UDK: 519d.673(047.1)=20
COSATI: 13-11

Method for modeling hyperplain problems and model evaluation

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A definition of a hyper plane class problems, modeling methods and model assessment have been given. Methods for modeling valves as well as hyper plane problems and model assessment are defined on the example of modeling of the similar valves shutting time under the effect of nuclear explosion. The paper defines methods of modeling shutting times of blast-controllable valve systems as well as methods of evaluating model $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ in accordance with the results of experimental research. The analytical expressions of multiple correlation coefficients have been defined for the qualitative model evaluation. Algorithms are the basis of a program for devising analytical expression-model of valve shuting times, numerical values of coefficients of partial as well as multiple corelation of the $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ functional relation for the model qualitative evaluation. An original way defined methods presented here are useful in some other technical areas (hydraulics, pneumatics, aerodynamics, automatic control, etc.) for defining and solving the problems which can not be exactly described by laws of physics, what among other things, adds to the scientific dimension of this work.

Key words: **Pneumatic valve, shuting times, modeling, algorithms, model, model evaluation, correlation coefficient, method application**

Symbols:

T_{zi}	– shuting time under direct effects of nuclear explosion air-blast waves (ABW)
VUT	– air-blast wave (ABW)
A, a, b, c, d, e, g	– constants
$A_1, g_1, g_2, g_3, g_4, g_5, g_6$	– equation system solutions
r	– valve fin semi-radius
γ	– valve fin material density
δ	– valve fin thickness
φ	– valve fin rotation angle
α	– fin rotational axis inclincation angle with respect to vertical plane
p_f	– direct shock wave front pressure
ε	– experimental errors
N	– experimental units number
$R_{i\cdot 123456}$	– multiple correlation coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$
$r_{i2\cdot 1}$	– partial corelation characteristics coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b$
$r_{i3\cdot 12}$	– partial corelation characteristics coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c$
$r_{i4\cdot 123}$	– partial corelation characteristics coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d$
$r_{i5\cdot 1234}$	– partial corelation characteristics coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e$
$r_{i1\cdot 23456}, r_{i2\cdot 13456}, r_{i3\cdot 12456}$	– partial corelation characteristics coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$
$r_{i4\cdot 12356}, r_{i5\cdot 12346}, r_{i6\cdot 12345}$	– partial corelation characteristics coefficient functionals relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$
n_r	– level number of the valve fin semi-radius
n_γ	– level number of the valve fin material density
n_δ	– level number of the valve fin thickness
n_φ	– level number of the valve fin rotation angle
n_α	– level number of the valve fin rotational axis inclincation angle with respect to vertical plane
n_{p_f}	– level number of pressure in the direct shock wave front (number of resistance valves level).

Introduction

PROBLEMS and phenomenons class of hyper plane problems.

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Plain analytically define multiple correlation coeficients between the obtained models and experimental results research. Hyper plains are adequate.

For the need of the multi-criteria analysis and optimization process it is necessary to define the probabilities of transmitting ion-reduction of hyper plain models into the plain models.

These probabilities are defined thought partial correlation characteristics coefficients functional relationship parameters problem that researched, and relevant system parameters.

The probabilities of transmitting ion-reduction of hyper plain models into plain models simultaneously define the probability of influence of a relevant system parameter to that research problem parameters, in case the other characteristic system parameters are constant.

Multi-dimensional models are also considered hyper plain models. Because of their multi-dimensionality, they are not "perceptible" to humans.

This is the reason why it is necessary to transmit hyper plain models into the plain models. Special plain hyper plains cross, and the section curve become plain curves.

Analitically defining the probability of transmitting ion-reduction of hyper plain models into the plain models, the correlation between the hyper plain models the and plain models are estabilished.

Models for the parameters problem-phenomenon can be obtained by mathematical statistic, utilizing the least sqare method as well as experimental research data.

The physics of the problem is in the results of experimental research, primarily.

In case of high corellation between the hyper plain models and experimental results, the obtained models with the same corellation represent the physics of the problem.

Establishing correlations between the hyper plain models and the plain models the correlations between the plain models and experimental results research i.e. the correlation between the plain model and the physics of the process, is established.

Estimation of the agreement level between the hyper plain models and the experimental results is made on the basis of the numerical values of the overall multiple correlation coeficients.

Estimation of the agreement level between the model in the plain and the experimental results is made on the basis of the numerical values of the partial correlation characteristics coefficients, or the model in the plain and the hyper plain model.

Estimation of the effect of one o the relevant system parameters on the parameters of the investigated problem is made on the basis of numerical values of the partial correlation characteristics coefficients, when the other relevant parameters are constant.

The given modeling and model evaluation methods can be applied to the problems that can be solved using analytical or numerical method, for the purpose of transforming in qualitative and quantitative the results into hyper plains based on which the necessary analysis can easily be performed, model evaluation, multi-criteria analysis and the process optimization.

Very often it is necesssry to introduce shift and approximation into solving (differential) equations describing the phenomenon.

Any simplification leads further away from solving the problem, i.e. physics of the problem. Very often the results of theorethical research differ from the experimental results, the amount being 30% or more.

How to relate the theory and exeprimental results so the physics of the phenomenon remains preserved is a question that impose itself.

Since, it is necessary to define:

- method of mathematical modeling of the phenomenon,
- experimental reserch,
- methods for the qualitative model evaluation and experimental research data,
- methods to analytically define the probability of the influence of the system characteristics parameters which make phenomenon parameters optimal.

Before modeling it is necessay to:

- determine the phenomenon physics,
- identify the system characteristics values,
- identify the parameters to investigate,
- draw up a global plan of the experimental research,
- define the system for measuring and experimental research methods of the parameters phenomenon that is being researched.

Mathematical modelling of the parameters is done in the following way:

- utilizing the classical method of mathematical statistic, and
- the least square method.

Estimation of agreement between the model and the experimental data is based on:

- the Fisher criterion (compare coefficient of the dispersion relation with Fisher criterion for selecting adequate level, 95 do 99%),
- the numerical values of the interdependence of multiple correlation coefficients of multiple-dimensional functional relationship parameters problem and relevant system the parameters,
- the numerical values of the partial correlation characteristics coefficients of multiple-dimensional functional relationship parameters problem and relevant system the parameters,

In the evaluation of models suitability, based on the Fisher criterion when a model happens to be inadequate, it is necessary to define methods of the analytical-qualitative assessment of the obtained models adequacy level and the testing results agreement.

Multicriteria analysis, and optimization problem-phenomenon parameters is extracted on the basis of the obtained model and the partial correlation characteristics coefficients system numerical values, i.e. the propabylity of influence on the characteristic parameters on the problem-phenomenon parameters being researched.

Therefore it is necessary to define the probabilities of transmitting of the hyper plain models into the plain models, i.e. the probabilities level of agreement between the plain model and the experimental results.

The experimental research creates a data bank containing the parameters of the phenomenon, that is being investigated in order to find the model parameters of the problem phenomenon, the model adequacy estimation, using algorithms and software devised.

On the example of modeling of the similar valves shutting of time under direct effects of nuclear explosion air blast waves, of the working and dimensional characteristics $(r, \gamma, \delta, \varphi, \alpha, p_f)$ of the valves, the modeling methods of the valves as well as hyper plane problems and the model estimation are defined.

Goals and structure of the research

In designing valves an important requirement is for the valve construction to prevent the penetration of air-blast waves (ABW) into the interior of special purpose buildings thus preventing a possible damage to parts of ventilation and air-conditioning systems.

