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Efficiency analysis of the planetary gear trains functional modules

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The efficiency analysis of the planetary gear trains functional modules was performed. The obtained results are very important in the design process of planetary gear trains. Analytical expressions for the determination of efficiency for several existing methods were shown. The correlation of the considered methods was determined from the standpoint of the defined values of efficiency and the influence of relevant parameters on the functional modules efficiency was considered.

Key words: **planetary gear train, functional module, gear ratio, efficiency.**

Introduction

THE power transmission in a planetary gear train occurs after the creation of a kinematic connection between an input and output element. In that case, the constant gear ratio is established and the gear train has one ratio of the freedom of movement. One of the aims of train design is to provide several determined gear ratios. The distinct configurations of gear train elements are created in order to provide gear ratios. The created configuration which transmits the power (functions), other than by the gear ratio, is characterized by other kinematic, dynamic and energetic parameters (angular velocity and peripheral speed of elements, torques, flows of effective and circulating power and power losses).

The configuration of a gear train, which performs the determined gear ratio and cannot be separated functionally into simpler configurations, is defined as a functional module [1].

The definition of functional modules enables the simplification of the analysis of the current planetary trains kinematic schemes referring to kinematics, dynamics and energy losses. On the other hand, the functional modules have an important role during the creation of new kinematic schemes, due to the simplified creation of kinematical schemes during the planetary gear trains design.

Theoretical bases, kinematic and structure schemes kinematic and dynamic analysis of the functional modules are given in [2] and [3]. The analysis of efficiency in a gear mesh, using the following methods: the method of mesh power, method of M. A. Krejnes, method of I. N. Kornilaev and Combination method, will be carried out in this paper. Analytical expressions of efficiency, determined by the mentioned methods for all considered functional modules were derived from the initial expressions given in [1]. The influence of relevant parameters on the efficiency value was analyzed by the obtained expressions. The analysis showed that in most cases, the value of basic gear ratios, has an important influence on efficiency, regardless of the applied type of a functional module (one-set, two-set, etc.) and the method used for the determination of efficiency. Its dependence on the influence of other relevant parameters (P_u – input power, n_u – input speed, Z_a – number of sun gear teeth, m – gear module, n – lubrication coefficient) on efficiency can be determined efficiently using the Combination method.

Considering the basic gear ratios influence on efficiency, the following parameters were used for all types of functional modules: P_u=500 kW; n_u=2600 min^{-1}; Z_a=36; m=5 mm; n=1.4. During the consideration of the influence of other parameters, adequate values of the parameters which were constant during that consideration were chosen.

The basic aim of the analysis is to determine those influential parameters values which contribute the maximum value of efficiency. Besides the correlation between the mentioned methods from the standpoint of the achieved efficiency values and the comparison of functional modules of the same type was carried out.

Efficiency of the single-set functional modules

The kinematic schemes and power flows for the single-set functional modules are shown in Fig.1.

The derived analytical expressions for the efficiency of functional modules are given in Table 1.

The meaning of the parameters given in Table 1 is as follows:

- k – basic gear ratio of a single planetary gear train (k=1.5–4.5),
- η_0=0.95 – efficiency of a single planetary gear train in case the carrier is fixed,
- A – experimentally obtained coefficient (A=0.005/1.36),
- C – constant ($C = \pi/60000$) and
- c_r – coefficient of power transmission by relative motion (c_r =0–1).

The analysis of the expressions given in Table 1 shows that the efficiency of functional modules with a fixed carrier (JM2, JM4), determined by the mesh power method and the method of M. A. Krejnes, has a constant value ($\eta=\eta_0$=0.95), while for other functional modules it depends

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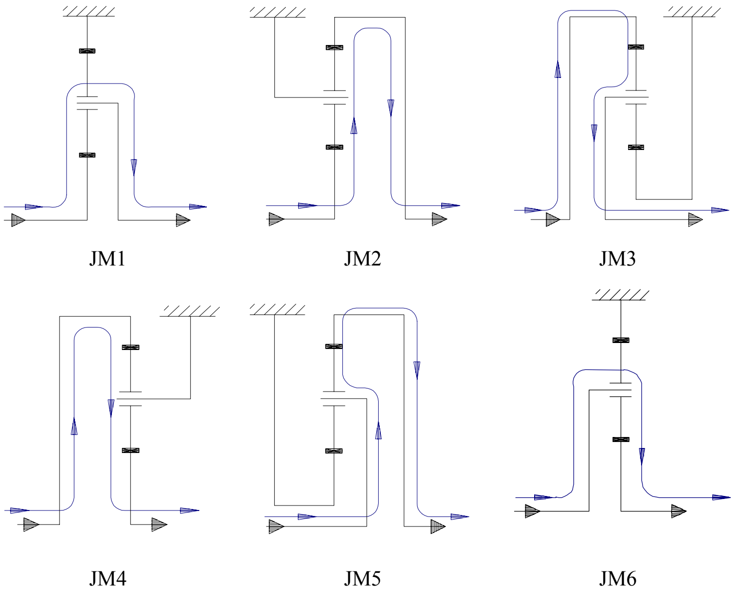

Figure 1. Single-set functional modules - kinematic schemes and power flows

Table 1

Module label	Power in gear mesh method	Krejnes method	Combination method
JM1	$1-0.05\,\frac{k}{1+k}$	$\frac{1+k\,\eta_0}{1+k}$	$1-\frac{2Am\,(Cn_u\,\frac{k}{1+k}\,mZ_a)^n+0.015P_u\,\frac{k}{1+k}}{c_r\,P_u}$
JM2	η_0	η_0	$1-\frac{2Am\,(Cn_u\,mZ_a)^n+0.015P_u}{c_r\,P_u}$
JM3	$1-0.05\,\frac{1}{1+k}$	$\frac{k+\eta_0}{1+k}$	$1-\frac{2Am\,(Cn_u\,\frac{k}{1+k}\,mZ_a)^n+0.015P_u\,\frac{1}{1+k}}{c_r\,P_u}$
JM4	η_0	η_0	$1-\frac{2Am\,(Cn_u\,kmZ_a)^n+0.015P_u}{c_r\,P_u}$
JM5	$1-0.05\,\frac{1}{1+k}$	$\frac{(1+k)\,\eta_0}{1+k\,\eta_0}$	$1-\frac{2Am\,(Cn_u\,mZ_a)^n+0.015P_u\,\frac{1}{1+k}}{c_r\,P_u}$
JM6	$1-0.05\,\frac{k}{1+k}$	$\frac{\eta_0\,(1+k)}{\eta_0+k}$	$1-\frac{2Am\,(Cn_u\,k\,m\,Z_a)^n+0.015P_u\,\frac{k}{1+k}}{c_r\,P_u}$

on the basic gear ratio. The efficiency determined by the Combination method for all functional modules, except for the module JM2, depends on the basic gear ratio. This dependence is shown in Fig.2.

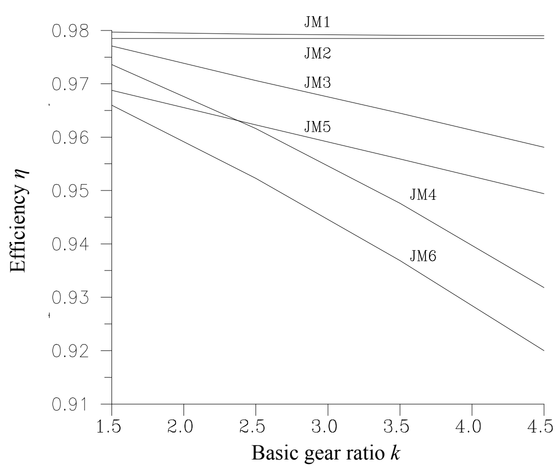

Figure 2. Efficiency of the single-set functional modules

As noticed from the diagram (Fig.2), the JM1 functional module is among others, the most convenient from the standpoint of efficiency. Its value of efficiency slightly varies in the whole range of the basic gear ratio change.

The minimum values of efficiency were achieved in the JM6 functional module. This happens due to kinematic and dynamic performances of the JM6 functional module.

The overall gear ratio, as the most important kinematic parameter, among all functional modules, for the same basic gear ratio, has the minimum value in the JM6 functional module. The maximum pheripheral speeds occur in the JM6 functional module elements due to the previous fact, and the maximum slide velocities happen on the gear teeth, so the maximum energy is consumed on the overcoming of frictional resistance in the gear mesh.

