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A semi-empirical method for determining aerodynamic load distribution along the helicopter blade span

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A semi-empirical method for determining aerodynamic load distribution along the helicopter blade span in translational flight is presented. The method is based on the classical approach stating that the flow around the blade section is two-dimensional and quasisteady. The vortex wake effects and compressibility are approximated with a constant induced velocity in the plane perpendicular to the no-feathering axis and the Prandtl-Glauert rule, respectively. The modification involves the effect of blade flapping by an additional term in the local resultant velocity and the blade twist along the span. The presented method was applied in determining the Mi-8 helicopter blade load, and the obtained results were compared with those from the FLUENT 6.1. code that solves the averaged Navier-Stokes equations. The noticed differences at the tip of the blade occur due to the vortex wake and transonic flow nonlinear effects.

Key words: **aerodynamics, helicopter, semi-empirical methods, computational fluid dynamics.**

Introduction

SEMI-empirical methods are most widely used in the determination of helicopter blade aerodynamic loads, such as the lifting line theory combined with two-dimensional airfoil experimental data as a function of the local angle of attack and the Mach number. A few analyses also incorporate approximate or semi-empirical corrections for the dynamic stoll, three-dimensional and compressibility flow effects at the tip, and the blade interaction with the trailing vortex. Empirical corrections are made either because the existing aerodynamic theories are not able to take into consideration all effects of compressible viscous flow around the blade, or because the rigorous application of these theories leads to considerable numerical problems.

The basic purpose of these methods is to enable determination of aerodynamic forces at desired sections along the blade span. The standard approach is used in the presented method, assuming two-dimensional flow at each blade section. It means that the local velocity and the angle of attack are calculated, and then the empirically determined aerodynamic characteristics for the airfoil section are used. It has to be pointed out that even if the flow parameters are precisely determined for a considered section, the results will depend on the reliability of the airfoil section data. These mainly depend on Reynolds number differences, tunnel turbulence level and model surface quality.

The effect of blade flapping during each revolution is taken into account by the method using the additional term in the local resultant velocity relative to the considered section, as well as the blade twist along the span.

Two additional assumptions are implicitly introduced in to the method as well. First, a very complex shape of the vortex wake is not modelled, and its effect is involved by the induced velocity that is a part of the local resultant velocity. The induced velocity is assumed to be constant in the plane perpendicular to the no-feathering axis, in the first approximation. Second, the translational velocity component along the chord of the considered section has unsteady character because of the blade rotation. The reduced frequency of this flow is a low value of the order l/R, where l and R are the airfoil chord and the blade span respectively, so this unsteady flow can be approximated by a quasisteady one.

Calculation method

Before being able to calculate the aerodynamic force on a blade, one has to determine the components of the resultant undisturbed velocity relative to any section perpendicular to the longitudinal axis of the blade. Accordingly, an element of the blade, defined by the chord l, of the width dr, at a distance r from the axis of rotation, is considered. The resultant undisturbed velocity relative to the considered element is designated by W, while U_P and U_T are its components along the no-feathering axis and in the plane perpendicular to it (Fig.1).

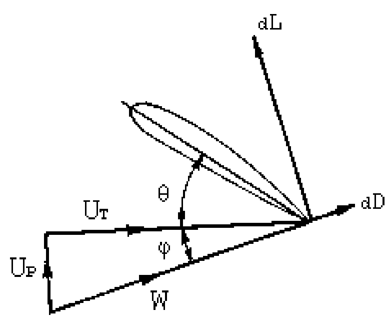

Figure 1. The components of the resultant undisturbed velocity relative to any section perpendicular to the longitudinal axis of the blade.

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The local geometric pitch angle of the blade element and the local inflow angle relative to the plane perpendicular to the no-feathering axis are θ and φ respectively, while the local incidence angle is obtained as

$$\alpha = \theta + \varphi \tag{1}$$

In translational flight, the resultant velocity W involves the velocity components due to translation as well as blade flapping. The helicopter translational velocity and the rotor incidence angle are designated with V and α_{nf} respectively, the latter is an angle between the flight path tangent and the plane perpendicular to the no-feathering axis, i.e. an angle between the plane perpendicular to the flight path and the no-feathering axis (Fig.2).