The conception and design of the valves has to ensure their resistance and operation ability under the circumstances of high impulsive loads, meaning, they must shut down under the action of an air-blast wave, and open up when the action is ended in order to enable the polluted air to leave the building.

Figure **1**, [1], gives a schematic view of the valve.

When the air-blast wave approaches a shelter building, the directed wave, through the vane fitted anti shock valve channel, acts by its pressure on the surface of the vanes, causing the sudden closure of the exhausting passage.

While the valve is closing, a small amount of the air-blast pressure impulse will pass through the vane.

The valve is closed until the outer atmosphere is over pressurized. At the moment when the air-blast wave stops its action, the inside air flow through the valve vanes brings them into opened position.

Within the spectrum of the ventilation it is always imperative to maintain some overpressure inside the building with respect to the surrounding atmosphere.

The overpressure is necessary to prevent nuclear, chemical or biological contamination of the buldings inner space.

Furthemore, it is also necessary to maintain overpressuration between the rooms inside the building.

In order to keep that overpressure within desirable limits, the overpressure control valves are fitted up to the air exhausts.

The task of valves is to control the mass of the used air to be exhausted to control the level of overpressure as well, because the air flow through the valves is conditioned by the overpressure itself.

To perform the specified functions, the valves have to satisfy a number of technical requirements.

It is of utmost importance that the valve closing time under the effect of blast waves, in case of automatic mode failure, should be as short as possible, and that resistances flow of an through valves in the conditions of ventilations of building be as small as possible.

The goals of the research is to define the methods of modeling of closing time of blast-controllable pneumatic similar valves, closing due to the effects of blast waves as functions of the geometry and working characteristics $(r,\gamma,\delta,\varphi,\alpha,p_f)$ as well as evaluation of model $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ accordance with the bank experimental results of the research.

In other words, the goals of the research are to define:

- modeling methods of shutting time T_{z_i} of similary valves under the action of the air-blast waves explosion,
- multiple correlation coefficient relationships in the form $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$, and
- the presentation plain of experimental results.

For the functional relationship in the form $T_{z_i} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ the best approximate plane coefficients least square for the method, of the appropriate regressive plane can be obtained by regression analysis using experimental data for T_{z_i}.

Based the least sqare method, regression analysis, defines analytical expressions partial correlation coefficients and multiple correlation coefficient functional relationships in the form $T_{z_i} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ by the following expression, algorithms, for finding the following are defined:

- model closing times of the similar valves under the action of the air-blast waves,
- numerical values of partial correlation coefficients and multiple correlation coefficients, applied in the evalution of the qualitative model $T_{z_i} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ with experimental data for T_{z_i}.

Method for defining the shuting times of the similar valves modeling method under the action of the air-blast waves explosion

In general, pneumatics valves shuting times T_{z_i} is a functions consisting of six parameters, which can be expressed as following [1-15]:

$$T_{zi} = Ar^a \gamma^b \delta^c \varphi^d \alpha^e p_f^g \tag{1}$$

Models for the pneumatic valves shuting time T_{z_i} depending on $(r,\gamma,\delta,\varphi,\alpha,p_f)$ can be obtained by regression analysis, utilizing the least sqare method as well as experimental data for T_{z_i}. If we find the logarithms of equations (1):

$$1nT_{z_i} = 1nA + a1nr + b1n\gamma + c1n\delta + d1n\varphi + e1n\alpha + g1np_f \tag{2}$$

introduce the substitutions:

$$\begin{aligned} &Y_i = 1nT_{z_i};\ a_0 = 1nA;\ a_1 = a; \\ &X_{1i} = 1nr;\ a_2 = b;\ X_{2i} = 1n\gamma;\ a_3 = c; \\ &X_{3i} = 1n\delta;\ a_4 = d;\ X_{4i} = 1n\varphi;\ a_5 = e; \\ &X_{5i} = 1n\alpha;\ a_6 = g;\ X_{6i} = 1np_f \end{aligned} \tag{3}$$

take into acconnt the test error ε, we will get the linear regression equation:

$$Y_i = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 + a_5X_5 + \varepsilon \tag{4}$$

Constants $(a_0,a_1,a_2,a_3,a_4,a_5)$ determination could be done by processing the experimental data using the least sqare method [15-21]. The method minimizes the test results dispersion from the regresion polynomial:

$$\begin{aligned} &\left|\varepsilon\left(Y_i - a_0 - a_1X_{1i} - a_2X_{2i} - a_3X_{3i} -\right.\right. \\ &\left.\left. -a_4X_{4i} - a_5X_{5i} - a_6X_{6i}\right)\right|_{\min} = \left(\varepsilon^2\right)\Big|_{\min} \end{aligned} \tag{5}$$

The minimal dispersions can be obtained by polynomial (5) derivation with respect to the parameters needed, and by equalizing derivatives to zero. After this we will have the linear regression equations system represented in the matrix form:

$$\begin{bmatrix} N & \Sigma X_{1i} & \Sigma X_{2i} & \Sigma X_{3i} & \Sigma X_{4i} & \Sigma X_{5i} & \Sigma X_{6i} \\ \Sigma X_{1i} & \Sigma X_{1i}^2 & \Sigma X_{1i}X_{2i} & \Sigma X_{1i}X_{3i} & \Sigma X_{1i}X_{4i} & \Sigma X_{1i}X_{5i} & \Sigma X_{1i}X_{6i} \\ \Sigma X_{2i} & \Sigma X_{1i}X_{2i} & \Sigma X_{2i}^2 & \Sigma X_{2i}X_{3i} & \Sigma X_{2i}X_{4i} & \Sigma X_{2i}X_{5i} & \Sigma X_{2i}X_{6i} \\ \Sigma X_{3i} & \Sigma X_{1i}X_{3i} & \Sigma X_{2i}X_{3i} & \Sigma X_{3i}^2 & \Sigma X_{3i}X_{4i} & \Sigma X_{3i}X_{5i} & \Sigma X_{3i}X_{6i} \\ \Sigma X_{4i} & \Sigma X_{1i}X_{4i} & \Sigma X_{2i}X_{4i} & \Sigma X_{3i}X_{4i} & \Sigma X_{4i}^2 & \Sigma X_{4i}X_{5i} & \Sigma X_{4i}X_{6i} \\ \Sigma X_{5i} & \Sigma X_{1i}X_{5i} & \Sigma X_{2i}X_{5i} & \Sigma X_{3i}X_{5i} & \Sigma X_{4i}X_{5i} & \Sigma X_{5i}^2 & \Sigma X_{5i}X_{6i} \\ \Sigma X_{6i} & \Sigma X_{1i}X_{6i} & \Sigma X_{2i}X_{6i} & \Sigma X_{3i}X_{6i} & \Sigma X_{4i}X_{6i} & \Sigma X_{5i}X_{6i} & \Sigma X_{6i}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} \Sigma X_{1i} \\ \Sigma X_{1i}Y_i \\ \Sigma X_{2i}Y_i \\ \Sigma X_{3i}Y_i \\ \Sigma X_{4i}Y_i \\ \Sigma X_{5i}Y_i \\ \Sigma X_{6i}Y_i \end{bmatrix} \equiv \begin{bmatrix} B(0) \\ B(1) \\ B(2) \\ B(3) \\ B(4) \\ B(5) \\ B(6) \end{bmatrix} \quad (6)$$

When the experimental data is sorted according to table 1, we get analytical expression for the pneumatics valves shuting time in the form:

$$T_{zi} = A_1 r^{g1} \gamma^{g2} \delta^{g3} \varphi^{g4} \alpha^{g5} p_f^{g6} \quad (7)$$

The adecuacy of the analytical expressions (7) for the similar pneumatics valves shuting time under the action of the air-blast waves explosion has be checked.