The common characteristics of JM3, JM4, JM5 and JM6 functional modules are the decreasing of their efficiency in case the basic gear ratio increases.

The shown dependance, considering the given teeth number of the sun gear (Z_a=36), has the practical application for the basic gear ratio range k=1.5–3.0 (the teeth number of epicycle is real for the application on vehicle gear trains). The increasing of the internal gear ratio range (from 3.0 to 4.5) was performed due to the theoretical consideration and the comparison of efficiency values, obtained using various methods.

In order to compare efficiency values obtained by the considered methods, for the same functional module and efficiency values of various functional modules obtained by the same method, they need to be determined for the same value of the basic gear ratio. However, this way of comparison is not useful for a number of basic gear ratio values. In order to simplify it and include the whole range of basic gear ratio values (1.5–4.5), the average value of efficiency (η_{sr}), needs to be determined which is defined by the following relation:

$$\eta_{sr} = \frac{\int_{k_{\min}}^{k_{\max}} \eta\,(k)\,dk}{k_{\max} - k_{\min}} \qquad (1)$$

An average value of efficiency determined by one of the above mentioned methods depends on the functional module. In order to consider a mutual relationship between functional modules from the standpoint of efficiency values, the gradation of them was carried out in such a way that the module with the highest value of efficiency was placed in the first position and got the grade marked by the lowest ordinal number.

The grade of the functional module for one method points out which module has to be selected in the phase of kinematic scheme creation of gear trains. The comparison of the same functional module grades for the various methods of efficiency determination provides the correlation between the used methods. In order to consider the grade of functional modules on the basis of relation (1), using the computer programme STEPKOR, the average efficiency values were determined and given in Table 2, together with the module grade.

Table 2

Module label	Power in gear mesh method		Krejnes method		Combination method	
	η_{sr}	grade	η_{sr}	grade	η_{sr}	grade
JM1	0.963	2.	0.963	3.	0.979	1.
JM2	0.950	3.	0.950	5.	0.978	2.
JM3	0.987	1.	0.987	1.	0.968	3.
JM4	0.950	3.	0.950	5.	0.954	5.
JM5	0.987	1.	0.987	2.	0.959	4.
JM6	0.963	2.	0.962	4.	0.944	6.

As shown in Table 2, there is no functional module of the same grade for all three used methods. The JM3 module has the same grade for the mesh power method and the method of M. A. Krejnes.

The difference between the average values of the efficiency coefficient calculated by the power method in gear mesh and the M. A. Krejnes method, for the same functional modules, is negligible (≤0.001). The average value of efficiency coefficient determined by the Combination method differs from the average values obtained by M. A. Krejnes method and power in gear mesh method. The difference in values is from 0.004 (for JM4) to 0.028 (for JM2).

Besides the knowledge of the functional module grade, for an applied method of the efficiency determination coefficient, it is necessary to know the value of the basic gear ratio the maximum efficiency is obtained for maximum and minimum efficiency values, as well as corresponding values of the basic gear ratio (Table 3), were determined numerically.

Table 3

Module label		Power in gear mesh method		Krejnes method		Combination method	
		η_{max}	η_{min}	η_{max}	η_{min}	η_{max}	η_{min}
		k	K	k	K	k	K
JM1		0.970	0.959	0.970	0.959	0.980	0.979
		1.5	4.5	1.5	4.5	1.5	4.5
JM2		0.950		0.950		0.978	
		1.5 - 4.5		1.5 - 4.5		1.5 - 4.5	
JM3		0.991	0.980	0.991	0.980	0.977	0.958
		4.5	1.5	4.5	1.5	1.5	4.5
JM4		0.950		0.950		0,969	0,949
		1.5 - 4.5		1.5 - 4.5		1,5	4,5
JM5		0.991	0.980	0.990	0.979	0.974	0.932
		4.5	1.5	4.5	1.5	1.5	4.5
JM6		0.970	0.959	0.969	0.959	0.966	0.920
		1.5	4.5	1.5	4.5	1.5	4.5

According to the Combination method, for all functional modules except for the JM2 (efficiency does not depend on the basic ratio), with the raise in the value of the basic gear ratio, efficiency decreases. The change of efficiency is also the same for the other two methods for the modules JM1 and JM6, while for the modules JM3 and JM5 it is inversely proportional. The efficiency determined by the method of mesh power and the M. A. Krejnes method for the functional modules JM1 and JM2 does not depend on the basic gear ratio.

During the analysis of the influence of other parameters on efficiency, the value of the basic basic gear ratio was k=2.0. The following was found:

- By increasing the input speed (from 1500 to 3000 min^{-1}), efficiency decreases negligibly within all functional modules. The efficiency decreases because in the gear mesh there is an increase in slipping speed, and also in slipping work, which yields to the increase of power consumption.
  The greatest influence of the input speed on the value of efficiency is in the functional module JM6 and the greatest difference in the value of the efficiency coefficient, which corresponds to the given speed, is 0.019.
- By increasing the gear module value (from 3 to 6 mm), efficiency decreases. The change in gear module (while other parameters were constant) increases the gear diameter and so the peripheral speed and the slipping speed, which directly influences the power losses in gear mesh. The increasing in the gear module value, in the above mentioned range, has the greatest influence on the efficiency change of the functional module JM6, and its value is 0.032.
- By increasing the input power (from 50 to 500 kW), i.e. the input torque (at the constant input speed) the efficiency value significantly rises. To determine the influence of the moment of load on gear mesh efficiency, the character of the change in internal resistance needs to be determined.
  When there is no external torque on output shaft, the torque introduced in train design is equal to the sum of moments of all internal resistances (frictional resistance in gear mesh, hydraulic and ventilation resistance, etc.) at the input shaft. In this case, efficiency is equal to zero.
  In the case when the speed and lubrication temperatures are constant, all internal resistances, except the frictional resistance in gear mesh, are constant and independent of the load (frictional resistance in bearings is very low because there is no load in planetary gear bearing). Significant influence on the efficiency coefficient has the difference between the torque on the input shaft and the torque which includes all resistances of the constant value. If the input torque is higher, the difference will also be higher as well as efficiency coeficient. In the opposite case by decreasing the value of the input torque, efficiency coefficient decreases.
  In the analyzed case, the greatest influence of torque on the efficiency of one-set functional modules is the greatest in the JM6 module. In this case, the change of efficiency is 0.230.
- By increasing the factor of lubrication from 1.4 to 1.7, the efficiency of all functional modules decreases. In this case, the module JM6 also has the highest efficiency reduction and its value is 0.056.
  The lubrication factor includes the oil effect on the power losses on gear contact surfaces. In the analytical expression for determining the power losses, this parameter is on exponent, so its increase causes the increase of power losses and the decrease of efficiency.

On the basis of the given data, it was found that the input torque, that is the moment of load has the greatest influence on the efficiency value, while the input speed has the least influence.

Among all one-set functional modules, the one named JM1 was chosen for the I. N. Kornilaev method. This method is used to determine the efficiency in all gear meshes of the functional module.

On the basis of the expression given in [3] and the established energy balance, the following analytical expression for efficiency in all gear meshes is deduced:

$$\eta = (D - \frac{EK}{P_u})(F - \frac{EK}{P_u}\frac{1}{D - \frac{EK}{P_u}}) \tag{2}$$

The values of the auxiliary parameters D, E, F, and K from expression (2) are:

$$E = A\,m\,(C\,m\,Z_a n_u \frac{k}{1+k})^n \text{ [kW]} \tag{3}$$

$$K = \frac{1+k}{k} \tag{4}$$

$$D=0.99,\ F=0.995$$

By using expression (2) and the same values for the parameters, used in the Combination method, the efficiency values were determined for the internal gear ratio (1.5–4.5). The efficiency change character is that with the increase of the basic gear ratio efficiency slightly decreases, so it is practically constant for the value of 0.985.

By comparing the efficiency values obtained by means of the I. N. Kornilaev method and the Combination method, it can be concluded that the character of the efficiency change is identical in both cases. The value of the basic gear ratio has little effect on efficiency. The value obtained by the I. N. Kornilaev method is slightly higher (about 0.005) than the values obtained by the Combination method.

The simpler mathematical expression for determining the efficiency by the Combination method and a very small difference in values obtained by both methods, give a great advantage to the Combination method.

