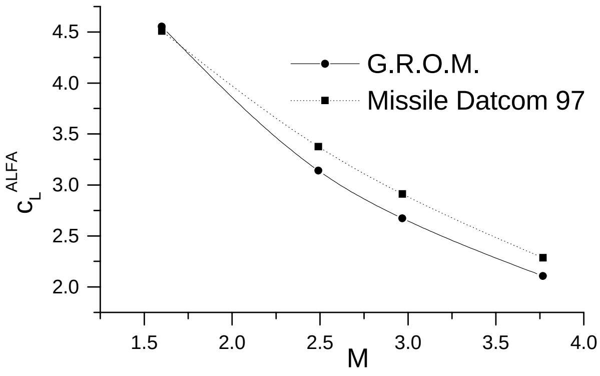


Diagram 2. Comparative review of the change of the aerodynamic derivative of the lift force with respect to the angle of attack for the PSNI family of wings

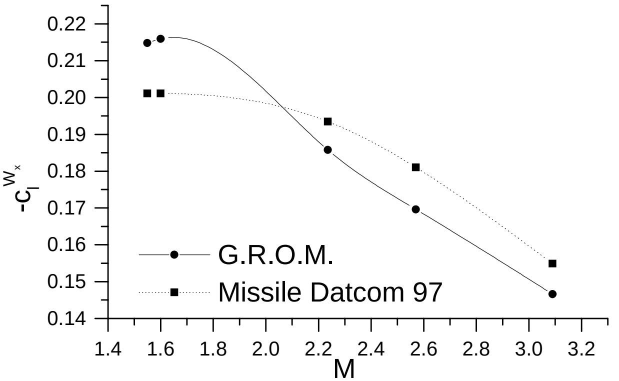


Diagram 3. Comparative review of the change of the aerodynamic derivative of the roll-dumping moment with respect to the rate of roll for the PNNI family of wings

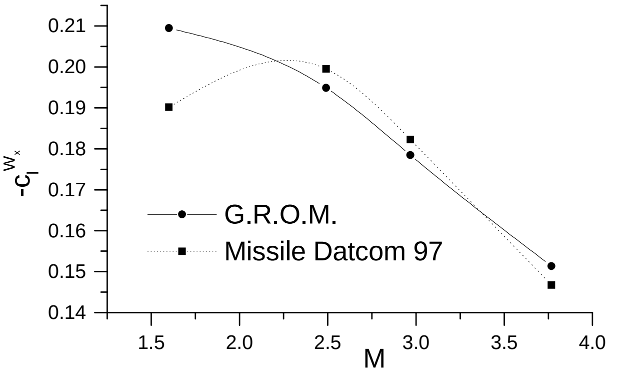


Diagram 4. Comparative review of the change of the aerodynamic derivative of the roll-dumping moment with respect to the rate of roll for the PSNI family of wings

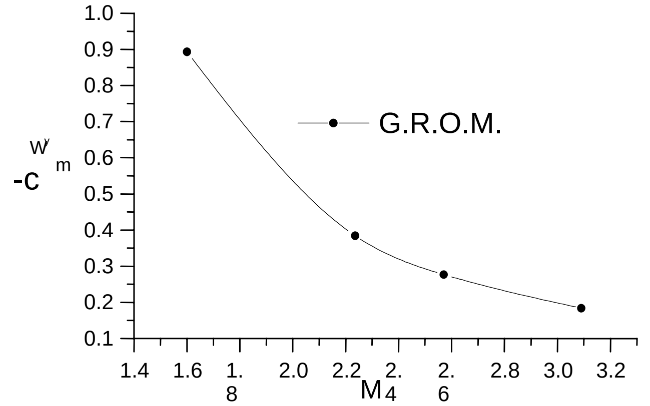


Diagram 5. Comparative review of the change of the aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch for the PNNI family of wings

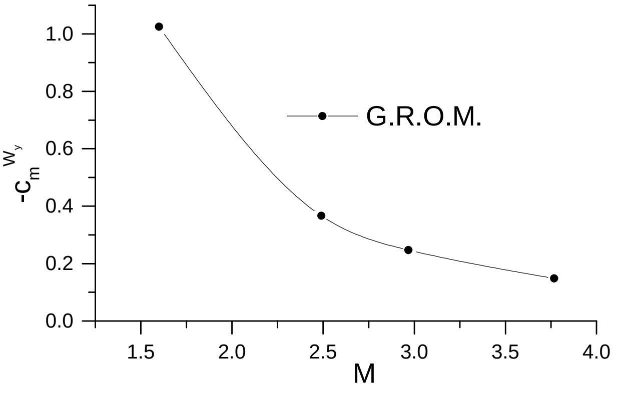


Diagram 6. Comparative review of the change of the aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch for the PSNI family of wings

It should be noted that diagrams for the aerodynamic derivative of the drag force with respect to the angle of attack on the squire, for PNNI and PSNI families of wings, are the same as diagrams D.1. and D.2., the only difference being that instead of tag c_L^{ALFA}, $c_D^{\mathbf{ALFASQUARE}}$ should stand.