Similar valves shutting time model adequacy check method

The adequacy check of shuting times of the similar valves model cold be done using the numerical values of the multiple correlation coefficients $R_{i\cdot123456}$ between functional relationship (1), which have to be analyticaly defined.

To do this, the following subscripts were introduced into expression (1): valve vane semi radius r got subscript 1, valve vane material density γ got subscript 2, valve vane δ thickness got subscript 3, valve fin rotation angle φ got subscript 4, valve vane rotation axis inclination angle with respect to the vertical plane α got subscript 5 and the pressure p_f in front of the nuclear explosion direct air-blast wave got subscript 6.

$$T_{zi} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g \quad (8)$$

Multiple correlation coefficients $R_{i\cdot12345}$ for relation $T_{zi} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$

Multiple correlation coefficient $R_{i\cdot123456}$ is defined by the following expression:

$$R_{i\cdot123456} = \sqrt{1-\left[\left(1-r_{i1}^2\right)\left(1-r_{i2\cdot1}^2\right)\left(1-r_{i3\cdot12}^2\right)\left(1-r_{i4\cdot123}^2\right)\left(1-r_{i5\cdot1234}^2\right)\left(1-r_{i6\cdot12345}^2\right)\right]} \quad (9)$$

The partial correlation coefficients which exist within expression (9) need to be analytically defined.

Partial correlation coefficient $r_{i2\cdot1}$

A partial correlation coefficient $r_{i2\cdot1}$ is defined by expression:

$$r_{i2\cdot1} = \frac{r_{i2} - r_{i1} r_{21}}{\sqrt{\left(1-r_{i1}^2\right)\left(1-r_{21}^2\right)}} \quad (10)$$

Partial correlation coefficient $r_{i3\cdot12}$

The partial correlation coefficient $r_{i3\cdot12}$ is defined by expression, from [4]:

$$r_{i3\cdot12} = \frac{r_{i3\cdot2} - r_{i1\cdot2} r_{31\cdot2}}{\sqrt{\left(1-r_{i1\cdot2}^2\right)\left(1-r_{31\cdot2}^2\right)}} \quad (11)$$

The partial correlation coefficients $r_{i3\cdot2}$, $r_{i1\cdot2}$, and $r_{31\cdot2}$, can be defined by the expressions:

$$r_{i3\cdot2} = \frac{r_{i3} - r_{i2} r_{32}}{\sqrt{\left(1-r_{i2}^2\right)\left(1-r_{32}^2\right)}} \quad (12)$$

$$r_{i1\cdot2} = \frac{r_{i1} - r_{i2}\ r_{12}}{\sqrt{\left(1-r_{i2}^2\right)\left(1-r_{12}^2\right)}} \quad (13)$$

$$r_{31\cdot2} = \frac{r_{31} - r_{32}\ r_{12}}{\sqrt{\left(1-r_{32}^2\right)\left(1-r_{12}^2\right)}} \quad (14)$$

Partial correlation coefficient $r_{i4\cdot123}$

The partial correlation coefficient $r_{i4\cdot123}$ is defined by expression, from [4]:

$$r_{i4\cdot123} = \frac{r_{i4\cdot23} - r_{i1\cdot23}\ r_{41\cdot23}}{\sqrt{\left(1-r_{i1\cdot23}^2\right)\left(1-r_{41\cdot23}^2\right)}} \quad (15)$$

The partial correlation coefficients $r_{i4\cdot23}$, $r_{i1\cdot23}$ and $r_{41\cdot23}$, existing in expression (15) can be defined by the expressions:

$$r_{i4\cdot23} = \frac{r_{i4\cdot3} - r_{i2\cdot3}\ r_{42\cdot3}}{\sqrt{\left(1-r_{i2\cdot3}^2\right)\left(1-r_{42\cdot3}^2\right)}} \quad (16)$$

$$r_{i1\cdot23} = \frac{r_{i1\cdot3} - r_{i2\cdot3}\ r_{12\cdot3}}{\sqrt{\left(1-r_{i2\cdot3}^2\right)\left(1-r_{12\cdot3}^2\right)}} \quad (17)$$

$$r_{41\cdot23} = \frac{r_{41\cdot3} - r_{42\cdot3}\ r_{12\cdot3}}{\sqrt{\left(1-r_{42\cdot3}^2\right)\left(1-r_{12\cdot3}^2\right)}} \quad (18)$$

The partial correlation coefficients $r_{i4\cdot3}$, $r_{i2\cdot3}$ and $r_{42\cdot3}$, existing in expressions (16), (17) and (18), can be defined by the expressions:

$$r_{i4\cdot3}=\frac{r_{i4}-r_{i3}\ r_{43}}{\sqrt{\left(1-r_{i3}^2\right)\left(1-r_{43}^2\right)}} \tag{19}$$

$$r_{i2\cdot3}=\frac{r_{i2}-r_{i3}\ r_{23}}{\sqrt{\left(1-r_{i3}^2\right)\left(1-r_{23}^2\right)}} \tag{20}$$

$$r_{42\cdot3}=\frac{r_{42}-r_{43}\ r_{23}}{\sqrt{\left(1-r_{43}^2\right)\left(1-r_{23}^2\right)}} \tag{21}$$

The partial correlation coefficients $r_{i1\cdot3}$, $r_{12\cdot3}$ and $r_{41\cdot3}$, can be defined by the expressions:

$$r_{i1\cdot3}=\frac{r_{i1}-r_{i3}\ r_{13}}{\sqrt{\left(1-r_{i3}^2\right)\left(1-r_{13}^2\right)}} \tag{22}$$

$$r_{12\cdot3}=\frac{r_{12}-r_{13}\ r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}} \tag{23}$$

$$r_{41\cdot3}=\frac{r_{41}-r_{43}\ r_{13}}{\sqrt{\left(1-r_{43}^2\right)\left(1-r_{13}^2\right)}} \tag{24}$$

Partial correlation coefficient $r_{i5\cdot1234}$ for relation form $T_{zi}=Ar_1^a\gamma_2^b\delta_3^c\varphi_4^d\alpha_5^e$

The partial correlation coefficient $r_{i5\cdot1234}$ can be defined by the expressions:

$$r_{i5\cdot1234}=\frac{r_{i5\cdot234}-r_{i1\cdot234}\ r_{51\cdot234}}{\sqrt{\left(1-r_{i1\cdot234}^2\right)\left(1-r_{51\cdot234}^2\right)}} \tag{25}$$

Partial correlation coefficients $r_{i5\cdot234}$, $r_{i1\cdot234}$ and $r_{51\cdot234}$

The partial correlation coefficients $r_{i5\cdot234}$, $r_{i1\cdot234}$ and $r_{51\cdot234}$, existing in expression (25) can be defined by the expressions:

$$r_{i5\cdot234}=\frac{r_{i5\cdot34}-r_{i2\cdot34}\ r_{52\cdot34}}{\sqrt{\left(1-r_{i2\cdot34}^2\right)\left(1-r_{52\cdot34}^2\right)}} \tag{26}$$

$$r_{i1\cdot234}=\frac{r_{i1\cdot34}-r_{i2\cdot34}\ r_{12\cdot34}}{\sqrt{\left(1-r_{i2\cdot34}^2\right)\left(1-r_{12\cdot34}^2\right)}} \tag{27}$$

$$r_{51\cdot234}=\frac{r_{51\cdot34}-r_{52\cdot34}\ r_{12\cdot34}}{\sqrt{\left(1-r_{52\cdot34}^2\right)\left(1-r_{12\cdot34}^2\right)}} \tag{28}$$

