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Analysis of the detection threshold of pulsed laser tracking systems

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A threshold of a pulse detection is derived on the basis of three criteria: equal both false-alarm probability and missed-pulse probability, minimum probability of the total error, and constant false-alarm probability. The obtained semi-optimal, optimal and Neyman-Pearson thresholds are analyzed as a function of both signal-to-noise ratio (SNR) and gate-to-pulse duration ratio (t_R/τ). The number of false-alarms and the number of missed-pulses at the gate time an analyzed for all derived thresholds. The number of false-alarms is more critical than the number of missed-pulses in pulsed laser tracking systems. The number of false-alarms at the gate time is the biggest for small SNRs. But, the number of false-alarms for semi-optimal thresholds is bigger than for other thresholds for the same signal-to-noise ratio. Optimal and Neyman-Pearson thresholds are suitable when the SNR is lowr than 15 dB while for the SNR higher than 15 dB all analyzed thresholds can be used.

Key words: **threshold of detection, values of threshold, false-alarm rate, missed-pulse rate.**

Introduction

PULSE threshold detection is a specific problem in laser tracking systems, laser guidance and laser rangefinders. In these systems the pulse period is longer than the pulse duration, and the level of signal power is very similar to the background power. On the other hand, the optical pulse to receive for unknown time, but the pulse is expected in the period of transmitted pulses. Detection of a pulse in laser tracking systems involves two types of errors. The first one is a loss of the received pulse and the second one is known as a false-alarm. Both errors can be described by the lost-pulse probability and the false-alarm probability.

The theory of signal detection in noise [1] gives two criteria for finding the threshold value, the criterion of the minimum total error and the criterion of the maximum pulse detection probability, for a constant false alarm rate probability. The probability of total error in digital signal transmission is analyzed [2], and threshold detection is obtained from the condition of both false-alarm and missed-pulse probabilities. Detections of single pulses in white noise by the optimized receiver are analyzed in [3]. The optimization of laser tracking optical receivers is given in [4], and the pulse detection threshold is derived on the basis of the minimum error probability.

In this paper the pulse detection threshold is derived on the basis of three criteria: equal false-alarm and missed-pulse probability, minimum probability of the total error, and maximum probability of pulse detection for a given false-alarm probability. Both false-alarm rate and missed-pulse rate are analyzed for derived thresholds.

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Detection of pulses

A block diagram of a typical pulse laser tracking system is shown in Fig.1.

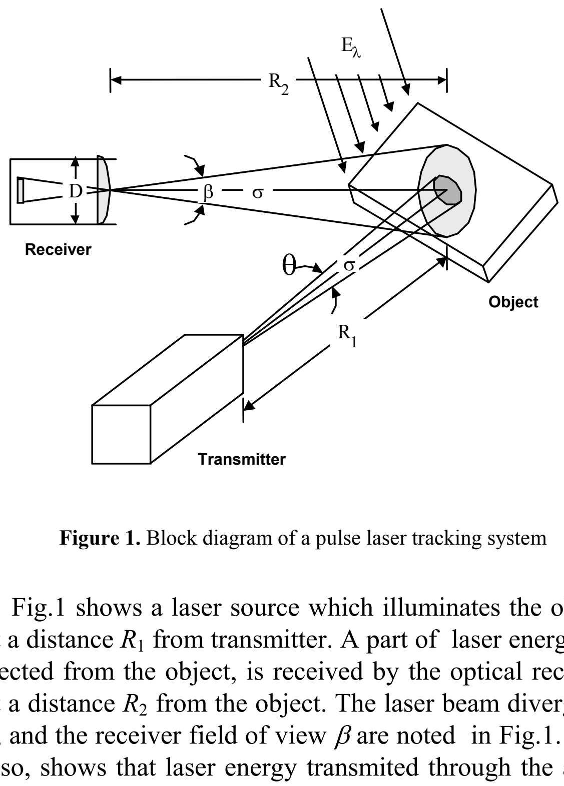

Figure 1. Block diagram of a pulse laser tracking system

Fig.1 shows a laser source which illuminates the object at a distance R_1 from transmitter. A part of laser energy, reflected from the object, is received by the optical receiver, at a distance R_2 from the object. The laser beam divergence θ, and the receiver field of view β are noted in Fig.1. Fig.1 also, shows that laser energy transmited through the atmosphere with the extinction coefficient σ, and the spectral irradiance of the Sun, E_λ comes on to the object surface.