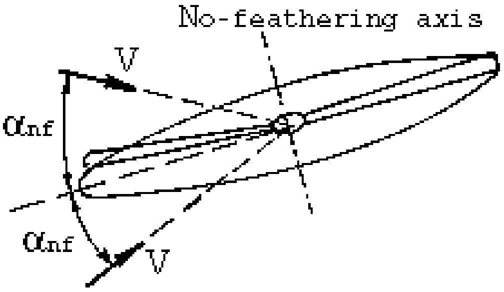

Figure 2. Helicopter translational velocity and the rotor incidence

The projection of the helicopter translational velocity in the plane perpendicular to the no-feathering axis, is decomposed in to two components relative to the blade. These components change with the azimuth due to blade rotation, so the component perpendicular to the longitudinal axis of the blade is

$$V \cos\alpha_{nf} \sin\psi$$

while the component along the blade becomes

$$V \cos\alpha_{nf} \cos\psi$$

These components are shown in Fig.3.

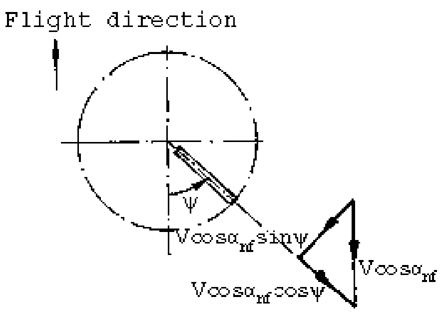

Figure 3. Projection of the helicopter translational velocity in the plane normal to the no-feathering axis, decomposed in two components relative to the blade

Due to blade flapping, a velocity component perpendicular to the longitudinal axis of the blade appears, with the magnitude equal to

$$r\frac{d\beta}{dt}$$

and shown in Fig.4, where the flapping angle β is given relative to the plane perpendicular to the no-feathering axis.

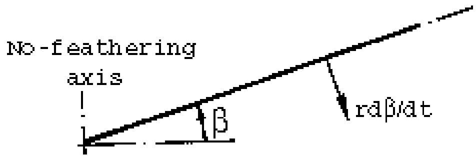

Figure 4. The velocity component due to blade flapping

The components of the resultant undisturbed velocity relative to the considered blade element can be expressed now as

$$U_T = r\Omega + V\cos\alpha_{nf}\sin\psi \tag{2}$$

$$\begin{aligned} U_P = {} & V\sin\alpha_{nf}\cos\beta - r\frac{d\beta}{dt} - w - \\ & -V\cos\alpha_{nf}\cos\psi\sin\beta \end{aligned} \tag{3}$$

while the resultant velocity component along the longitudinal axis of the blade is neglected. Thus, the three-dimensional flow around the blade is approximated by the two-dimensional one, and this is acceptable except near the blade tip. The intensity of induced velocity in the direction of the no-feathering axis is defined as w in eq.(3).

By the introduction of parameters that determine the rotor flow regime, i.e. the advance ratio μ and the inflow ratio λ, defined in [1] by expressions

$$\mu = \frac{V\cos\alpha_{nf}}{\Omega R} \tag{4}$$

$$\lambda = \frac{V\sin\alpha_{nf} - w}{\Omega R} \tag{5}$$

the resultant velocity components become

$$U_T = r\Omega + \mu\Omega R\sin\psi \tag{6}$$

$$U_P = \lambda\Omega R - r\frac{d\beta}{dt} - \mu\Omega R\beta\cos\psi \tag{7}$$

The elementary aerodynamic force dR acts on the considered blade element, with a component along the no-feathering axis equal to the elementary thrust, i.e.