Partial correlation coefficients $r_{i5\cdot34}$, $r_{i2\cdot34}$ and $r_{52\cdot34}$,

The partial correlation coefficients $r_{i5\cdot34}$, $r_{i2\cdot34}$ and $r_{52\cdot34}$, existing in expression (26) can be defined by the expressions:

$$r_{i5\cdot34}=\frac{r_{i5\cdot4}-r_{i3\cdot4}\ r_{53\cdot4}}{\sqrt{\left(1-r_{i3\cdot4}^2\right)\left(1-r_{53\cdot4}^2\right)}} \tag{29}$$

$$r_{i2\cdot34}=\frac{r_{i2\cdot4}-r_{i3\cdot4}\ r_{23\cdot4}}{\sqrt{\left(1-r_{i3\cdot4}^2\right)\left(1-r_{23\cdot4}^2\right)}} \tag{30}$$

$$r_{52\cdot34}=\frac{r_{52\cdot4}-r_{53\cdot4}\ r_{23\cdot4}}{\sqrt{\left(1-r_{53\cdot4}^2\right)\left(1-r_{23\cdot4}^2\right)}} \tag{31}$$

Partial correlation coefficients $r_{i1\cdot34}$ and $r_{12\cdot34}$

The partial correlation coefficients $r_{i1\cdot34}$ and $r_{12\cdot34}$, existing in expression (27) can be defined by the expressions:

$$r_{i1\cdot34}=\frac{r_{i1\cdot4}-r_{i3\cdot4}\ r_{13\cdot4}}{\sqrt{\left(1-r_{i3\cdot4}^2\right)\left(1-r_{13\cdot4}^2\right)}} \tag{32}$$

$$r_{12\cdot34}=\frac{r_{12\cdot4}-r_{13\cdot4}\ r_{23\cdot4}}{\sqrt{\left(1-r_{13\cdot4}^2\right)\left(1-r_{23\cdot4}^2\right)}} \tag{33}$$

Partial correlation coefficient $r_{51\cdot34}$

The partial correlation coefficient $r_{51\cdot34}$, existing in expression (28) can be defined by the expression:

$$r_{51\cdot34}=\frac{r_{51\cdot4}-r_{53\cdot4}\ r_{13\cdot4}}{\sqrt{\left(1-r_{53\cdot4}^2\right)\left(1-r_{13\cdot4}^2\right)}} \tag{34}$$

Partial correlation coefficients $r_{i5\cdot4}$, $r_{i3\cdot4}$ and $r_{53\cdot4}$

The partial correlation coefficients $r_{i5\cdot4}$, $r_{i3\cdot4}$ and $r_{53\cdot4}$, existing in expression (29) can be defined by the expressions:

$$r_{i5\cdot4}=\frac{r_{i5}-r_{i4}\ r_{54}}{\sqrt{\left(1-r_{i4}^2\right)\left(1-r_{54}^2\right)}} \tag{35}$$

$$r_{i3\cdot4}=\frac{r_{i3}-r_{i4}\ r_{34}}{\sqrt{\left(1-r_{i4}^2\right)\left(1-r_{34}^2\right)}} \tag{36}$$

$$r_{53\cdot4}=\frac{r_{53}-r_{54}\ r_{34}}{\sqrt{\left(1-r_{54}^2\right)\left(1-r_{34}^2\right)}} \tag{37}$$

Partial correlation coefficients $r_{i2\cdot4}$ and $r_{23\cdot4}$

The partial correlation coefficients $r_{i2\cdot4}$ and $r_{23\cdot4}$, existing in expression (30) can be defined by the expressions:

$$r_{i2\cdot4}=\frac{r_{i2}-r_{i4}\ r_{24}}{\sqrt{\left(1-r_{i4}^2\right)\left(1-r_{24}^2\right)}} \tag{38}$$

$$r_{23\cdot4}=\frac{r_{23}-r_{34}\ r_{24}}{\sqrt{\left(1-r_{34}^2\right)\left(1-r_{24}^2\right)}} \tag{39}$$

Partial correlation coefficient $r_{52\cdot4}$

The partial correlation coefficient $r_{52\cdot4}$, existing in expression (31) can be defined by the expression:

$$r_{52\cdot 4} = \frac{r_{52} - r_{54}\ r_{24}}{\sqrt{\left(1-r_{54}^2\right)\left(1-r_{24}^2\right)}} \quad (40)$$

Partial correlation coefficients $r_{i1\cdot 4}$ *and* $r_{13\cdot 4}$

The partial correlation coefficients $r_{i1\cdot 4}$ and $r_{13\cdot 4}$, existing in expression (32) can be defined by the expressions:

$$r_{i1\cdot 4} = \frac{r_{i1} - r_{i4}\ r_{14}}{\sqrt{\left(1-r_{i4}^2\right)\left(1-r_{14}^2\right)}} \quad (41)$$

$$r_{13\cdot 4} = \frac{r_{13} - r_{14}\ r_{34}}{\sqrt{\left(1-r_{14}^2\right)\left(1-r_{34}^2\right)}} \quad (42)$$

Partial correlation coefficient $r_{12\cdot 4}$

The partial correlation coefficient $r_{12\cdot 4}$, existing in expression (33) can be defined by the expression:

$$r_{12\cdot 4} = \frac{r_{12} - r_{24}\ r_{14}}{\sqrt{\left(1-r_{24}^2\right)\left(1-r_{14}^2\right)}} \quad (43)$$

Partial correlation coefficient $r_{51\cdot 4}$

The partial correlation coefficient $r_{51\cdot 4}$, existing in expression (34) can be defined by the analytical expression:

$$r_{51\cdot 4} = \frac{r_{51} - r_{54}\ r_{14}}{\sqrt{\left(1-r_{54}^2\right)\left(1-r_{14}^2\right)}} \quad (44)$$

Partial correlation coefficient $r_{i6\cdot 12345}$ *for relation form* $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$

The partial correlation coefficient $r_{i6\cdot 12345}$ of the functional relationship (1) and (8), defines the level of agreement between the dependent variable T_{zi} and independent parameter p_f, when the other independent parameters $(r,\gamma,\delta,\varphi,\alpha)$ are presumed to be constant. The partial correlation coefficient $r_{i6\cdot 12345}$ of the functional relationship (8), defines of the probability of transmit ion-reduction of model (8) into the plain $p_f T_{zi}$ models, when the other independent parameters $(r,\gamma,\delta,\varphi,\alpha)$ are presumed to be constant. The partial correlation coefficient $r_{i6\cdot 12345}$ can be defined by the expressions:

$$r_{i6\cdot 12345} = \frac{r_{i6\cdot 2345} - r_{i1\cdot 2345} r_{61\cdot 2345}}{\sqrt{\left(1-r_{i1\cdot 2345}^2\right)\left(1-r_{61\cdot 2345}^2\right)}} \quad (45)$$

Partial correlation coefficients $r_{i6\cdot 2345}$, $r_{i1\cdot 2345}$ *and* $r_{61\cdot 2345}$

The partial correlation coefficients $r_{i6\cdot 2345}$, $r_{i1\cdot 2345}$ and $r_{61\cdot 2345}$ existing in expression (45) can be defined by the expressions:

$$r_{i6\cdot 2345} = \frac{r_{i6\cdot 345} - r_{i2\cdot 345} r_{62\cdot 345}}{\sqrt{\left(1-r_{i2\cdot 345}^2\right)\left(1-r_{62\cdot 345}^2\right)}} \quad (46)$$

$$r_{i1\cdot 2345} = \frac{r_{i1\cdot 345} - r_{i2\cdot 345} r_{12\cdot 345}}{\sqrt{\left(1-r_{i2\cdot 345}^2\right)\left(1-r_{12\cdot 345}^2\right)}} \quad (47)$$

$$r_{61\cdot 2345} = \frac{r_{61\cdot 345} - r_{62\cdot 345} r_{12\cdot 345}}{\sqrt{\left(1-r_{62\cdot 345}^2\right)\left(1-r_{12\cdot 345}^2\right)}} \quad (48)$$