Test results - comments

"INTERNAL" TESTING: While performing sets of "internal" testing, mistakes causing discontinuity of functions for velocity circulation arose, thus violating the basic principle of continuity of these functions, bringing about the necessity for modification of the functions. Testing results presented in this paper are obtained by the use of modified functional expressions.

COMPARATIVE TESTING: For this testing Missile Datcom 97 software was used and results presented in numerical form (Tables 25 - 30) and graphical form (Diagrams 1 – 6):

Aerodynamic derivative of the lift force with respect to the angle of attack and aerodynamic derivative of the drag force with respect to the angle of attack on the square

- Family of the PNNI wings – wings with sweptback leading and unswept trailing edges (Diagram 1). For Mach number value for which the leading edge is subsonic (M=1.6) values obtained by the use of both programs are practically the same. For the flow conditions causing the leading edge to be subsonic, G.R.O.M. software shows sharper gradient of decrease for derivative value. Beginning with $M = 2.2$ derivative values show the tendency of parallel curves with approximately constant deviation. Practically, this tendency of parallel curves and the same gradients of the derivative value decrease occur almost immediately after establishing the conditions for transition from flow conditions for which the leading edge is subsonic to conditions for which the leading edge is supersonic.
- Family of the PSNI wings – wings with sweptback leading and trailing edges (Diagram 2). For Mach number value for which the leading edge is subsonic (M=1.6) values obtained by the use of program G.R.O.M. shows slightly greater value than the one obtained using control program. Beginning with $M = 2.1$, practically almost immediately after establishing the range of Mach numbers for which the leading edge is subsonic to the range of Mach numbers for which the leading edge is supersonic, derivative values show the tendency of parallel curves with approximately constant deviation.

General conclusion is that both methods show very similar results, with small deviations and the same gradient in the range of Mach numbers 2.2-3.1 for the PNNI family of wings, and in the range of $M = 2.2$ -3.8 for the PSNI family of wings, i.e. for the flow conditions for which the leading edge is supersonic. In the range of the Mach numbers for which the leading edge is subsonic the values are very close to each other, but gradient characters are somewhat different.

This character difference in the zone that corresponds, or is very near, to the conditions for sonic leading edge is in concordance with transitional regime of projectile acceleration and its durance is very short.

Aerodynamic derivative of the roll-dumping moment with respect to the rate of roll

- Family of the PNNI wings – wings with sweptback leading and unswept trailing edges (Diagram 3). While testing this family of wings, the need for conducting the testing for one more value of Mach number arose in order to show the character of the diagram of values for this derivative obtained through the use of software G.R.O.M., so only for this derivative and this family of wings the value for $M = 1.55$ was taken into consideration.

For flow conditions for which the leading edge is subsonic it can be said that:

- In the range of the Mach numbers $M = 1.55$ to $M = 1.6$ curve gathered by G.R.O.M. has a tendency to decrease in dumping while the curve obtained by Missile Datcom stays practically constant, thus showing a difference in the gradients of derivative value change i.e. divergence of the results. This divergence remains within 8%.
- In the range of $M = 1.6$ to $M = 2.05$ (i.e. untill the existence of flow conditions for which the leading edge is supersonic) curves have the tendency of convergence. For the flow conditions for which the leading edge is sonic both programs show the same values.

For the flow conditions for which the leading edge is supersonic (M>2.05) it can be concluded that:

- Immediately after transition into the range of the Mach number for which the leading edge is supersonic, up to Mach number $M = 2.3$, slight divergence of the results i.e. slight change of the gradients occurs,.
- After that, Mach number value, up to the value of $M = 3.1$ curves stay almost parallel, with almost the same gradient of the derivative value change.
- Family of the PSNI wings – wings with sweptback leading and trailing edges (Diagram 4). From the diagram it can be noted that the curve for roll-dumping derivative values gathered by G.R.O.M. program is somewhat shifted to the right compared to the curve obtained by the use of Missile Datcom software.