For the detection of periodical pulses, transmitted and received signals in time domain can be used, as shown in Fig.2.

In Fig.2 the pulse period T_i and the pulse width τ are shown. The delay time t_k is the total time difference between the received and the emitted pulse. This time consists of two parts: the first one represents the distance R_1, and the second part represents R_2. The time duration t_R is the time in which to expect the pulse at the receiving side, known as a gate in time.

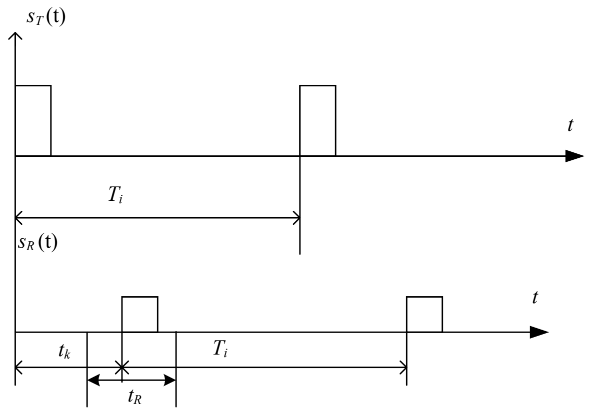

Figure 2. Signal in time domain

On the basis of known theory, the probability of total error, for pulse detection, is given as [2]

$$P_E = P(1)P(0/1) + P(0)P(1/0) \tag{1}$$

where: $P(1)$ is the probability of pulse existence at the gate time, $P(0)$ is the probability of pulse absence at the gate time, $P(0/1)$ is the missed pulse probability, and $P(1/0)$ is the false alarm probability.

From the diagrams given in Fig.2, the probabilities $P(1)=\tau/t_R$ and $P(0)=1\text{-}P(1)$. can be derived. The probability of false-alarm $P(1/0)$, for the Gaussian noise distribution, can be written as

$$P(1/0) = \frac{1}{\sqrt{2\pi}\sigma_0}\int_{I_p}^{\infty}\exp(-\frac{(i-I_b)^2}{2\sigma_0^2}di = \frac{1}{2}erfc\left(\frac{I_p - I_b}{\sqrt{2}\sigma_0}\right) \tag{2}$$

where: i is the signal amplitude value, I_p is the threshold value, I_b is the average background value, and σ_0 is the standard deviation of the signal when the pulse is not present.

The probability of missed-pulse $P(1/0)$, for the Gaussian signal plus the noise distribution, can be written as

$$P(0/1) = \frac{1}{\sqrt{2\pi}\sigma_1}\int_{-\infty}^{I_p}\exp(-\frac{(i-I_s)^2}{2\sigma_1^2}di = \frac{1}{2}erfc\left(\frac{I_s - I_p}{\sqrt{2}\sigma_1}\right) \tag{3}$$

where: I_s is the average value of the signal plus noise, and σ_1 is the standard signal deviation when the pulse is present.

Finally, after substituting (2) and (3) in (1), the probability of total error is obtained as

$$P_E = \frac{\tau}{t_R}\frac{1}{2}erfc\left(\frac{I_s - I_p}{\sqrt{2}\sigma_1}\right) + \left(1 - \frac{\tau}{t_R}\right)\frac{1}{2}erfc\left(\frac{I_p - I_b}{\sqrt{2}\sigma_0}\right) \tag{4}$$

The probability of total error is reduced for equal arguments of the complimentary error function (4). Then, for $P(1/0)=P(0/1)$, the probability of total error becomes

$$P_{E1} = P_{FA} = \frac{1}{2}erfc\left(\frac{I_p - I_b}{\sqrt{2}\sigma_0}\right) = \frac{1}{2}erfc\left(\frac{I_s - I_p}{\sqrt{2}\sigma_1}\right) \tag{5}$$

The threshold value from (5), for $P(1/0)=P(0/1)$ can be obtained as

$$I_{p1} = \frac{I_s\sigma_0 + I_b\sigma_1}{\sigma_0 + \sigma_1} \approx \frac{1}{2}I_s(1+a) \tag{6}$$

where: $\sigma_1=\sigma_0=\sigma_n$, and $a=I_b/I_s$.