Partial correlation coefficients $r_{i6\cdot 345}$, $r_{i2\cdot 345}$ *and* $r_{62\cdot 345}$

The partial correlation coefficients $r_{i6\cdot 345}$, $r_{i2\cdot 345}$ and $r_{62\cdot 345}$, existing in expression (46) can be defined by the expressions:

$$r_{i6\cdot 345} = \frac{r_{i6\cdot 45} - r_{i3\cdot 45} r_{63\cdot 45}}{\sqrt{\left(1-r_{i3\cdot 45}^2\right)\left(1-r_{63\cdot 45}^2\right)}} \quad (49)$$

$$r_{i2\cdot 345} = \frac{r_{i2\cdot 45} - r_{i3\cdot 45} r_{23\cdot 45}}{\sqrt{\left(1-r_{i3\cdot 45}^2\right)\left(1-r_{23\cdot 45}^2\right)}} \quad (50)$$

$$r_{62\cdot 345} = \frac{r_{62\cdot 45} - r_{63\cdot 45} r_{23\cdot 45}}{\sqrt{\left(1-r_{63\cdot 45}^2\right)\left(1-r_{23\cdot 45}^2\right)}} \quad (51)$$

Partial correlation coefficients $r_{i1\cdot 345}$ *and* $r_{12\cdot 345}$

The partial correlation coefficients $r_{i1\cdot 345}$ and $r_{12\cdot 345}$ existing in expression (47) can be defined by the expressions:

$$r_{i1\cdot 345} = \frac{r_{i1\cdot 45} - r_{i3\cdot 45} r_{13\cdot 45}}{\sqrt{\left(1-r_{i3\cdot 45}^2\right)\left(1-r_{13\cdot 45}^2\right)}} \quad (52)$$

$$r_{12\cdot 345} = \frac{r_{12\cdot 45} - r_{13\cdot 45} r_{23\cdot 45}}{\sqrt{\left(1-r_{13\cdot 45}^2\right)\left(1-r_{23\cdot 45}^2\right)}} \quad (53)$$

Partial correlation coefficient $r_{61\cdot 345}$

The partial correlation coefficients $r_{61\cdot 345}$ existing in expression (48) can be defined by the expression:

$$r_{61\cdot 345} = \frac{r_{61\cdot 45} - r_{63\cdot 45} r_{13\cdot 45}}{\sqrt{\left(1-r_{63\cdot 45}^2\right)\left(1-r_{13\cdot 45}^2\right)}} \quad (54)$$

Partial correlation coefficients $r_{i6\cdot 45}$, $r_{i3\cdot 45}$ *and* $r_{63\cdot 45}$

The partial correlation coefficients $r_{i6\cdot 45}$, $r_{i3\cdot 45}$ and $r_{63\cdot 45}$, existing in expression (49) can be defined by the expressions:

$$r_{i6\cdot 45} = \frac{r_{i6\cdot 5} - r_{i4\cdot 5} r_{64\cdot 5}}{\sqrt{\left(1-r_{i4\cdot 5}^2\right)\left(1-r_{64\cdot 5}^2\right)}} \quad (55)$$

$$r_{i3\cdot 45} = \frac{r_{i3\cdot 5} - r_{i4\cdot 5} r_{34\cdot 5}}{\sqrt{\left(1-r_{i4\cdot 5}^2\right)\left(1-r_{34\cdot 5}^2\right)}} \quad (56)$$

$$r_{63\cdot 45} = \frac{r_{63\cdot 5} - r_{64\cdot 5} r_{34\cdot 5}}{\sqrt{\left(1-r_{64\cdot 5}^2\right)\left(1-r_{34\cdot 5}^2\right)}} \quad (57)$$

Partial correlation coefficients $r_{i2\cdot 45}$ and $r_{23\cdot 45}$

The partial correlation coefficients $r_{i2\cdot 45}$ and $r_{23\cdot 45}$, existing in expression (50) can be defined by the expressions:

$$r_{i2\cdot 45} = \frac{r_{i2\cdot 5} - r_{i4\cdot 5} r_{24\cdot 5}}{\sqrt{\left(1 - r_{i4\cdot 5}^2\right)\left(1 - r_{24\cdot 5}^2\right)}} \tag{58}$$

$$r_{23\cdot 45} = \frac{r_{23\cdot 5} - r_{24\cdot 5} r_{34\cdot 5}}{\sqrt{\left(1 - r_{24\cdot 5}^2\right)\left(1 - r_{34\cdot 5}^2\right)}} \tag{59}$$

Partial correlation coefficient $r_{62\cdot 45}$

The partial correlation coefficient $r_{62\cdot 45}$, existing in expression (51) can be defined by the expressions:

$$r_{62\cdot 45} = \frac{r_{62\cdot 5} - r_{64\cdot 5} r_{24\cdot 5}}{\sqrt{\left(1 - r_{64\cdot 5}^2\right)\left(1 - r_{24\cdot 5}^2\right)}} \tag{60}$$

Partial correlation coefficients $r_{i1\cdot 45}$ and $r_{13\cdot 45}$

The partial correlation coefficient $r_{i1\cdot 45}$ and $r_{13\cdot 45}$, existing in expression (52) can be defined by the expressions:

$$r_{i1\cdot 45} = \frac{r_{i1\cdot 5} - r_{i4\cdot 5} r_{14\cdot 5}}{\sqrt{\left(1 - r_{i4\cdot 5}^2\right)\left(1 - r_{14\cdot 5}^2\right)}} \tag{61}$$

$$r_{13\cdot 45} = \frac{r_{13\cdot 5} - r_{14\cdot 5} r_{34\cdot 5}}{\sqrt{\left(1 - r_{14\cdot 5}^2\right)\left(1 - r_{34\cdot 5}^2\right)}} \tag{62}$$

Partial correlation coefficient $r_{12\cdot 45}$

The partial correlation coefficient $r_{12\cdot 45}$, existing in expression (53) can be defined by the expressions:

$$r_{12\cdot 45} = \frac{r_{12\cdot 5} - r_{14\cdot 5} r_{24\cdot 5}}{\sqrt{\left(1 - r_{14\cdot 5}^2\right)\left(1 - r_{24\cdot 5}^2\right)}} \tag{63}$$

Partial correlation coefficient $r_{61\cdot 45}$

The partial correlation coefficient $r_{61\cdot 45}$, existing in expression (54) can be defined by the expressions

$$r_{61\cdot 45} = \frac{r_{61\cdot 5} - r_{64\cdot 5} r_{14\cdot 5}}{\sqrt{\left(1 - r_{64\cdot 5}^2\right)\left(1 - r_{14\cdot 5}^2\right)}} \tag{64}$$

Partial correlation coefficients $r_{i6\cdot 5}$, $r_{i4\cdot 5}$ and $r_{64\cdot 5}$