$$dT = dR_P \tag{8}$$

The component along the no-feathering axis of the elementary aerodynamic force can be expressed in terms of the elementary lift and drag as

$$dR_P = dR_L\cos\varphi - dR_D\sin\varphi \tag{9}$$

The elementary lift and drag depend on the local aerodynamic coefficients and the resultant undisturbed velocity relative to the considered blade element, and they are given by

$$dR_L = \frac{1}{2}\rho W^2 c_L l dr \tag{10}$$

$$dR_D = \frac{1}{2}\rho W^2 c_D l dr \tag{11}$$

so the elementary thrust becomes

$$dT = \frac{1}{2}\rho W^2 \left(c_L - c_D \varphi\right) l dr \tag{12}$$

The local lift coefficient is determined by using the lift slope of the considered blade element section and the local incidence angle expressed as

$$\alpha = \theta + \frac{U_P}{U_T} \tag{13}$$

If the product of the local drag coefficient and the local inflow angle compared to the local lift coefficient is neglected, and since $W \approx U_T$ in translational flight, the elementary thrust is

$$dT = \frac{1}{2}\rho a\left(\theta U_T^2 + U_P U_T\right) l dr \tag{14}$$

Because of

$$\frac{d\beta}{dt} = \frac{d\beta}{d\psi}\frac{d\psi}{dt} = \frac{d\beta}{d\psi}\Omega$$

the resultant velocity components become

$$W_T = r\Omega + \mu\Omega R \sin\psi \tag{15}$$

$$W_P = \lambda\Omega R - r\Omega\frac{d\beta}{d\psi} - \mu\Omega R\beta\cos\psi \tag{16}$$

The horizontal component of velocity creates the flow field asimmetry for the advancing and retreating blade in translational flight. This results in different lift forces, and the rolling moment appears. This moment is compensated by a horizontal hinge that provides blade flapping and equilibrium about the longitudinal axis in articulated rotors. It is possible to determine the dependance of the blade flapping angle on the azimuth at each flight regime. The blade describes a conical surface about the axis tilted towards the no-feathering axis. The flapping angle is a periodic function of the azimuth angle, expressed in [2] by the Fourier series of the form

$$\beta = a_0 - a_1\cos\psi - b_1\sin\psi - a_2\cos 2\psi - b_2\sin 2\psi - ... \tag{17}$$

Experimental investigations show that coefficients beyond the second order are small quantities compared to coefficient of the first and second order and can be neglected. For most steady regimes coefficients of the second order can be neglected, and calculation is considerably simplified. As a rough rule it can be taken that the value of a coefficient of some order is about one tenth of the value of that of the previous lower order.

The coefficients a_0, a_1, and b_1 depend on the specific weight of blade material and the rotor flow regime, and under the condition that the flapping-hinge offset is neglected, they are given in [3] by expressions

$$a_0 = \gamma\left[\frac{\theta_{0.75}}{4}\left(1+\mu^2\right) + \frac{\lambda}{3}\right] \tag{18}$$

$$a_1 = \frac{2\mu\left(\frac{4}{3}\theta_{0.75} + \lambda\right)}{1 - \frac{\mu^2}{2}} \tag{19}$$

$$b_1 = \frac{4\mu a_0}{3\left(1 + \frac{\mu^2}{2}\right)} \tag{20}$$

where γ is the specific weight of blade material. It should be mentioned that the expression for the coefficient b_1 holds under the assumption of a constant induced velocity in the plane perpendicular to the no-feathering axis.

Using eq.(17) the resulting velocity components become

$$W_T = r\Omega + \mu\Omega R\sin\psi \tag{21}$$

$$\begin{aligned} W_P = {} & \lambda\Omega R - r\Omega\left(a_1\sin\psi - b_1\cos\psi\right) - \\ & -\mu\Omega R\left(a_0 - a_1\cos\psi - b_1\sin\psi\right)\cos\psi \end{aligned} \tag{22}$$

In a numerical procedure, the segments of the finite width Δr are considered, so thrust at each segment becomes

$$\Delta T = \frac{1}{2}\rho a\left(\theta W_T^2 + W_P W_P\right) l \Delta r \tag{23}$$

taking into account that all quantities are constant at the considered segment, and their values are calculated in the middle of the segment.

The resultant velocity increases toward the blade tip due to rotation, and the effect of fluid compressibility must be taken into account. In the presented method this effect is determined by the correction of the section lift slope using the Glauert rule

$$a = \frac{a_0}{\sqrt{1 - M^2}} \tag{24}$$

The Mach number M of the resulting undisturbed stream relative to the considered section is given by the expression

$$M = \frac{W}{c_\infty} = \frac{\sqrt{W_P^2 + W_T^2}}{c\infty} \tag{25}$$

where c_∞ is the speed of sound in the undisturbed streem.