Efficiency of the two-set functional modules

Kinematic schemes and the power flows of the two-set functional modules are shown in Fig.3.

The analytical expression for the functional module efficiency obtained by the Combination method is given in Table 4 and the one obtained by the M. A. Krejnes method and power in mesh method in Table 5.

In this kind of functional modules, the efficiency in gear mesh changes depending on the value of basic gear ratios for all functional modules, regardless of the applied method. In relation to the basic gear ratio efficiency, in an analytical way, it represents a function with two variables (k_1 and k_2) and in a graphical way-surface in space. In contrast to the one-set functional modules, the graphic representation of several functional modules efficiency on the same diagram is practically impossible.

The efficiency dependence of the functional module DM5, on internal gear ratios k_1 and k_2, determined by the Combination method, is shown in Fig.4.

As seen in the diagram (Fig.4), the efficiency rises with the increase of the basic gear ratio of the first planetary set (k_1) and decreases while the basic gear ratio of the second planetary set (k_2) rises. Determining the average value of efficiency for the two-set functional modules is much more complicated than in the case of one-set functional modules, because the field of the efficiency definition, considering the basic gear ratio, is a surface.

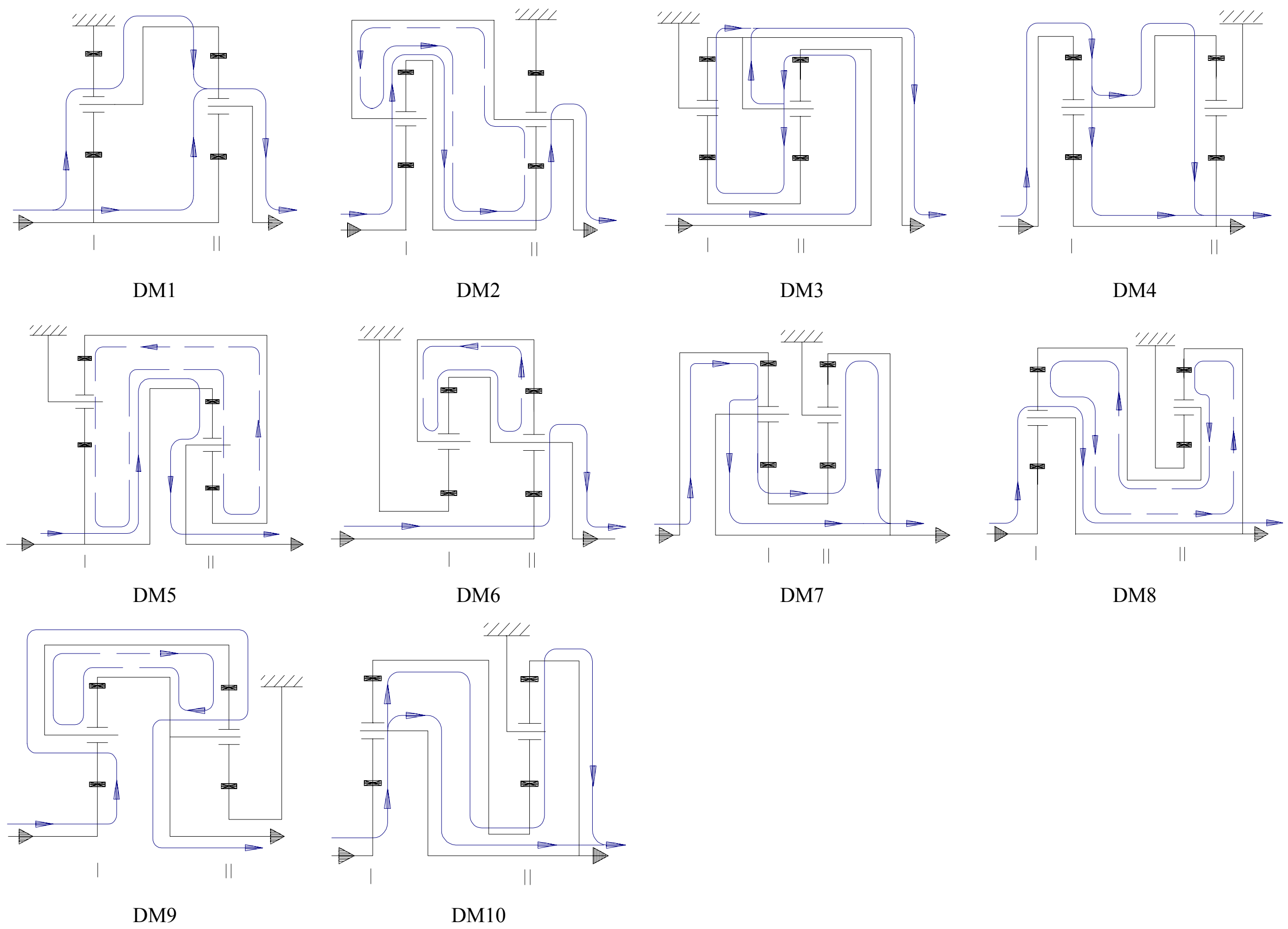

Figure 3. Two-set functional modules - kinematic schemes and power flows

Table 4

Module label	Combination method
DM2	$1-\dfrac{2Am\,[\,(\,CmZ_{a1}n_u\frac{k_1k_2}{k_1k_2-1}\,)^n+(\,CmZ_{a2}\,n_u\,\frac{k_2}{k_1k_2-1}\,)^n\,]+0.03P_u\,\frac{k_1k_2}{k_1k_2-1}}{c_r\,P_u}$
DM3	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}\,n_u\,\frac{k_1k_2}{1+k_1+k_2}\,)^n+(\,CmZ_{a_2}\,n_u\,\frac{k_2\,(1+k_1)}{1+k_1+k_2}\,)^n\,]+0.015P_u\,\frac{1+2k_1}{1+k_1+k_2}}{c_r\,P_u}$
DM4	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,\frac{k_1\,(1+k_2)}{1+k_1+k_2}\,)^n+(\,CmZ_{a_2}n_u\,\frac{k_1k_2}{1+k_1+k_2}\,)^n\,]+0.015P_u\,\frac{2+k_1+k_2}{1+k_1+k_2}}{c_r\,P_u}$
DM5	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,)^n+(\,CmZ_{a_2}n_u\,\frac{k_2\,(1+k_1)}{k_1\,(1+k_2)}\,)^n\,]+0.015P_u\,\frac{1+k_2\,(2+k_1)}{(k_1k_2-1)\,(1+k_2)}}{c_r\,P_u}$
DM6	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,\frac{k_1}{1+k_1+k_2}\,)^n+(\,CmZ_{a_2}n_u\,\frac{k_2}{1+k_1+k_2}\,)^n\,]+0.015P_u\,\frac{k_2\,(1+k_1)}{1+k_1+k_2}}{c_r\,P_u}$
DM7	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,\frac{k_1\,(1+k_2)}{1+k_1+k_2}\,)^n\,(\,CmZ_{a_2}n_u\,\frac{k_1k_2}{1+k_1+k_2}\,)^n+0.015P_u\,\frac{1+2k_2}{1+k_1+k_2}}{c_r\,P_u}$
DM8	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,\frac{k_1}{1+k_1+k_2}\,)^n+(\,CmZ_{a_2}n_u\,\frac{k_2}{1+k_1+k_2}\,)^n\,]+0.015P_u\,\frac{k_1\,(1+2k_2)}{(1+k_2)\,(1+k_1+k_2)}}{c_r\,P_u}$
DM9	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,\frac{k_1}{1+k_1+k_2}\,)^n+(\,CmZ_{a_2}n_u\,\frac{k_2}{1+k_1+k_2}\,)^n\,]+0.015P_u\,\frac{1+2k_1}{1+k_1+k_2}}{c_r\,P_u}$
DM10	$1-\dfrac{2Am\,[\,(\,CmZ_{a_1}n_u\,\frac{k_1+k_1k_2}{1+k_1+k_1k_2}\,)^n+(\,CmZ_{a_2}n_u\,\frac{k_2}{1+k_1+k_1k_2}\,)^n+0.015P_u\,\frac{k_1+k_1k_2}{1+k_1+k_1k_2}}{c_r\,P_u}$