The partial correlation coefficients $r_{i6\cdot 5}$, $r_{i4\cdot 5}$ and $r_{64\cdot 5}$, existing in expression (52) can be defined by the expressions:

$$r_{i6\cdot 5} = \frac{r_{i6} - r_{i5} r_{65}}{\sqrt{\left(1 - r_{i5}^2\right)\left(1 - r_{65}^2\right)}} \tag{65}$$

$$r_{i4\cdot 5} = \frac{r_{i4} - r_{i5} r_{45}}{\sqrt{\left(1 - r_{i5}^2\right)\left(1 - r_{45}^2\right)}} \tag{66}$$

$$r_{64\cdot 5} = \frac{r_{64} - r_{65} r_{45}}{\sqrt{\left(1 - r_{65}^2\right)\left(1 - r_{45}^2\right)}} \tag{67}$$

Partial correlation coefficients $r_{i3\cdot 5}$ and $r_{34\cdot 5}$

The partial correlation coefficients $r_{i3\cdot 5}$ and $r_{34\cdot 5}$, existing in expression (56) can be defined by the expressions:

$$r_{i3\cdot 5} = \frac{r_{i3} - r_{i5} r_{35}}{\sqrt{\left(1 - r_{i5}^2\right)\left(1 - r_{35}^2\right)}} \tag{68}$$

$$r_{34\cdot 5} = \frac{r_{34} - r_{35} r_{45}}{\sqrt{\left(1 - r_{35}^2\right)\left(1 - r_{45}^2\right)}} \tag{69}$$

Partial correlation coefficient $r_{63\cdot 5}$

The partial correlation coefficients $r_{63\cdot 5}$, existing in expression (57) can be defined by the expressions:

$$r_{63\cdot 5} = \frac{r_{63} - r_{65} r_{35}}{\sqrt{\left(1 - r_{65}^2\right)\left(1 - r_{35}^2\right)}} \tag{70}$$

Partial correlation coefficients $r_{i2\cdot 5}$ and $r_{24\cdot 5}$

The partial correlation coefficients $r_{i2\cdot 5}$ and $r_{24\cdot 5}$, existing in expression (58) can be defined by the expressions:

$$r_{i2\cdot 5} = \frac{r_{i2} - r_{i5} r_{25}}{\sqrt{\left(1 - r_{i5}^2\right)\left(1 - r_{25}^2\right)}} \tag{71}$$

$$r_{24\cdot 5} = \frac{r_{24} - r_{25} r_{45}}{\sqrt{\left(1 - r_{25}^2\right)\left(1 - r_{45}^2\right)}} \tag{72}$$

Partial correlation coefficient $r_{23\cdot 5}$

The partial correlation coefficient $r_{23\cdot 5}$, existing in expression (59) can be defined by the expressions:

$$r_{23\cdot 5} = \frac{r_{23} - r_{23} r_{35}}{\sqrt{\left(1 - r_{25}^2\right)\left(1 - r_{35}^2\right)}} \tag{73}$$

Partial correlation coefficient $r_{62\cdot 5}$

The partial correlation coefficient $r_{62\cdot 5}$, existing in expression (60) can be defined by the expressions:

$$r_{62\cdot 5} = \frac{r_{62} - r_{65} r_{25}}{\sqrt{\left(1 - r_{65}^2\right)\left(1 - r_{25}^2\right)}} \tag{74}$$

Partial correlation coefficients $r_{i1\cdot 5}$ and $r_{14\cdot 5}$

The partial correlation coefficients $r_{i1\cdot 5}$ and $r_{14\cdot 5}$, existing in expression (61) can be defined by the expressions:

$$r_{i1\cdot 5} = \frac{r_{i1} - r_{i5} r_{15}}{\sqrt{\left(1 - r_{i5}^2\right)\left(1 - r_{15}^2\right)}} \tag{75}$$

$$r_{14\cdot5} = \frac{r_{14} - r_{15}r_{45}}{\sqrt{\left(1-r_{15}^2\right)\left(1-r_{45}^2\right)}} \quad (76)$$

Partial correlation coefficient $r_{13\cdot5}$

The partial correlation coefficient $r_{13\cdot5}$, existing in expression (62) can be defined by the expressions:

$$r_{13\cdot5} = \frac{r_{13} - r_{15}r_{35}}{\sqrt{\left(1-r_{15}^2\right)\left(1-r_{35}^2\right)}} \quad (77)$$

Partial correlation coefficient $r_{12\cdot5}$

The partial correlation coefficient $r_{12\cdot5}$, existing in expression (63) can be defined by the expressions:

$$r_{12\cdot5} = \frac{r_{12} - r_{15}r_{25}}{\sqrt{\left(1-r_{15}^2\right)\left(1-r_{25}^2\right)}} \quad (78)$$

Partial correlation coefficient $r_{61\cdot5}$

The partial correlation coefficient $r_{61\cdot5}$, existing in expression (64) can be defined by the expressions:

$$r_{61\cdot5} = \frac{r_{61} - r_{65}r_{15}}{\sqrt{\left(1-r_{65}^2\right)\left(1-r_{15}^2\right)}} \quad (79)$$

Partial correlation coefficients $r_{i1}, r_{i2}, r_{i3}, r_{i4}, r_{i5}$ *and* r_{i6}

The partial correlation coefficients $r_{i1}, r_{i2}, r_{i3}, r_{i4}, r_{i5}$ and r_{i6} can be defined by the expressions:

$$r_{i1} = \frac{\sum Y_i X_{1i}}{\sqrt{\left(\sum X_{1i}^2\right)\left(\sum Y_i^2\right)}} = \frac{B(1)}{\sqrt{X(1,1)C(0)}} \quad (80)$$

$$r_{i2} = \frac{\sum Y_i X_{2i}}{\sqrt{\left(\sum X_{2i}^2\right)\left(\sum Y_i^2\right)}} = \frac{B(2)}{\sqrt{X(2,2)C(0)}} \quad (81)$$

$$r_{i3} = \frac{\sum Y_i X_{3i}}{\sqrt{\left(\sum X_{3i}^2\right)\left(\sum Y_i^2\right)}} = \frac{B(3)}{\sqrt{X(3,3)C(0)}} \quad (82)$$

$$r_{i4} = \frac{\sum Y_i X_{4i}}{\sqrt{\left(\sum X_{4i}^2\right)\left(\sum Y_i^2\right)}} = \frac{B(4)}{\sqrt{X(4,4)C(0)}} \quad (83)$$

$$r_{i5} = \frac{\sum Y_i X_{5i}}{\sqrt{\left(\sum X_{5i}^2\right)\left(\sum Y_i^2\right)}} = \frac{B(5)}{\sqrt{X(5,5)C(0)}} \quad (84)$$

$$r_{i6} = \frac{\Sigma X_{6i} Y_i}{\sqrt{\left(\Sigma X_{6i}^2\right)\left(\Sigma Y_i^2\right)}} = \frac{B(6)}{\sqrt{X(6,6)C(0)}} \quad (85)$$

Partial correlation coefficients $r_{12}, r_{13}, r_{14}, r_{15}$ and r_{16}

The partial correlation coefficients $r_{12}, r_{13}, r_{14}, r_{15}$ and r_{16} can be defined by the expressions:

$$r_{12} \equiv r_{21} = \frac{\sum X_{1i} X_{2i}}{\sqrt{\left(\sum X_{1i}^2\right)\left(\sum X_{2i}^2\right)}} = \frac{X(1,2)}{\sqrt{X(1,1)X(2,2)}} \quad (86)$$

$$r_{13} \equiv r_{31} = \frac{\sum X_{1i} X_{3i}}{\sqrt{\left(\sum X_{1i}^2\right)\left(\sum X_{3i}^2\right)}} = \frac{X(1,3)}{\sqrt{X(1,1)X(3,3)}} \quad (87)$$

$$r_{14} \equiv r_{41} = \frac{\sum X_{4i} X_{1i}}{\sqrt{\left(\sum X_{4i}^2\right)\left(\sum X_{1i}^2\right)}} = \frac{X(4,1)}{\sqrt{X(4,4)X(1,1)}} \quad (88)$$

$$r_{15} \equiv r_{51} = \frac{\sum X_{5i} X_{1i}}{\sqrt{\left(\sum X_{5i}^2\right)\left(\sum X_{1i}^2\right)}} = \frac{X(5,1)}{\sqrt{X(5,5)X(1,1)}} \quad (89)$$

$$r_{16} \equiv r_{61} = \frac{\sum X_{6i} X_{1i}}{\sqrt{\left(\sum X_{6i}^2\right)\left(\sum X_{1i}^2\right)}} = \frac{X(6,1)}{\sqrt{X(6,6)X(1,1)}} \quad (90)$$