Results

The presented method was applied in order to determine the thrust distribution along the span of the Mi-8 helicopter main rotor blade. The purpose was the static strength check, because the part of the blade behind the spar originally made of metal, was replaced with composite material.

The following input data are given for the helicopter:

helicopter mass	m = 11100 kg
rotor area	S =355.7 m2
rotor diameter	D = 21.29 m
number of blades	5
blade airfoil	NACA 23012M
blade chord	l=0.52 m
blade twist	-5°
rotor solidity	σ=0.0777
rotor rotational speed	Ω=20.94 rad/sec
max. speed	V=69.44 m/s (250 km/h)

The blade is divided into 21 segments of equal width. Both thrust and blade loading along the span are determined at each segment, and the latter is given as

$$q_L = \frac{\Delta T}{\Delta r} \qquad (26)$$

The test case is defined by:

load factor $n = n_{max} = 2.5$
velocity of translation $V = V_{max} = 69.44$m/s (250 km/h)

The resultant helicopter thrust for the given load factor is

$$T = nG \qquad (27)$$

while the average thrust at each blade is obtained as

$$T_b = \frac{T}{5} \qquad (28)$$

For the Mi-8 helicopter in a level flight at maximum speed, the thrust change of the main rotor blade during one revolution, is given in [4] (Fig.5). It can be noticed firstly, that the maximum blade thrust achieves about 150 % of the mean value, and, secondly that, blade thrust changes as the second harmonic. It means that the blade thrust has two oscillations during each revolution. The dashed

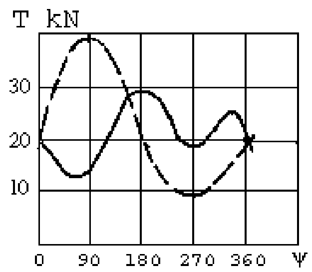

Figure 5. Mi-8 main rotor blade thrust change during one revolution

line gives the blade thrust change in the hingeless rotor, with much more irregularity of thrust changing as the first harmonic.

In order to determine thrust at each blade segment, the components of the resultant velocity relative to a considered segment have to be determined. These components depend on the parameters μ and λ, varying with the change of the helicopter angle of attack α_{nf}, expressed in [3] as

$$\alpha_{nf} = \frac{\lambda}{\mu} + \frac{1}{2}\frac{C_T}{\mu\sqrt{\lambda^2 + \mu^2}} \qquad (29)$$

Since the helicopter angle of attack depends on μ and λ, the quantities μ, λ, α_{nf} and the induced velocity w are determined iteratively.

The thrust coefficient C_T is defined in [1] as

$$C_T = \frac{T}{\rho(\Omega R)^2 S} \qquad (30)$$

The induced velocity is assumed to be constant in the plane perpendicular to the no-feathering axis, given in [3] as

$$w = \frac{C_T V}{2\mu^2} \qquad (31)$$

The test case is characterized by the thrust coefficient C_T=0.012187, the translational velocity V=69.44 m/s and the rotor angular velocity Ω=20.94 rad/s. The values of the requested quantities are obtained in a few iterations:

$$\alpha_{nf} = -0.1^{\circ}$$
$$\mu = 0.3175$$
$$\lambda = -0.0180$$
$$w = 4.2 \text{ m/s}$$

The thrust distributions along the blade span are given in D-1 for the azimuth angle equal to 180º (Fig.6). They are obtained by the presented metod and the FLUENT software 6.1. that solves Reynolds averaged Navier-Stokes equations. The largest differences appear at the blade tip because of nonlinearity due to vortex wake and transonic flow.