Table 5

Module label	Power in gear mesh method	Krejnes method
DM1	$1-0.05\,\frac{k_1k_2}{1+k_1+k_2}\,(\,\frac{1}{1+k_1}+\frac{1}{1+k_2}$	$\frac{(1+k_1+k_2)\,(1+k_1\eta_0)\,(1+k_2\eta_0)}{(1+k_1\eta_0+k_2\eta_0)\,(1+k_1)\,(1+k_2)}$
DM2	$1-0.1\,\frac{k_1k_2}{k_1k_2-1}$	$\frac{1-k_1k_2\,\eta_0}{1-k_1\,k_2}$
DM3	$1-0.05\,\frac{1+2k_1}{1+k_1+k_2}$	$\frac{\eta_0+k_1\,\eta_0^2+k_2}{1+k_1+k_2}$
DM4	$1-0.05\,\frac{2+k_1+k_2}{1+k_1+k_2}$	$\frac{\eta_0\,(\eta_0+k_1+k_2)}{1+k_1+k_2}$
DM5	$1-0.05\,\frac{1+k_2\,(2+k_1)}{(k_1k_2-1)\,(1+k_2)}$	$\frac{(\eta_0+k_2)\,(k_1k_2-1)}{(1+k_2)\,(k_1k_2-\eta_0^2)}$
DM6	$1-0.05\,\frac{k_2\,(1+2k_1)}{(1+k_1)\,(1+k_1+k_2)}$	$\frac{(1+k_1)\,(\eta_0+k_1+k_2\eta_0^2)}{(\eta_0+k_1)\,(1+k_1+k_2)}$
DM7	$1-0.05\,\frac{1+2k_2}{1+k_1+k_2}$	$\frac{\eta_0+k_1+k_2\eta_0^2}{1+k_1+k_2}$
DM8	$1-0.05\,\frac{k_1\,(1+2k_2)}{(1+k_2)\,(1+k_1+k_2)}$	$\frac{(1+k_2)\,(\eta_0+k_1\eta_0^2+k_2)}{(\eta_0+k_{52})\,(1+k_1+k_2)}$
DM9	$1-0.05\,\frac{1+2k_1}{1+k_1+k_2}$	$\frac{\eta_0+k_1\eta_0^2+k_2}{1+k_1+k_2}$
DM10	$1-0.05\,\frac{k_1+k_1k_2}{1+k_1+k_1k_2}$	$\frac{1+k_1\eta_0+k_1k_2\eta_0^2}{1+k_1+k_1k_2}$

The average efficiency value is determined by:

$$\eta_{sr}=\frac{\int_{k_{1\min}}^{k_{1\max}}\int_{k_{2\min}}^{k_{2\max}}\eta(k_1,k_2)\,dk_1dk_2}{(k_{1\max}-k_{1\min})\,(k_{2\max}-k_{2\min})} \quad (5)$$

On the basis of the program STEPKOR and expression (5), average efficiency values were determined (the interval of the basic gear ratio values for both planetary sets was the same 1.5–4.5) and given in Table 6 together with the functional module grade.

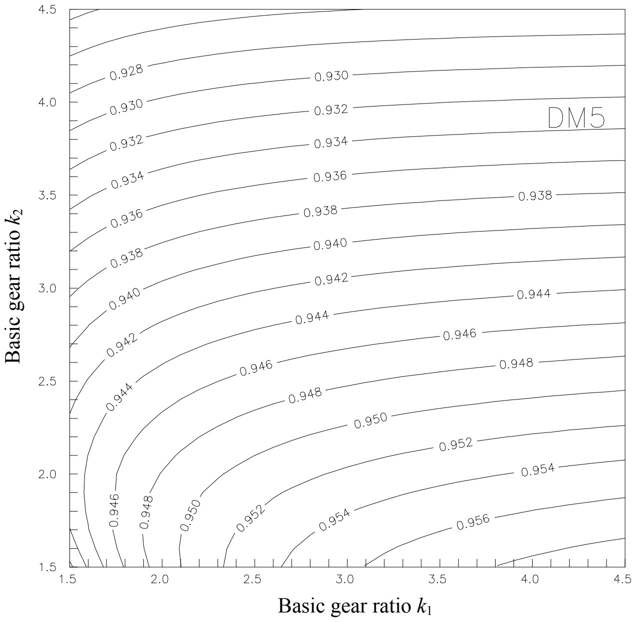

Figure 4. Efficiency of the functional module DM5

According to the data given in Table 6, it can be seen that the functional module DM1 has the same grade for all three methods. The functional module grade of DM5 and DM6 is the same for the first two methods and the functional modules DM2, DM3, DM4 and DM8 have the same grade for the power met-hod in gear mesh and the Combination method, while for the M. A. Krejnes method and the Combination method there is no correlation in average efficiency value in gear mesh. The difference in the same average efficiency value, calculated by the power method in gear mesh and the M. A. Krejnes method, is the highest in the DM 10 module and equals 0.0032.

Table 6

Module label	Power in gear mesh method		Krejnes method		Combination method	
	η_{sr}	grade	η_{sr}	grade	η_{sr}	grade
DM1	0,969	2.	0.969	2.	0.978	2
DM2	0,884	8.	0.887	7.	0.944	8.
DM3	0,95	6.	0.951	4.	0.950	6
DM4	0,943	7.	0.943	5.	0.945	7.
DM5	0,971	1.	0.972	1.	0.953	5.
DM6	0,963	3.	0.964	3.	0.934	9.
DM7	0,95	6.	0.951	4.	0.932	10.
DM8	0,963	4	0.964	3.	0.956	4.
DM9	0,95	6.	0.951	4.	0.970	3.
DM10	0,954	5.	0.922	6.	0.980	1.

The average efficiency value calculated by the Combination method differs from the average values calculated by the two other methods. The difference is the highest in DM2 and equals 0.06.

The maximum and minimum efficiency values were numerically determined for related gear ratios (Table 7).

According to Tables 6 and 7, and in respect to the efficiency, it is clear that when the Combination method is applied, it is the most rational to use the functional module DM 10. The maximum value of the efficiency for this functional module equals 0.980 and is realized with the basic gear ratios k_1=1.5 and k_2=1.68.

On the basis of the data given in the same Tables, it is clear that the values and the efficiency character of the change, obtained from the Power method in gear mesh and the M.A. Krejnes method are the same for the functional modules DM3, DM7 and DM9 even though their kinematic schemes are different. The reason for this is that the analytical expressions for total gear ratios of these functional modules have the *same* mathematical form in respect to basic gear ratios (this was depicted in the kinematic analysis) and because the interval of values of all basic gear ratios are the same (1.5–4.5).

While analyzing the dependences of main efficiency parameters it was assumed that the values of basic gear ratios equal k_1=k_2=2.0, hence the following was concluded:

- By increasing the input speed from 1500 to 3000 min^{-1} the efficiency value of all functional modules decreases from 0.00248 (DM9) to 0.01356 (DM7), which means that the efficiency of the module DM9 least depends, in relation to the other two-step modules, on the change of the input speed. Among all two-step modules, the greatest influence of the input speed on efficiency exists in the functional module DM7 because its kinematic scheme was chosen in a way that chosen values of basic gear ratios (k_1,k_2), relative tangential velocities, and the slipping velocities on gear meshes, depend mostly, in respect to other described two-step modules, on the input speed.
- After the changing of the gear module from 3 to 6 mm, the efficiency value of all functional modules decreases. The decrease of the efficiency value is especially low in the functional module DM7 (0.0410), while in the functional module DM9 there is the lowest change of efficiency (0.0075). If the functional modules JM6 and DM7 were compared, having the greatest influence of gear module on efficiency, in respect to the values for which there is a decrease in efficiency, (for the chosen range of the gear module), it is noticeable that the greatest change happens in the module DM7. The absolute efficiency values in these functional modules are lower, and the reason for this is the existence of a greater number of teeth in mesh.
- By increasing the input power (from 50 to 500 kW), while the input speed is constant, there is a significant increase in efficiency in all functional modules. The highest increase is in the module DM7 and it equals 0.2945, while in the module DM9 there is the lowest change in efficiency and its value is 0.0538. The least influence of the input torque on efficiency is in the functional module DM9 because the structure of its kinematic scheme and the value of basic gear ratios influence in such a way that the difference of the input torque and the resistance change the least in respect to other functional modules. On the other hand, the greatest change in the input torque and internal resistance exists in the functional module DM7, which also depends on the kinematic concept and a chosen value of basic gear ratios.
- By increasing the lubrication factor (from 1.4 to 1.7), the efficiency decreases due to the increasing power losses in gear teeth interaction. The efficiency decreases the most in the functional module DM7 (0.05377), while the lowest efficiency decrease is in the module DM9 and its value is 0.0059.