Partial correlation coefficients r_{23}, r_{24}, r_{25} and r_{26}

The partial correlation coefficients r_{23}, r_{24}, r_{25} and r_{26} can be defined by the expressions:

$$r_{23} \equiv r_{32} = \frac{\sum X_{2i} X_{3i}}{\sqrt{\left(\sum X_{2i}^2\right)\left(\sum X_{3i}^2\right)}} = \frac{X(2,3)}{\sqrt{X(2,2)X(3,3)}} \quad (91)$$

$$r_{24} \equiv r_{42} = \frac{\sum X_{4i} X_{2i}}{\sqrt{\left(\sum X_{4i}^2\right)\left(\sum X_{2i}^2\right)}} = \frac{X(4,2)}{\sqrt{X(4,4)X(2,2)}} \quad (92)$$

$$r_{25} \equiv r_{52} = \frac{\sum X_{5i} X_{2i}}{\sqrt{\left(\sum X_{5i}^2\right)\left(\sum X_{2i}^2\right)}} = \frac{X(5,2)}{\sqrt{X(5,5)X(2,2)}} \quad (93)$$

$$r_{26} \equiv r_{62} = \frac{\sum X_{6i} X_{2i}}{\sqrt{\left(\sum X_{6i}^2\right)\left(\sum X_{2i}^2\right)}} = \frac{X(6,2)}{\sqrt{X(6,6)X(2,2)}} \quad (94)$$

Partial correlation coefficients r_{34}, r_{35} and r_{36}

The partial correlation coefficients r_{34}, r_{35} and r_{36} can be defined by the expressions:

$$r_{34} \equiv r_{43} = \frac{\sum X_{4i} X_{3i}}{\sqrt{\left(\sum X_{4i}^2\right)\left(\sum X_{3i}^2\right)}} = \frac{X(4,3)}{\sqrt{X(4,4)X(3,3)}} \quad (95)$$

$$r_{35} \equiv r_{53} = \frac{\sum X_{5i} X_{3i}}{\sqrt{\left(\sum X_{5i}^2\right)\left(\sum X_{3i}^2\right)}} = \frac{X(5,3)}{\sqrt{X(5,5)X(3,3)}} \quad (96)$$

$$r_{36} \equiv r_{63} = \frac{\sum X_{6i} X_{3i}}{\sqrt{\left(\sum X_{6i}^2\right)\left(\sum X_{3i}^2\right)}} = \frac{X(6,3)}{\sqrt{X(6,6)X(3,3)}} \quad (97)$$

Partial correlation coefficients r_{45} and r_{46}

The partial correlation coefficients r_{45} and r_{46} can be defined by the expressions:

$$r_{45} \equiv r_{54} = \frac{\sum X_{5i} X_{4i}}{\sqrt{\left(\sum X_{5i}^2\right)\left(\sum X_{4i}^2\right)}} = \frac{X(5,4)}{\sqrt{X(5,5)X(4,4)}} \quad (98)$$

$$r_{46} \equiv r_{64} = \frac{\sum X_{6i} X_{4i}}{\sqrt{\left(\sum X_{6i}^2\right)\left(\sum X_{4i}^2\right)}} = \frac{X(6,4)}{\sqrt{X(6,6)X(4,4)}} \quad (99)$$

Partial correlation coefficient r_{56}

The partial correlation coefficients r_{56} can be defined by the expressions:

$$r_{56} \equiv r_{65} = \frac{\sum X_{6i} X_{5i}}{\sqrt{\left(\sum X_{6i}^2\right)\left(\sum X_{5i}^2\right)}} = \frac{X(6,5)}{\sqrt{X(6,6)X(5,5)}} \quad (100)$$

Presentation plain of experiment results

For the purpose of performing experiments and defining the model of similar valves shutting time T_{z_i} under the action of the air-blast waves explosion of the working and dimensional characteristics $(r, \gamma, \delta, \varphi, \alpha, p_f)$ of the valves, presentation plain of experiment results is defined.

Presentation plain of experimental research functional relationships in the form $T_{zi} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$, is given table 1.

Experimental investigation description

For the purpose of discovering the analytical expression

Table 1. Plan for experimental investigation of the functional relationship $T_{zi} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$

Valve fin semi-radius r[m]	Valve fin material density γ[kg/m^3]	Valve fin thickness δ[mm]	Valve fin rotation angle φ [°]	Valve fin rotational axis inclination angle α [°]	Direct shock wave front pressure p_f [kPa]			
					p_{f1}	p_{f2}	...	p_{fnpf}
					Closing time of the valve T_{zi} [ms]			
r_{n_r}	γ_1	δ_1	φ_1	α_1				
				...				
				α_{n_α}				
			...					
			φ_{n_φ}	α_1				
				...				
				α_{n_α}				
		...						
		δ_{n_δ}	φ_1	α_1				
				...				
				α_{n_α}				
			...					
			φ_{n_φ}	α_1				
				...				
				α_{n_α}				
	γ_{n_γ}	δ_1	φ_1	α_1				
				...				
				α_{n_α}				
			...					
			φ_{n_φ}	α_1				
				...				
				α_{n_α}				
		...						
		δ_{n_δ}	φ_1	α_1				
				...				
				α_{n_α}				
			...					
			φ_{n_φ}	α_1				
				...				
				α_{n_α}				

so shuting times T_{z_i} of the valves model, it is necessary to determine the influence of the working and dimensional characteristics $(r,\gamma,\delta,\varphi,\alpha,p_f)$ of the similar design valves, (with optimum aerodynamic-characteristics), on the valve shuting time under the action of the air-blast waves explosion. The aim is to determine the valve shuting times T_{z_i} under the action of the air-blast waves, for levels $(n_r,n_\gamma,n_\delta,n_\varphi,n_\alpha)$ of characteristic values $(r,\gamma,\delta,\varphi,\alpha)$ of the system with the resistance valve level n_{pf}. We are dealing with the six-factorized experiment. The number of the experimental units is:

$$N = n_r n_\gamma n_\delta n_\varphi n_\alpha n_{pf} \tag{101}$$

Where:

n_r - level number of the characteristic parameter r,

n_γ - level number of the characteristic parameter γ,

n_δ - level number of the characteristic parameter δ,

n_φ - level number of the characteristic parameter φ,

n_α - level number of the characteristic parameter α and

n_{pf} - level number of characteristic parameter p_f

The experiment is performed according to investigations the choice has been the plan given in table 1.

Numerical data of the valve shutting time is entered in the plan of the experiment, table 1.

The experimental research creates a data bank containing the similar valves shutting times which, could also be used for the valves shuting times under the action of the air-blast waves modelling and the model adequacy estimation, in accordance with the algorithms presended above.