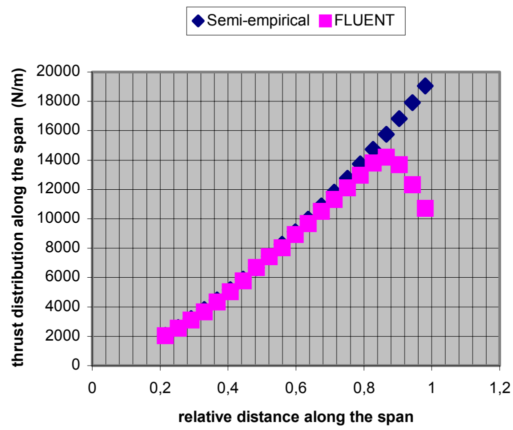

Figure 6. D-1 Thrust distribution along the blade span obtained by two methods

Conclusion

The presented method gives the preliminary aerodynamic loads i.e. the thrust distribution along the span of the twisted helicopter blade in translational flight, taking into account blade flapping.

The method is based on the assumptions that the flow around the blade sections is two-dimensional, quasisteady and without flow separation.

The main limitation of the method is its inability to calculate unsteady loads, in the case of a close vortex-blade interaction (loads at higher harmonics).

The results obtained by the method were compared to the ones from a code solving averaged Navier-Stokes equations, and it is concluded that the method can usefully serve for a fast determination of preliminary aerodynamic blade loads needed for static strength checks.

References

[1] NENADOVIĆ,M. Osnovi projektovanja i konstruisanja helikoptera, izdavač Beograd, 1982.

[2] SAVEZNI SEKRETARIJAT ZA NARODNU ODBRANU, RV i PVO-227, Teorija leta helikoptera, (prevod sa ruskog),1972.

[3] BRAMWELL,A.R.S. *Helicopter Dynamics*. Edward Arnold, 1976.

[4] VOLDKO,A.M. *Osnovy letnoj ehkcpluatatsii vertoletov*. Moskva, Transport, 1984.

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Poluempirijska metoda za određivanje raspodele aerodinamičkog opterećenja duž razmaha lopatice helikoptera

Izložena je poluempirijska metoda za preliminarno određivanje raspodele aerodinamičkog opterećenja duž razmaha lopatice helikoptera u translatornom letu. Metod je zasnovan na klasičnom pristupu da je strujanje oko preseka lopatice dvodimenzijsko i kvazistacionarno, efekat vrtložnog traga se aproksimira konstantnom indukovanom brzinom u ravni cikloprstena, a uticaj stišljivosti Prantl-Glauertovim pravilom. Modifikacija obuhvata efekat mahanja lopatice preko dodatnog člana u lokalnoj rezultujućoj brzini, kao i vitoperenje duž razmaha. Izloženi metod primenjen je pri određivanju opterećenja lopatice helikoptera Mi-8, a dobijeni rezultati upoređeni su sa rezultatima softvera FLUENT 6.1., koji rešava usrednjene Navije – Stoksove jednačine. Uočene razlike na kraju lopatice javljaju se usled nelinearnih efekata vrtložnog traga i transsoničnog strujanja.

Ključne reči: **aerodinamika, helikopter, poluempirijske metode, numerička dinamika fluida.**

Une méthode sémi-empirique pour la détermination de la distribution de charge aérodynamique le long de l’envergure de pale de l’hélicoptère

L'article présente une méthode sémi-empirique pour la détermination préliminaire de la distribution de charge aérodynamique le long de l’envergure de pale de l’hélicoptère en vol en translation. La méthode est basée sur l’approche classique disant que l’écoulement autour du profile de pale est bidimensionnel et quasi-stationnaire. L’effet du sillage tourbillonnaire est déterminé approximativement par la vitesse constante induite dans le plan perpendiculaire à l’axe normal au plan de base rotor et la compressibilité par la règle de Prandtl-Glauert. La modification comprend l’effet du battement de pale par un terme supplémentaire dans la vitesse résultante locale aussi bien que le vrillage de pane le long de l’envergure. La méthode présentée était appliquée pour déterminer la charge le long d’une pale de l’hélicoptère Mi-8 et les résultats obtenus sont comparés aux résultats du logiciel FLUENT 6.1. qui résout les équations de Navier-Stokes rendues moyennes. Les différences observées, au bout de la pale se produisent à cause des effets non-linéaires du sillage tourbillonnaire et de l’écoulement transsonique.

Mots-clés: **aérodynamique, hélicoptère, méthodes sémi-empiriques, dynamique numérique de fluide.**