For flow conditions for which the leading edge is subsonic it can be said that:

- In the range of $M = 1.6$ to $M = 2.05$ (i.e. untill existence of flow conditions for which the leading edge is supersonic) curves have the tendency of convergence.

For the flow conditions for which the leading edge is supersonic (M>2.05) it can be concluded that:

- Immediately after transition into the range of the Mach number for which the leading edge is supersonic, up to Mach number $M = 2.5$, slight divergence of the results, i.e. slight change of the gradients occurs.
- After that, Mach number value, up to the value of $M = 3.0$ curves stay almost parallel, with almost the same gradient of the derivative value change.
- In the range of Mach numbers $M = 3.0$ to $M = 3.768$ (value up to which testing has been carried out) curve of the data for derivative value obtained by the method G.R.O.M. shows sharper gradient compared to the curve of the data obtained by Missile Datcom 97. Intersection point of these two curves is in the middle of this range of Mach numbers. However, values obtained by these two methods stay very close.

Generally, it can be said that the deviation of the results obtained by two methods are small for all the flow conditions. Any significant difference in the value change occurs only for the low supersonic Mach numbers, and it is linked to flight velocities which are achieved and exceeded very quickly (practically buster phase). This deviation is also the consequence of the control program. Within Missile Datcom software interpolation of the data obtained by experimental testing in aero tunnel for different Mach number values and different wing geometry is used for calculation of this derivative, resulting with the influences of: the existing vortex emanating from the nose of the projectile, body-wing interference, deviations of the productional and constructive sweep angle of the leading edge, wing build-in deviations of the sweep angle, unsymmetrical position of the wings on the body of the projectile, non-uniform mass distribution, wing roughness, possible existence of the aerodynamic impurities, leading edge is never sharp, wing is not of vanishingly small thickness, on high Mach numbers (over 3.2) influence of aerodynamic heating increases as approaching the area of hypersonic-which software G.R.O.M. does not take into account. All these influences (except the last one) decrease with the Mach number increase.

Aerodynamic derivative of the pitch-dumping moment with respect to the rate of pitch

As mentioned before, due to inability to find referent data, for general comparison theoretical distribution of the change of value for this derivative with respect to the change of the Mach number for delta wings shown in Fig.8, ref. [2] can be used. Comparing this diagram to Diagrams 5 - 6 presented in this work, whose data are obtained by the use of G.R.O.M. software for two families of the tested wings, it can be concluded that the shapes and trends of diagrams are similar.

The change in position of the point around which the wing rotates does not change the shape of the curve, but it only translates it vertically. Any significant changes in derivative gradient with the change of the point about which the wing rotates should not be expected.

Conclusion

In the frame of this paper the developed program G.R.O.M. is presented. This software is used for determining the values of aerodynamic derivatives of the lift force with respect to the angle of attack and drag force with respect to the angle of attack on the square as static derivatives, roll-dumping moment with respect to the rate of roll and pitch-dumping moment with respect to the rate of pitch as dynamic derivatives for isolated wing of vanishingly small thickness with the end of the wing parallel to free stream, in supersonic flow. The method is particularly applicable for determining the influence of the wing on the tail surfaces.

Software was developed based on the linearized theory of the wing in supersonic flow and functional forms for the circulation of perturbation velocity potential, presented in ref. [1] and partly modified, with the condition that the leading and trailing edges must be supersonic with respect to free-stream velocity component normal to the edge. The leading edge is sweptback, while the trailing edge can be sweptback, swepforward or unswept. It was assumed that there is no wing-wing interference.

The presented test results show high level of correspondence for the calculated values of the aerodynamic derivative values with the results obtained by the use of control program Missile Datcom 97, proving use value of G.R.O.M. software for engineering purposes, when fast assessment of aerodynamic derivatives and load of isolated wing in supersonic flow is needed.

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Računarski program G.R.O.M. za određivanje vrednosti aerodinamičkih derivativa izolovanog krila supersoničnoj struji

Računarski program G.R.O.M. prezentovan u okviru ovog rada razvijen je u cilju određivanja vrednosti aerodinamičkih derivativa – 1. sile uzgona po napadnom uglu; 2 – sile otpora po kvadratu napadnog ugla, kao statičkih derivativa; 3 – prigušnog momenta valjanja po brzini valjanja i 4 – prigušnog momenta propinjanja po brzini propinjanja, kao dinamičkih derivativa. Na beskonačno tanko krilo, a terminezonom paralelnim neporemećenoj struji, primenjena je linearizovana teorija supersoničnog opstrujavanja, uz uslov da napadna i izlazna ivica moraju biti supersonične u odnosu na komponentu brzine neporemećene struje koja je normalna na ove ivice. Napadna ivica ima ugao prave strele, dok su testiranja vršena za dva tipa izlazne ivice (nezakošenu i izlaznu ivicu koja ima ugao prave strele).