For the analyzed two-set functional modules it is characteristic that there are two extremes: the functional module DM9, the efficiency of which does not depend on the

Table 7

Module label	Power in gear mesh method				Krejnes method				Combination method			
	η_{max}		η_{min}		η_{max}		η_{min}		η_{max}		η_{min}	
	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
DM1	0.977		0.963		0.977		0.963		0.979		0.977	
	1.5	1.5	4.5	4.5	1.5	1.5	4.5	4.5	1.5	3.98	4.5	1.5
DM2	0.895		0.820		0.897		0.824		0.956		0.887	
	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.5
DM3	0.971		0.928		0.972		0.930		0.967		0.929	
	1.5	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.5	1.5	4.5	4.5
DM4	0.945		0.937		0.945		0.938		0.967		0.921	
	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.5	1.5	4.5	4.5	1.5
DM5	0.986		0.900		0.986		0.909		0.955		0.919	
	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.5	4.5	1.5	1.5	1.5
DM6	0.980		0.948		0.981		0.949		0.958		0.906	
	4.5	1.5	1.5	4.5	4.5	1.5	1.5	4.5	1.5	4.5	4.5	1.5
DM7	0.971		0.928		0.972		0.930		0.953		0.907	
	4.5	1.5	1.5	4.5	4.5	1.5	1.5	4.5	1.5	4.5	4.5	1.5
DM8	0.980		0.948		0.981		0.949		0.951		0.951	
	1.5	4.5	4.5	1.5	1.5	4.5	4.5	1.5	4.5	4.5	1.5	1.5
DM9	0.971		0.928		0.972		0.930		0.973		0.968	
	1.5	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.5	1.5	4.5	4.4
DM10	0.960		0.952		0.938		0.914		0.980		0.979	
	1.5	1.5	4.5	4.5	1.5	1.5	4.5	4.5	1.5	1.68	4.5	1.5

change of influencing parameters and the functional module DM7, where the significant parameters influence efficiency most. Other functional modules are between these extremes.

In contrast to the one-set functional modules, where the analysis was conducted in relation to the applied methods (the I. N. Kornilaev and the Combination method) in the same functional modules, in the two-set functional modules, the effect of circulated power on efficiency regardless of the applied method was also taken into consideration. For this reason the DM1 functional module in which only the effective power exists was chosen, as well as the DM5 where, besides the effective power the circulated power was present. On the basis of the analysis, the following was concluded:

- The value of efficiency, determined by any of the above mentioned methods is higher in the functional module DM1 (0.97329 – I. N. Kornilaev method and 0.96493 determined by the Combination method) than the values of efficiency in the functional module DM5 (0.94196 – the I. N. Kornilaev method and 0.92009 – the Combination method). In respect to the absolute differences in efficiency values of the depicted functional modules (0.03133 – the I. N. Kornilaev method and 0.04484 – the Combination method), it is concluded that the circulating power has significant influence on the value of losses in gear mesh.
- For the same functional module, the efficiency differs depending on the applied method. The values obtained by the I. N. Kornilaev method are higher than those obtained by the Combination method (the absolute difference for the DM1 module is 0.00836, and for the DM5 module – 0,02187). In the functional modules where only the flow of power exists, this effect is negligible while it is somewhat higher in those modules where there is the circulating power.

Efficiency of the functional modules of the RAVIGNEAUX type

The kinematic schemes and the power flows for the RAVIGNEAUX functional type modules are given in Fig.5.

The analytical expressions, on the basis of which efficiency is determined by the power flow method in gear mesh, are given in Table 8.

Table 8

Module label	Power in gear mesh method	
RM1; RM13	$k_1 > \frac{k_2-1}{2}$	$1-\frac{k_1}{k_2-k_1}(0.03\frac{3k_1+2-k_2}{1+k_1}+0.02)$
	$k_1 = \frac{k_2-1}{2}$	$1-0.05\frac{k_1}{1+k_1}$
	$k_1 < \frac{k_2-1}{2}$	$1-\frac{k_1}{k_2-k_1}(0.03\frac{3k_2-5k_1-2}{1+k_1}+0.02)$
RM2; RM19		$1-0.09\frac{k_1}{1+k_1}$
RM3; RM9	$1-\frac{k_2}{(1+k_2)(k_2-k_1)}[0.03(3k_2+2-k_1)+0.02(1+k_1)]$	
RM4; RM24	$1-0.05\frac{k_2}{1+k_2}$	
RM5; RM18	0,95	
RM6; RM12	0.91	
RM7; RM15	$1-\frac{1}{1+k_2}(0.03\frac{3k_2+2-k_1}{1+k_1}+0.02)$	
RM8; RM21	$1-0.09\frac{1}{1+k_1}$	
RM10; RM23	$1-0.08\frac{k_2}{k_1-k_2}$	
RM11; RM17	0.92	
RM14; RM20	$1-0.08\frac{k_1}{k_2-k_1}$	
RM16; RM22	$1-0.05\frac{1}{1+k_2}$	