The derived threshold (6) is a function of the average value of the signal and the background-to-signal ratio ($a=I_b/I_s$). This threshold is callend the semi-optimal threshold.

The second criterion for the calculation of the threshold value is the criterion for minimizing the probability of total error. Then, after applying $dP_E/di=0$, the threshold is obtained from (1), (2) and (3) in the following form

$$I_{pop} = I_s(1+a)\left(\frac{1}{2} + \frac{\ln((t_R - \tau)/\tau)}{SNR(1-a^2)}\right) \tag{7}$$

where $SNR=(I_s/\sigma_n)^2$.

The obtained threshold (7) is a function of the signal-to-noise ratio (SNR), and the gate-to-pulse duration ratio (t_R/τ). This threshold is called the optimal threshold (I_{pop}).

The third criterion is known as the Neyman-Pearson performance criterion [1]. The threshold detection of pulse can be found from the given probability of false-alarm. The probability of false-alarm P(1/0) from (2), for erfc(x)=1-erf(x), can be written in the form

$$P_{FA} = \frac{1}{2}\left(1 - erf\left(\frac{I_p - I_b}{\sqrt{2}\sigma_0}\right)\right) \tag{8}$$

The threshold obtained from (8) for the Neyman-Pearson performance criterion can be written as

$$I_{pNP} = I_b + \sqrt{2}\sigma_0 erfinv(1 - 2P_{FA}) \tag{9}$$

where *erfinv*(x) is the inversion function of *erf*(x).

The obtained threshold (9) is a function of the noise standard deviation, for the given probability of false-alarm. This threshold is called the Neyman-Pearson threshold (I_{pNP}).

Suitable parameters for analyzing the threshold value are the number of false-alarm and the number of missed-pulse in the given time. The false-alarm rate [5] represents the average false-alarm rate (FAR -False-Alarm Rate)

$$N_{FA} = FAR = \frac{P_{FA}}{T_E} \tag{10}$$

where T_E is the filter time constant.

The filter time constant T_E and the filter equvivalent bandwidth B_E are inversly proportional.

The equvivalent bandwidth BE can be estimated on the basis of the pulse rise time. The rise time can be calculated from 10% and 90% of the pulse amplitude [6]. The rise time tr and the pulse width τ are related, for the Gaussian pulse, as

$$t_r = 1{,}19\sigma_i \quad \text{and} \quad \tau = \sigma_i 2\sqrt{\ln 2} \tag{11}$$

where τ is the pulse width at half amplitude.

Now, the filter equivalent bandwidth becomes

$$B_E = \frac{1}{t_r} = \frac{2\sqrt{\ln 2}}{1{,}19\tau} \approx \frac{1{,}4}{\tau} \tag{12}$$

Finally, the false-alarm rate becomes

$$N_{FA} = \frac{1,4}{\tau} P_{FA} \tag{13}$$

The missed-pulse rate is the average number of missed pulses per second denoted as MPR (Missed-Pulse Rate)

$$N_{MP} = MPR = f_i P(0/1) = f_i (1 - P_D) \tag{14}$$

where: f_i is the pulse rate, and P_D is the probability of pulse detection.

The probability of detection is 1-P(0/1), and can be written on the basis of equation (3) as

$$P_D = 1 - P(0/1) = 1 - \frac{1}{2} erfc\left(\frac{I_s - I_p}{\sqrt{2}\sigma_1}\right) \tag{15}$$

After rearanging (15), the probability of detection becomes

$$P_D = \frac{1}{2}\left(1 + erf\left(\sqrt{\frac{SNR}{2}} - \frac{I_p}{\sqrt{2}\sigma_1}\right)\right) \tag{16}$$

where $SNR=(I_s/\sigma_n)^2$ is the signal-to-noise ratio for $\sigma_1=\sigma_n$.

From (13) and (14) the required P_{FA} and P_D for the given N_{FA} and N_{MP} can be calculated. For example, if only one missed pulse of received 20 pulses per second is needed, from (14) one can obtaine P_D=1-1/20=0.95. Also, if only one false alarm at 20 pulse periods is required, from (13) one can obtaine $P_{FA}=0.0357\,\tau/t_R=1.071\cdot10^{-4}$, at $20t_R$, where $t_R=R_2/c=(10/3)\cdot10^{-5}$ s, and τ =100 ns. This means that numbers of false alarms and numbers of missed pulses are limited: $P_D\geq0.95$ and $1.071\cdot10^{-4}\geq P_{FA}$.