Vrednosti aerodinamičkih derivativa dobijenih programom G.R.O.M. upoređeni su sa rezultatima dobijenim upotrebom programa Missile Datcom 97. Visoko slaganje rezultata dobijeno putem ova dva programa predstavlja verifikaciju programa G.R.O.M., karakterišući njegovu upotrebnu vrednost u inženjerskoj praksi, pri određivanju aerodinamičkog opterećenja krila u supersoničnoj struji.

Ključne reči: **aerodinamički derivativi, supersonično strujanje, krilo, aerodinamičko opterećenje, sila uzgona, sile otpora, moment valjanja, moment propinjanja, moment prigušenja, softver.**

Программное обеспечение Г.Р.О.М. для определения величин аэродинамических дериватиов изолированного крыла в сверхзвуковом потоке

Программное обеспечение Г.Р.О.М., представленное в рамках этой работы, разработано с целью определения величин аэродинамических дериватиов - 1. подъемной силы по углу атаки; 2. силы сопротивления по квадрату угла атаки, как статических дериватиов; 3. демпфирующего момента крена по скорости крена и 4. демпфирующего момента кабрирования по скорости кабрирования, как динамических дериватиов. На бесконечно тонком крыле, со терминезоном параллельным с устойчивым потоком, применена линейная теория сверхзвукового потока, под условием, чтобы передняя и задняя кромки должны быть сверхзвуковые по отношению к компоненту скорости устойчивого потока, который перпендикулярен на эти кромки. У передней кромки угол прямой стреловидности, пока испытания проведены на двух типах задней кромки (на прямой кромке и на задней кромке с прямой стреловидностью). Величины аэродинамических дериватиов, получены программным обеспечением Г.Р.О.М., сравнены с результатами получеными употреблением програмы Миссиле Датцом 97. Высокое согласование результатов полученое этими двумя программамы представляет верификацию программного обеспечения Г.Р.О.М., характеризируя его употребляемую величину в инженерной практике, при определении аэродинамической нагрузки крыла в сверхзвуковом потоке.

Ключевые слова: аэродинамические дериватиы, сверхзвуковой поток, крыло, аэродинамическая нагрузка, подъемная сила, сила сопротивления, момент крена, кабрирующий момент, демпфирующий момент, программное обеспечение.

Le programme informatique G.R.O.M. pour la détermination des valeurs des dérivés aérodynamiques de l'aile isolée dans le courant supersonique

Le programme informatique G.R.O.M. présenté dans ce papier est développé dans le but de déterminer les valeurs des dérivés aérodynamiques – 1-forces de propulsion quant à l'angle d'attaque; 2-forces de résistence quant au carré de l'angle d'attaque, comme les dérivés statiques; 3-moment d'amortissement du roulement quant à la vitesse de roulement; 4-moment d'amortissement de tangage quant à la vitesse de tangage, comme les dérivés dynamiques. La théorie linéarisée du courant supersonique est appliquée sur une aile infiniment mince, avec la fin parallèle dans le courant non perturbé,à condition que les bords d'attaque et sortant soient supersoniques par rapport à la composante de la vitesse du courant non perturbé, qui est normale sur ces bords. Le bord d'attaque a l'angle d'une flèche droite, alors que les tests ont été effectués pour deux types de bords sortants (non incliné et sortant, à l'angle de flèche droite).Les valeurs des dérivés aérodynamiques, obtenues à l'aide du programme G.R.O.M., sont comparées aux résultats obtenus pour l'utilisation du programme Missile Datcom 97. Le grand accord des résultats obtenus au moyen des ces deux programmes représente la vérification du programme G.R.O.M. soulignant son utilité dans le travail pratique des ingénieurs lors de la détermination du chargé dynamique de l'aile dans le courant supersonique.

Mots clés: **dérivés aérodynamiques, courant supersonique, aile, charge aérodynamique, force de propulsion, force de résistence, moment de roulement, moment de tangage, moment d'amortissement, logiciel.**