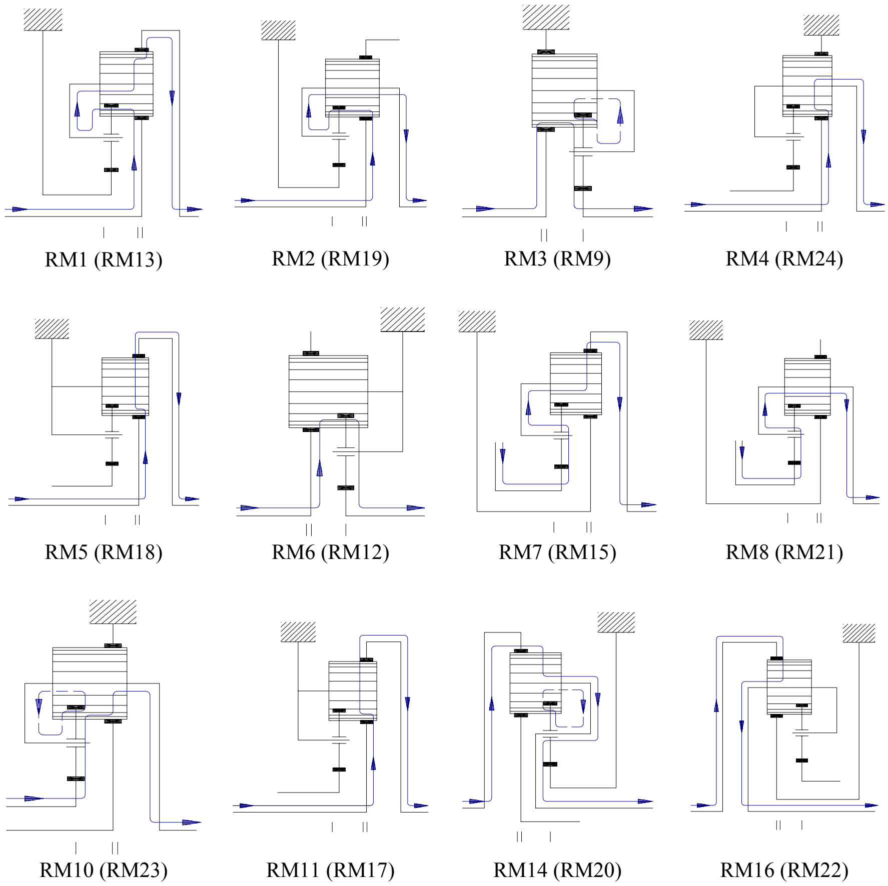

Figure 5. RAVIGNEAUX functional type modules - kinematic schemes and power flows

Table 9

Module label	Krejnes method	Module label	Krejnes method
RM1	$\frac{(k_2-k_1)\,(1+k_1\eta_0)}{(1+k_1)\,(k_2-k_1\eta_0)}$	RM13	$\frac{(1+k_1)\,(k_2\eta_0^2-k_1)}{(k_2-k_1)\,\eta_0\,(\eta_0+k_1)}$
RM2	$\frac{1+k_1\eta_0}{1+k_1}$	RM14	$\frac{k_2\eta_0^2-k_1}{\eta_0^2(k_2-k_1)}$
RM3	$\frac{\eta_0(\eta_0+k_2)\,(k_1-k_2)}{(1+k_2)\,(k_1\eta_0^2-k_2)}$	RM15	$\frac{(1+k_1)\,(\eta_0+k_2)}{(1+k_2)\,(1+k_1\eta_0)}$
RM4	$\frac{1+k_2\eta_0}{1+k_2}$	RM16	$\frac{\eta_0+k_2}{1+k_2}$
RM5	η_0	RM17	η_0^2
RM6	η_0	RM18	η_0
RM7	$\frac{k_1\,(1+k_2)\,\eta_0\,(\eta_0+k_1)}{(1+k_1)\,(1+k_2\eta_0)}$	RM19	$\frac{\eta_0\,(1+k_1)}{\eta_0+k_1}$
RM8	$\frac{\eta_0+k_1}{1+k_1}$	RM20	$\frac{k_2-k_1}{k_2-k_1\,\eta_0^2}$
RM9	$\frac{(1+k_2)\,(k_1-k_2\eta_0^2)}{(k_1-k_2)\,(1+k_2\eta_0)}$	RM21	$\frac{\eta_0(1+k_1)}{1+k_1\eta_0}$
RM10	$\frac{k_1-k_2\eta_0^2}{k_1-k_2}$	RM22	$\frac{\eta_0(1+k_2)}{1+k_2\eta_0}$
RM11	η_0^2	RM23	$\frac{\eta_0^2\,(k_1-k_2)}{k_1\eta_0^2-k_2}$
RM12	η_0	RM24	$\frac{\eta_0\,(1+k_2)}{\eta_0+k_2}$

On the basis of the formulas in Table 8, it is evident that the efficiency of the functional module RM1 and its inverse module RM13 depends not only on the value of internal gear ratios, but also on their mutual interaction. For the functional modules with the fixed carrier (RM5, RM6, RM11, RM12, RM17, and RM18), the efficiency has a constant value. For ten modules of this kind, it is characteristic that efficiency depends on both basic gear ratios, and for the eight rest modules only one of the basic gear ratios has an effect on the efficiency.

The obtained analytical expressions for the efficiency determined by the M. A. Krejnes method are shown in Table 9.

The type of efficiency change of functional modules, depending on the basic gear ratios, is the same as in the power method in gear mesh, except that for the functional modules RM1 and RM13 it does not depend on mutual relations of basic gear ratios.

The obtained analytical expressions for the efficiency determined by the Combination method are shown in Table 10.

The efficiency change character, determined by this method, in relation to basic gear ratios, is the same as in the power method in gear mesh, except for the modules RM11, RM12, RM17 and RM 18, where the efficiency is not constant but changes in relation to the basic gear ratio of one of two planetary sets.

Analyzing the expressions given in Table 10, it is evident that there are four groups of functional modules, regarding the dependences of their efficiency on basic gear ratios. In the first group, there are functional modules the efficiency of which does not depend on planetary sets (RM5 and RM6) basic gear ratios. The second group consists of modules the efficiency of which depends on the basic gear ratio of the first planetary set (RM2, RM8, RM11,

Tabela 10

Module label	Condition	Combination method
1		2
RM1	$k_1 > \frac{k_2-1}{2}$	$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1}{1+k_1})^n+P_u\,\frac{k_1}{k_2-k_1}\,0.01\,(\frac{3.5k_1-k_2+2.5}{1+k_1})}{c_r\,P_u}$
	$k_1 = \frac{k_2-1}{2}$	$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1}{1+k_1})^n+0.015P_u\,\frac{k_1}{1+k_1}}{c_r\,P_u}$
	$k_1 < \frac{k_2-1}{2}$	$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1}{1+k_1})^n+0.01P_u\,\frac{k_1}{k_2-k_1}\,\frac{3k_2-4.5k_1-1.5}{1+k_1}}{c_r\,P_u}$
RM2		$1-\frac{Am\,(CZ_{a_2}\,mn_u\,\frac{k_1}{1+k_1})^n\,(3+\frac{1}{0.995})+0.03P_u\,\frac{k_1}{1+k_1}}{c_r\,P_u}$
RM3		$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_2}{1+k_2})^n+P_u\,\frac{k_2}{(1+k_2)\,(k_2-k_1)}[\,0.01\,(k_2-k_1)+0.02\,(1+k_2)+0.005\,(1+k_1)\,]}{c_r\,P_u}$
RM4		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,\frac{k_2}{1+k_2})^n\,(1+\frac{1}{0.99})+0.015P_u\,\frac{k_2}{1+k_2}}{c_r\,P_u}$
RM5		$1-\frac{Am\,(CmZ_{a_2}\,n_u)^n\,(1+\frac{1}{0.99})+0.015P_u}{c_r\,P_u}$
RM6		$1-\frac{Am\,(CmZ_{a_2}\,n_u)^n\,(3+\frac{1}{0.995})+0.03P_u}{c_r\,P_u}$
RM7		$1-\frac{4Am\,(CZ_{a_2}\,mn_u\,\frac{k_1}{1+k_1})^n+P_u\,\frac{1}{(1+k_1)\,(1+k_2)}[\,0.01\,(k_2-k_1)+0.02\,(1+k_2)+0.005\,(1+k_1)\,]}{c_r\,P_u}$
RM8		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,\frac{k_1}{1+k_1})^n\,(3+\frac{1}{0.995})+0.03P_u\,\frac{1}{1+k_1}}{c_r\,P_u}$
RM9		$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1k_2}{k_2-k_1})^n+P_u\,\frac{k_2}{(1+k_2)\,(k_2-k_1)}[\,0.01\,(k_2-k_1)+0.02\,(1+k_2)+0.005\,(1+k_1)\,]}{c_r\,P_u}$
RM10		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,\frac{k_1k_2}{k_1-k_2})^n\,(3+\frac{1}{0.995})+0.025P_u\,\frac{k_2}{k_1-k_2}}{c_r\,P_u}$
RM11		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,k_1)^n\,(3+\frac{1}{0.99})+0.025P_u}{c_r\,P_u}$
RM12		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,k_1)^n\,(3+\frac{1}{0.995})+0.03P_u}{c_r\,P_u}$
RM13	$k_1 > \frac{k_2-1}{2}$	$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1k_2}{k_2-k_1})^n+P_u\,\frac{k_1}{(1+k_1)\,(k_2-k_1)}\,0.01(3.5k_1-k_2+2.5)}{c_r\,P_u}$
	$k_1 = \frac{k_2-1}{2}$	$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1k_2}{k_2-k_1})^n+0.015P_u\,\frac{k_1}{k_2-k_1}}{c_r\,P_u}$
	$k_1 < \frac{k_2-1}{2}$	$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_1k_2}{k_2-k_1})^n+0.01P_u\,\frac{k_1}{(1+k_1)\,(k_2-k_1)}(3k_2-4.5k_1)}{c_r\,P_u}$
RM14		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,\frac{k_1k_2}{k_2-k_1})^n\,(3+\frac{1}{0.99})+0.025P_u\,\frac{k_1}{k_2-k_1}}{c_r\,P_u}$
RM15		$1-\frac{4Am\,(CmZ_{a_2}\,n_u\,\frac{k_2}{1+k_2})^n+P_u\,\frac{1}{(1+k_1)\,(1+k_2)}[\,0.01\,(k_2-k_1)+0.02\,(1+k_2)+0.005\,(1+k_1)\,]}{c_r\,P_u}$
RM16		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,\frac{k_2}{1+k_2})^n\,(1+\frac{1}{0.99})+0.015P_u\,\frac{1}{1+k_2}}{c_r\,P_u}$
RM17		$1-\frac{Am\,(CmZ_{a_2}\,n_u\,k_2)^n\,(3+\frac{1}{0.995})+0.015P_u}{c_r\,P_u}$

Table 10 (continued)