Analysis of results

In Fig.3 the surface of I_p/I_s ratio as a function of the SNR and the t_R/τ, for the optimal threshold (7) is shown. Fig.3 (above) shows I_p/I_s for $a=I_b/I_s=0$, and Fig.3 (below) shows I_p/I_s, for $a=I_b/I_s=0.1$.

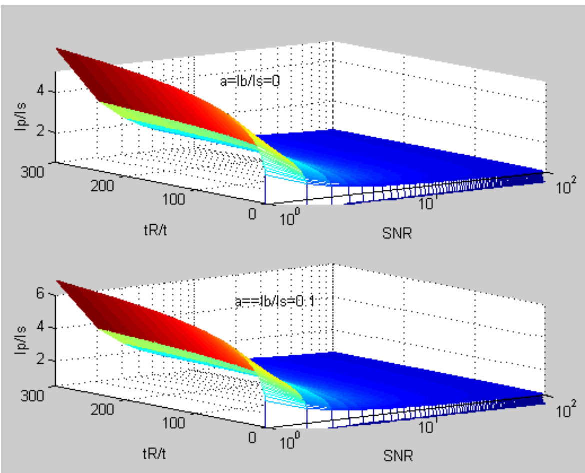

Figure 3. I_p/I_s as a function of the SNR and the t_R/τ

Fig.3 presents a very large change of I_p/I_s for a low SNR and a glint surface for77 a high SNR, which becomes constant and equal to 0.5 like the ratio for the semi-optimal treshold (6). On the other hand, the ratio I_p/I_s increase of the ratio t_R/τ, for a low SNR (Fig.3). From comparing diagrams shown in Fig.3 it can be seen that the ratio I_p/I_s is higher below (a=0.1) than alow (a=0).

The thresholds given in (6), (7) and (9) can be written as functions of the signal-to noise ratio (SNR), for the assumptions: a=0, t_R/τ=constant, and the given false-alarm probability. The threshold detection derived from the Neyman-Pearson performance criterion (9) can be written as function of the signal-to-noise ratio, for I_b=0, in the form of $I_{NP} = \sqrt{\frac{2}{SNR}} erfinv(1 - 2P_{FA})$.

In Fig.4, the curves I_p/Is for the Neyman-Pearson, semi-optimal and optimal thresholds are shown.

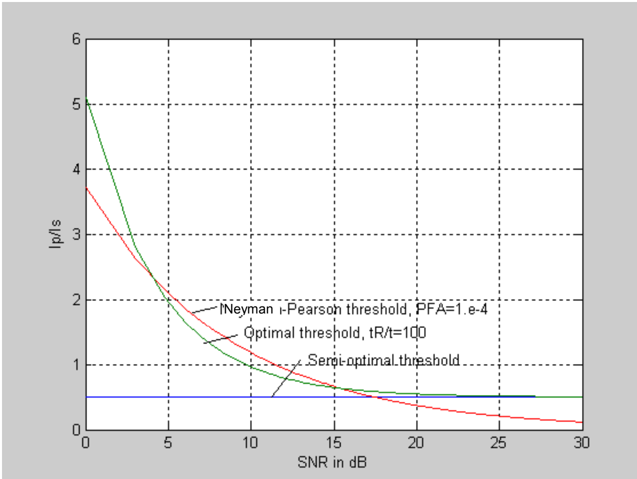

Figure 4. Ratio I_p/I_s as a function of the SNR

In Fig.4 the changing of the ratio I_p/I_s as a function of the SNR, for the optimal threshold (7) and the Neyman-Pearson threshold (9) is shown. The ratio I_p/I_s is constant for the semi-optimal threshold, and for the Neyman-Pearson and optimal threshold it is very changeable, when the SNR is low (<15 dB). Fig.4 shows that the ratio I_p/I_s is higher than one to interval for both optimal and Neyman-Pearson thresholds, for a SNR lower than 10 dB, and approximately between 1 and 0.5 for the SNR between 10 and 20 dB. Fig.4 shows that the value of the optimal threshold becomes equal to the value of the semi-optimal threshold for a SNR higher than 20 dB. On the other hand, the value of the Neyman-Pearson threshold is very similar to the value of the optimal threshold, for a low SNR (SNR==3-5 dB).