Model for the similar valves shuting time T_{z_i} under the action of the air-blast waves can be obtained by regression analysis, utilizing the least sqare method as well as experimental data for T_{z_i}.

Model for the similar valves shuting time T_{z_i} under the action of the air-blast waves can be obtained by regression analysis, utilizing the least sqare method as well as experimental data for T_{z_i}.

Estimation of agreement between the model and experimental results

Estimation of agreement between the shuting times of the similar valves model with experimental data for T_{z_i} is made on the basis of the numerical values of the multiple correlation coeficients $R_{i\cdot 123456}$.

Estimation of agreement between the models in the plain $p_f T_{z_i}$ with experimental data for T_{z_i} is made on the basis of the numerical values of the partial correlation characteristic coefficients $r_{i6\cdot 12345}$.

Conclusion

On the example of modeling the similar valves shutting time under direct effects of nuclear explosion air blast waves, the modeling methods of the valves as well as hyper plane problem and the model estimation are defined.

The methods for modeling closing times of similar valves, closing due to the effects of air-blast waves, have been defined, as well as evaluation of model accordance $T_{z_i} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ with experimental results.

In an original way, the analytical expressions of characteristic partial correlation coefficients $r_{i6\cdot 12345}$ and multiple correlation coefficients $R_{i\cdot 123456}$ has been defined for the qualitative model evaluation.

Using partial correlation coefficient $r_{i6\cdot 12345}$ for functional relationship (1) the probability of influence of the system characteristic parameter p_f to the valve response time T_{z_i} is defined for the case when the system other characteristic parameters $(r,\gamma,\delta,\varphi,\alpha)$ are constant.

On the basis of the originally developed analytical expression for characteristic partial correlation coefficient and original research plan, an algorithm for calculation of the numerical values of characteristic partial correlation coefficients and multiple correlation coefficients, is formed, in order to enable making the qualitative estimation of the order of harmony between the model and experimental data for T_{z_i}.

The qualitative analysis of the system (1), i.e. (7,8), characteristic parameters $(r,\gamma,\delta,\varphi,\alpha)$ influence upon the (parameters phenomenon) valves closing times T_{z_i}, under direct effects of nuclear explosion air blast waves, is made on the basis of analytical expressions of the partial correlation characteristic coefficients $r_{i1\cdot 23456}$, $r_{i2\cdot 13456}$, $r_{i3\cdot 12456}$, $r_{i4\cdot 12356}$ and $r_{i5\cdot 12346}$ of the functional relationship $T_{zi} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$. The coefficients $r_{i1\cdot 23456}$, $r_{i2\cdot 13456}$, $r_{i3\cdot 12456}$, $r_{i4\cdot 12356}$ and $r_{i5\cdot 12346}$ have to be analytically defined.

In other words it is necessary to define the probability of trasforming models $T_{z_i} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ into in the plane rT_{z_i}, γT_{z_i}, δT_{z_i}, φT_{z_i} and αT_{z_i} models.

The contribution of the author is in the orginality of defining: method of the similar valves shutting timce modeling, of the qualitative estimation model, analyticall expression of partial corelation characteristics coefficient $r_{i6\cdot 12345}$, and multiple correlation coefficient $R_{i\cdot 123456}$ functionals relationships in the form $T_{z_i} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ after qualitative estimation of the model agreement level with the experimental results for T_{z_i}.

Singular contribution of the author would be in defining the probability of transmitting ion-reduction $(r_{i1\cdot 23456}, r_{i2\cdot 13456}, r_{i3\cdot 12456}, r_{i4\cdot 12356}, r_{i5\cdot 12346}$ and $r_{i6\cdot 12345})$ of seven-dimensional models form $T_{z_i} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ into plain models.

Due to the excessive length, this analysis has been left out and will be the subject of another paper. Based on the obtained model of the valve shutig time and numerical values of characteristic partial corelation coefficients $(r_{i1\cdot 23456}, r_{i2\cdot 13456}, r_{i3\cdot 12456}, r_{i4\cdot 12356}$ and $r_{i5\cdot 12346})$ functionals relationships in the form $T_{z_i} = A r_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$, characteristic systems values are being chosen $(r,\gamma,\delta,\varphi,\alpha,p_f)$ so as to produce (phenomenon parameters) optimum (minimum) valve shutting time.

The described modeling and model evaluation methods can be applied in analytical or numericall solwing problems for the purpose of transforming the derived into appropiate hyper plains from which the necessary analysis, qualitative and quantitative model evaluation, multi-criteria analysis and the process optimization can easily be derived.

Algoritms presented here are useful in some other technical areas (hydraulics, fluidic, pneumatics, aerodynamics, automatic control, etc.) for defining and solving the problems which can not be exactly described by the laws of physics, which is one of the things that adds to the scientific dimension of this paper.

The topic of this paper has been treated es hyper plane problem.

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Received: 04.04.2004.

Metode modeliranja hiperravanskih problema i ocene modela

Data je definicija klase hiperravanskih problema, metode modeliranja i ocene modela. Na primeru modeliranja vremena zatvaranja ventila usled dejstva udarnih talasa definisane su metode modeliranja ventila kao hiperravanskih problema i ocene modela. Definisani su algoritmi matematičkog modeliranja vremena zatvaranja ventila, ocene saglasnosti modela $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ i eksperimentalnih rezultata istraživanja.Za kvalitativnu ocenu dobijenih modela definisani su analitički izrazi karakterističnih koeficijenata parcijalne korelacije i koeficijent višestruke korelacije.Na osnovu algoritama, plana eksperimenta, može se izraditi program za pronalaženje analitičkih izraza vremena odziva ventila, numeričkih iznosa koeficijenata parcijalne i koeficijenta višestruke korelacije funkcionalne veze, oblika $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$, za kvalitativnu ocenu modela. Originalno definisane metode primenjive su i u drugim oblastima (hidraulika, pneumatika, aerodinamika, pneumatika, automatsko upravljanje, obrada rezanjem itd.) u definisanju i rešavanju problema koji ne mogu egzaktno da se opišu direktnim korišćenjem zakona fizike, što između ostalog, daje naučnu dimenziju ovom radu.

Ključne reči: **hiperravanski problem, ventil, vreme zatvaranja, algoritam, matematičko modeliranje, model, koeficijent korelacije, ocena modela.**

Методы моделирования гиперровных проблем и оценки моделей

Здесь приведено определение класса гиперровных проблем, метода моделирования и оценки моделей. На примере моделирования времени выключения клапана вследствие действия скачка уплотнения определены методы моделирования клапана как гиперровных проблем и оценки модели. Тоже определены алгоритмы математического моделирования времени выключения клапана, оценки соответственности модели $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$ и опытных результатов исследования. Для качественной оценки полученых моделей определены аналитические выражения характерных коэффициентов частичной корреляции и коэффициент многократной корреляции. На основании алгоритма, плана опыта, возможно выработать программу для разыскивания аналитических выражений времени, выключения клапана, цифровых размеров коэффициентов частичной корреляции и коэффициента многократной корреляции функциональной зависимости формы $T_{zi} = Ar_1^a \gamma_2^b \delta_3^c \varphi_4^d \alpha_5^e p_{f6}^g$, для качественной оценки модели. Оригинально определенные методы применяемы и в других областях (гидравлика, пневматика, аэродинамика, автоматическая система управления, обработка резкой и.т.д.) в определении и разрешении проблем которых невозможно точно описать прямым пользованием законов физики, что между прочим, этой работе дает научнеы размеры.

Ключевые слова: гиперровная проблема, клапан, время выключения, алгоритм, математическое моделирование, модель, коэффициент корреляции, оценка модели.