RM18	$1-\dfrac{(\,CmZ_{a2}\;n_u\;k_2\,)^n\;\;(1+\frac{1}{0.99})+0.015P_u}{c_r\;P_u}$
RM19	$1-\dfrac{Am\;(\,CmZ_{a2}\;n_u\;\frac{1}{k_1}\,)^n\;\;(3+\frac{1}{0.995})+0.03P_u\;\frac{1}{1+k_1}}{c_r\;P_u}$
RM20	$1-\dfrac{Am\;(\,CmZ_{a2}\;n_u\;k_1\,)^n\;\;(3+\frac{1}{0.99}\,)+0.025P_u\;\frac{k_1}{k_2-k_1}}{c_r\;P_u}$
RM21	$1-\dfrac{Am\;(\,CmZ_{a2}\;n_u\,)^n\;\;(3+\frac{1}{0.995})+0.03P_u\;\frac{1}{1+k_1}}{c_r\;P_u}$
RM22	$1-\dfrac{Am\;(\,CmZ_{a2}\;n_u\,)^n\;\;(1+\frac{1}{0.99})+0.015P_u\;\frac{1}{1+k_2}}{c_r\;P_u}$
RM23	$1-\dfrac{Am\;(\,CmZ_{a2}\;n_u\;k_2\,)^n\;\;(3+\frac{1}{0.995})+0.025P_u\;\frac{k_2}{k_1-k_2}}{c_r\;P_u}$
RM24	$1-\dfrac{Am\;(\,CmZ_{a2}\;n_u\;k_2\,)^n\;\;(1+\frac{1}{0.99})+0.015P_u\;\frac{k_2}{1+k_2}}{c_r\;P_u}$

RM12, RM19 and RM21). The efficiency with the numerical value depending on the basic gear ratio of the second planetary grade k_2 is related to the functional modules of the third group, consisting of RM4, RM6, RM17, RM18, RM22 and RM24. The efficiency of the fourth module group depends on the basic gear ratios of both planetary sets k_1 and k_2. The functional modules in this group are: RM1, RM3, RM7, RM9, RM10, RM13, RM14, RM15, RM20, and RM23. For the functional modules RM1 and RM13, it is characteristic that the efficiency depends only on the parameter $\mathbf{k_1}$ when $k_1=(k_2\text{-}1)/2$.

The efficiency dependence of the functional modules of the second group on the basic gear ratio of the first planetary set is shown in Fig.6. The efficiency of the functional modules of this group, except RM19, slightly changes (decreases) with the increase in value of the internal gear ratio. Among all functional modules, the module RM11 has the highest efficiency value in the whole range of parameter $\mathbf{k_1}$. The greatest dependence of efficiency on the basic gear ratio exists in the functional module RM19, where the efficiency value also increases with the increase of $\mathbf{k_1}$.

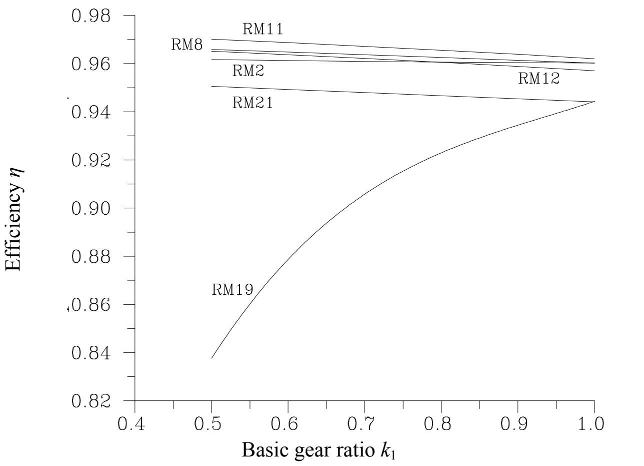

Figure 6. Efficiency of the second group of the functional modules of the RAVGNEAUX type

The efficiency dependence of the functional modules of he third group on the internal gear ratio of the secondary planetary grade is shown on the diagram in Fig.7. The character of efficiency change of the functional modules of this group is characterized in this way: with the increase of the basic gear ratio $\mathbf{k_2}$, the efficiency decreases. The efficiency of the functional module RM4 changes very little with the change of $\mathbf{k_2}$ parameter, and its value is the highest in respect to the efficiency of other functional modules in the whole range of $\mathbf{k_2}$. The efficiency of the functional modules RM17 and RM18 are almost identical for all values of the parameter $\mathbf{k_2}$.

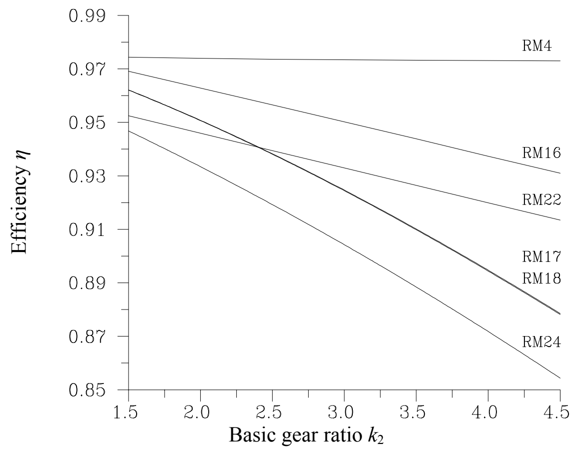

Figure 7. Efficiency of the third group of the functional modules of the RAVGNEAUX type

Comparing the efficiency for several functional modules of the fourth group by graphical means is rather difficult. That is the reason why only one of this functional modules (RM15) and the dependence of its efficiency on the parameters $\mathbf{k_1}$ and $\mathbf{k_2}$ is shown in Figure 8.

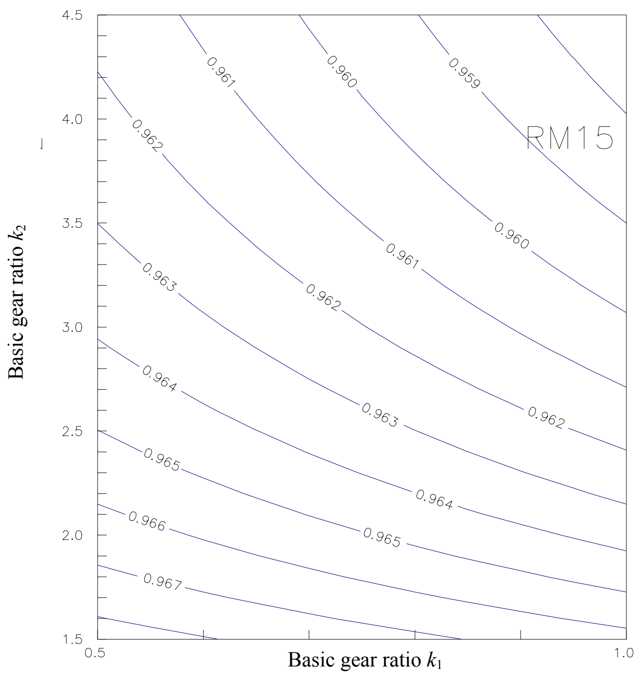

Figure 8. Efficiency of the functional module RM15

As shown on the diagram (Fig.8), the maximum value of efficiency is obtained from the range of basic gear ratios: k_1=0.5–0.8 and k_2=1.5–1.8.

By using expression (5), in the same way as in the two--set functional modules, the average values of efficiency were determined (the value of the basic gear ratio for the first planetary set was between 0.5 and 1.0, while for the second planetary set it was between 1.5 and 4.5), on the basis of which the functional module was graded (Table 11).

Table 11

Module label	Power in gear mesh method		Krejnes method		Combination method	
	η_{sr}	grade	η_{sr}	grade	η_{sr}	grade
RM1	0.968	2.	0.959	10.	0.971	3.
RM2	0.962	5.	0.979	3.	0.960	9.
RM3	0.880	12.	0.902	16.	0.935	16.
RM4	0.963	4.	0.963	7.	0.9735	1.
RM5	0.950	7.	0.950	13.	0.972	2.
RM6	0.910	10.	0.950	13.	0.952	12.
RM7	0.951	6.	0.714	20.	0.965	6.
RM8	0.948	8.	0.971	5.	0.963	7.
RM9	0.880	12.	0.895	17.	0.926	18.
RM10	0.887	11.	0.862	19.	0.917	21.
RM11	0.920	9.	0.903	15.	0.966	5.
RM12	0.910	10.	0.950	13.	0.961	8.
RM13	0.968	2	0.939	14.	0.943	15.
RM14	0.967	3.	0.955	12.	0.958	10.
RM15	0.951	6.	0.958	11.	0.957	11.
RM16	0.987	1.	0.987	1.	0.950	13.
RM17	0.920	9.	0.903	15.	0.9232	19.
RM18	0.950	7.	0.950	13.	0.9230	20.
RM19	0.962	5.	0.978	4.	0.906	22.
RM20	0.967	3.	0.961	9.	0.968	4.
RM21	0.948	8.	0.970	6.	0.947	14.
RM22	0.987	1.	0.986	2.	0.933	17.
RM23	0.887	11.	0.868	18.	0.838	24.
RM24	0.963	4.	0.962	8.	0.903	23.