Number of false alarms, at the gate time t_R is calculated from (13). Fig.5 shows the number of false alarms as a function of both SNR and t_R/τ, for the optimal threshold value.

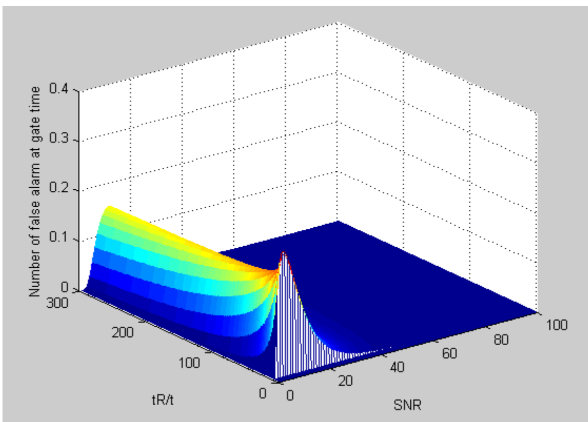

Figure 5. Number of the FAR at the gate time as a function of the SNR and the t_R/τ

Fig.5 shows that the number of false alarms, at the gate time increases with decrease of the SNR. The largest number of false alarms is obtained for the minimum SNR, for all gate-to-pulse duration rates.

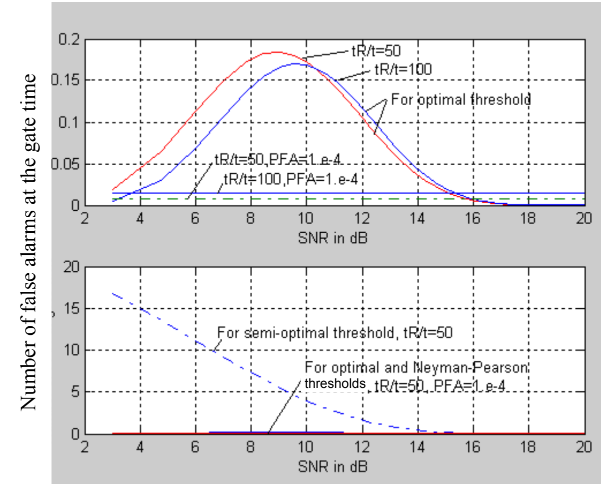

Figure 6. Number of FAR as a function of the SNR, for the parameters t_R/τ=50;100, and P_{FA}=10^{-4}

Fig.6 shows the number of false alarms at the gate time as a function of the signal-to-noise ratio, for the optimal (t_R/τ=50;100), semi-optimal (t_R/τ=50) and Neyman-Pearson (P_{FA}=10^{-4} and t_R/τ=50) threshold detection. The number of false alarms is the largest for the semi-optimal threshold for the SNR<15 dB.

The number of missed pulses at the gate time t_R is calculated from (14). Fig.7 shows the number of missed pulses at the gate time for the optimal threshold value.

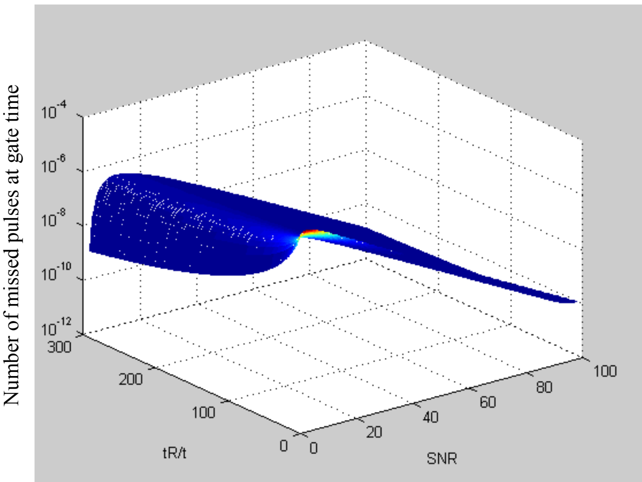

Figure 7. Number of missed pulses as a function of both SNR and t_R/τ, for t_R/T_i=1.0e^{-3}

The number of missed pulses at the gate time is very small for all values of the SNR and the t_R/τ. That means that the number of false alarms is a more critical parameter than the number of missed pulses in laser pulse tracking systems.

The probability of detection as a function of both SNR and t_R/τ is shown in Fig.8.

The detection probability is low for the SNR<10, for all values of the t_R/τ. The detection probability increases very fast with the increase of the SNR.

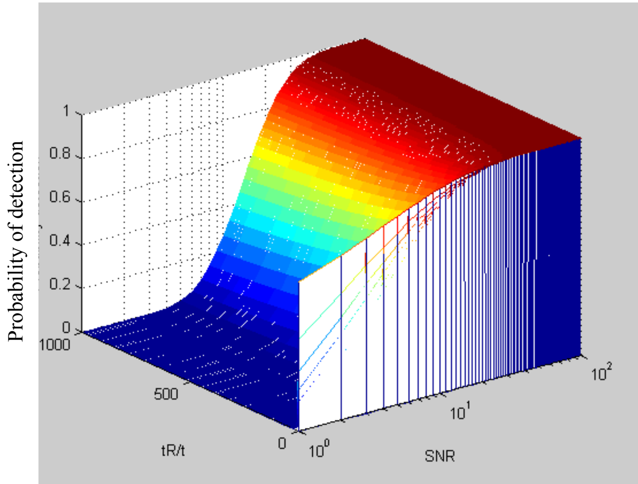

Figure 8. Probability of detection as a function of both SNR and t_R/τ

Fig.9 gives the curves of the probability of detection as a function of the signal-to-noise ratio, for semi-optimal, optimal (t_R/τ=100, and t_R/τ=500) and Neyman-Pearson (P_{FA}=10^{-3}, and P_{FA}=10^{-4}) threshold values.

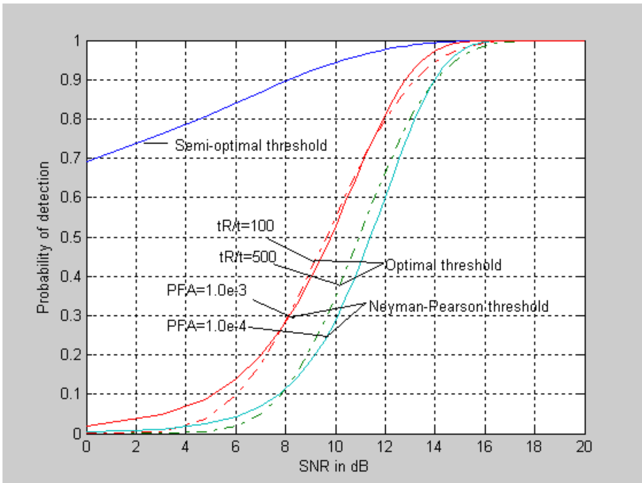

Figure 9. Probability of detection as a function of the SNR

From Fig.9 it can be seen that the detection probability for the semi-optimal threshold ($I_{p1}=I_s/2$) is the highest, for all SNR values. The detection probability is approximately the same at the optimal threshold (t_R/τ=1000) and at the Neyman-Pearson threshold (P_{FA}=10^{-4}). Also, the probability of pulse detection is higher than 0.95 for the SNR>15 dB.

Conclusion

The threshold of pulse detection is derived on the basis of three criteria: equal both false-alarm probability and missed-pulse probability, minimum probability of the total error, and maximum detection probability with the minimum false-alarm probability. The obtained semi-optimal, optimal and Neyman-Pearson thresholds are analyzed as a function of both signal-to-noise ratio (SNR) and gate-to-pulse duration rate (t_R/τ). The semi-optimal threshold-to-signal ratio I_p/I_s remains constant for all SNR and t_R/τ values. For the optimal and Neyman-Pearson thresholds, the ratio I_p/I_s showed changes with respect to both signal-to-noise ratio and gate-to-pulse ratio, but these changes are the biggest for a low signal-to- noise ratio.

The number of false alarm and the number of missed pulses at the gate time are analyzed for all derived thresholds. The number of false alarms is more critical than the number of missed pulses in pulsed laser tracking systems. The number of false alarms at the gate time is the largest for a low signal-to-noise ratio. However, the number of false alarms for the semi-optimal threshold is larger than the number of false alarms for other thresholds for the same signal-to-noise ratio. The optimal and Neyman-Pearson thresholds are suitable when the signal-to-noise ratio is lower than 15 dB. For signal-to-nose ratio higher than 15 dB, all analyzed thresholds can be used.

References

[1] MISCHA SCHWARTZ, LEONARD SHAW. *Signal Processing: Discrete Spectral Analysis, Detection, and Estimation*. McGraw-Hill, New York, 1975.

[2] MARINČIĆ,A. *Optičke telekomunikacije*. Univerzitet u Beogradu, 1997.

[3] RCA Corporation, *Electro-Optical Handbook*. Technical series EOH-11, 1974.

[4] MANOJLOVIĆ,L. *Analiza i optimizacija prijema optičkih signala sa optoelektronskog koordinatora u impulsnim laserskim sistemima praćenja.* Magistarski rad, Elektrotehnički fakultet, Beograd, 2002.

[5] SEYRAFI,K., HOVANESSIAN,S.A. *Introduction to Electro-Optical Imaging and Tracking Systems.* Artech House. Inc. Boston-London, 1993.

[6] BURNES,H.N., CHRISTODOULOU,C.G., GLENN D.BOREMAN. System design of a pulsed laser rangefinder. *Optical Engineering*, 1991, vol.30, no.3.

[7] BARBARIĆ,Ž. *Optimization of Optical Pulse Detection Threshold*. XI Telecommunications Forum, TELFOR 2003, Belgrade, 25,26 and 27 November 2003.

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Analiza praga detekcije impulsa u laserskim sistemima praćenja

Izveden je prag detekcije laserskog impulsa po tri kriterijuma: jednake verovatnoće lažnog alarma i gubitka impulsa, minimalne verovatnoće ukupne greške u odlučivanju i konstantne verovatnoće lažnog alarma. Analizirane su dobijene vrednosti kvazi-optimalnog, optimalnog i Nojman-Pirsonovog praga detekcije u funkciji odnosa signal-šum (SNR) i odnosa vremena otvorenosti prijemnika i trajanja impulsa (t_R/τ). Izvršena je analiza broja lažnih alarma i broja propuštenih impulsa u vremenu otvorenosti prijemnika, za izvedene pragove. Broj lažnih alarma je mnogo kritičniji od broja propuštenih impulsa, u laserskim sistemima za praćenje. Broj lažnih alarma u vremenu otvorenosti prijemnika je najveći za minimalan odnos signal--šum. Ali, broj lažnih alarma je veći za kvazi-optimalan nego za optimalan i Nojman-Pirsonov prag, za isti odnos signal-šum. Optimalan i Nojman-Pirsonov prag su pogodni za SNR manji od 15 dB, a za SNR veći od 15 dB mogu se koristiti svi anlizirani pragovi.

Ključne reči: **prag detekcije, vrednosti praga, broj lažnih alarma, broj propuštenih impulsa.**

Analyse du seuil de détection des impulsions chez les systèmes de poursuite laser

Le seuil de détection des impulsions est dérivé selon trois critères: probabilité egaux de fausse alarme et de la perte d'impulsions, probabilité minimale de l'erreur totale et la probabilité constante de fausse alarme. Les valeurs du seuil de détection quasi-optimal, optimal et celui de Neyman-Pearson sont analysées en fonction du rapport signal-bruit (SNR) et du rapport durée d'ouverture du récepteur–durée d'impulsion (t_R/τ). Le nombre de fausses alarmes et impulsions manquées pendant l'ouverture du récepteur est analysé pour tous les seuils derivés. Le nombre de fausses alarmes est plus critique que le nombre d'impulsions manquées chez les systèmes de poursuite laser. Le nombre de fausses alarmes pendant l'ouverture du récepteur est plus grand pour le rapport signal-bruit minimal. D'autre part, le nombre de fausses alarmes est plus grand pour le seuil quasi-optimal que pour le seuil optimal et le seuil de Neyman-Pearson pour le même rapport signal-bruit. Le seuil optimal et le seuil de Neyman--Pearson conviennent pour le SNR inférieur à 15dB et pour le SNR supérieur à 15dB tous les seuils analysés peuvent être utilisés.

Mots-clés: **seuil de détection, valeurs des seuils, nombre de fausse alarmes, nombre d'impulsions manqées.**