By analyzing the results obtained in Table 11, it is seen that there is no functional module with the same grade for all applied methods. The functional module RM16 has the same grade for the power in gear mesh method; the module RM17 has the same grade in the power in gear mesh method and in the Combination method, while the modules RM3 and RM15 have the same grade for the M. A. Krejnes method and the Combination method.

The difference between the average values of the efficiency determined by the power in gear mesh method and the M. A. Krejnes method is the greatest in the RM12 module and its value is 0.04. The average efficiency value determined by the Combination method for the module RM24 is significantly lower from the average efficiency values determined by the power in gear mesh method and the M. A. Krejnes method, so the difference is 0.06. The functional module RM4 has the highest grade obtained by the Combination method, which means this module should be used when determining the kinematic scheme, especially since efficiency slightly changes in the whole range of values of basic planetary gear ratios.

Testing the character of efficiency change in dependence on basic gear ratios the maximum and minimum values are determined, as well as the values of the basic gear ratios for which the losses in gear mesh are minimal or maximal (Table 12).

While analyzing the dependence of efficiency on other significant parameters the values of the basic gear ratios were k_1=0.5 and k_2=2.0. The following was concluded:

- By increasing the speed (from 1500 to 3000 min^{-1}), the value of efficiency of all functional modules decreases in the range from 0.00317 (RM1, RM7 and RM8) to 0.07778 (RM19). In functional modules RM1, RM7 and RM8 the values of efficiency decrease are the same because the analytical expressions for relative peripheral speed are identical (Table 10), and the values of power in gear mesh are very similar because of the mutual relationship of basic gear ratios and their chosen values.

  If the functional module RM19 is compared with the two-step functional module DM7 (with the efficiency also depending on the input speed), regarding the efficiency (for the given values of other parameters), higher values are obtained in the functional module DM7. This is explained by the fact that in this module exists a greater number of internal gear meshes where the losses are less than those in outer gear mesh. Comparing the values of the efficiency change of these two modules, it is evident that efficiency depends significantly on the input speed in the functional module RM19.
- By increasing the value of the gear module from 3 to 6 mm, efficiency decreases. The highest decrease of efficiency is in the functional module RM19 and its value is 0.12883, while the lowest change of efficiency is in the functional modules RM1, RM7 and RM8 (0.00524). As stated before, in this type of functional modules, with the change of gear module, the gear diameter also changes and so do the pheripheral speed and slipping speed in gear mesh.
- Among all significant factors, the input speed (input torque), when the speed is constant, affects most the change of efficiency in all functional modules. By increasing the input power from 50 to 500 kW, efficiency increases in all functional modules for a value with the range from 0.03754 (RM1, RM7 and RM8) to 0.83738 (RM19). In the functional module RM19 the dependence of efficiency on the input torque is evident, and the reason for this are the chosen values for basic gear ratios and the structure of kinematic scheme of this functional module.

Table 12

Module label	Power in gear mesh method				Krejnes method				Combination method			
	η_{max}		η_{min}		η_{max}		η_{min}		η_{max}		η_{min}	
	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2	k_1	k_2
1	2				3				4			
RM1	0,983		0,86		0,977		0,889		0.980		0.903	
	0.5	2.0	1.0	1.5	0.5	4.5	1.0	1.5	0.5	2.0	1.0	1.5
RM2	0.970		0.955		0.983		0.975		0,962		0,96	
	0.5	1.5-4.5	1.0	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5
RM3	0.902		0.759		0.930		0.783		0.949		0.862	
	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5
RM4	0.970		0.959		0.970		0.959		0.974		0.973	
	0.5-1.0	1.5	0.5-1.0	4.5	0.5-1.0	4.5	0.5-1.0	1.5	0.5-1.0	1.5	0.5-1.0	4.5
RM5	0.950				0.950				0.972			
	0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5			
RM6	0.910				0.950				0.957			
	0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5			
RM7	0.959		0.942		0.956		0.473		0.969		0.962	
	1.0	1.5	0.5	4.5	1.0	4.5	0.5	1.5	0.5	1.5	1.0	4.5
RM8	0.955		0.940		0.975		0.967		0.966		0.960	
	1.0	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5	0.5	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5
RM9	0.902		0.759		0.928		0.735		0.953		0.776	
	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5
RM10	0.910		0.765		0.890		0.713		0.943		0.781	
	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5
RM11	0.920				0.902				0.970		0.962	
	0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5				0.5	1.5-4.5	1.0	1.5-4.5
RM12	0.910				0.950				0.965		0.957	
	0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5				0.5	1.5-4.5	1.0	1.5-4.5
RM13	0.983		0,86		0.969		0.770		0.963		0.796	
	0.5	2.0	1.0	1.5	0.5	4.5	1.0	1.5	0.5	2.0	1.0	1,5
RM14	0.990		0.845		0.986		0.790		0.985		0.823	
	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5
RM15	0.959		0.942		0.965		0.947		0.963		0.952	
	1.0	1.5	0.5	4.5	1.0	4.5	0.5	1.5	0.5	1.5	1.0	4.5
RM16	0.991		0.980		0.991		0.980		0.969		0.931	
	0.5-1.0	4.5	0.5-1.0	1.5	0.5-1.0	4.5	0.5-1.0	1.5	1.5-4.5	1.5	1.5-4.5	4.5
RM17	0.920				0.902				0.962		0.878	
	0.5-1.0; 1,5-4.5				0.5-1.0; 1.5-4.5				0.5-1.0	1.5	0.5-1.0	4.5
RM18	0.950				0.950				0.962		0.878	
	0.5-1.0; 1.5-4.5				0.5-1.0; 1.5-4.5				0.5-1.0	1.5	0.5-1.0	4.5
RM19	0.970		0.955		0.983		0.974		0.943		0.837	
	0.5	1.5-4.5	1.0	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5	1.0	1.5-4.5	0.5	1.5-4.5
RM20	0.990		0.845		0.988		0.841		0.987		0.898	
	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5
RM21	0.955		0.940		0.974		0.966		0.950		0.944	
	1.0	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5	0.5	1.5-4.5	0.5	1.5-4.5	1.0	1.5-4.5
RM22	0.991		0.980		0.990		0.979		0.952		0.913	
	0.5-1.0	4.5	0.5-1.0	1.5	0.5-1.0	4.5	0.5-1.0	1.5	0.5-1.0	1.5	0.5-1.0	4.5
RM23	0,91		0.765		0.892		0.759		0.899		0.769	
	0.5	4.5	1.0	1.5	0.5	4.5	1.0	1.5	0.5	1.5	1.0	4.5
RM24	0.970		0.959		0.969		0.959		0.947		0.854	
	0.5-1.0	1.5	0.5-1.0	4.5	0.5-1.0	1.5	0.5-1.0	4.5	0.5-1.0	1.5	0.5-1.0	4.5

– By increasing the value of lubrication factor from 1.4 to 1.7, the efficiency for all functional modules decreases from values 0.00366 (RM1, RM7 and RM8) to 0.22718 (RM19). The lubrication factor has a greater influence on these functional modules than on the common two--set functional modules.

In these functional modules, in respect to major parameters, it is evident, that there are two extreme cases: functional modules RM1, RM7 and RM8 (efficiency depends least on major parameters) and functional module RM19 (the greatest dependence of efficiency on significant parameters).

Efficiency of the three-step functional modules and the functional modules of the RAVIGNEAUX type in combination

The kinematic schemes and power flows of the three-step functional modules are given in Fig.9.

The derived analytical expressions for efficiency, determined by the power method in gear mesh and the M. A. Krejnes method are given in Table 13, and fo the Combination method in Table 14.

The efficiency of these functional modules (no matter which method is applied) depends on the value of basic gear ratios of planetary sets which form the respective module.

While determining the efficiency of the three-set functional modules, no mather which method applied, the values of basic gear ratios (k_1, k_2 and k_3) are found in a range from $k_{\min} - k_{\max}$ for the modules TM2 and TM3, while for the functional module TM1 they are determined by:

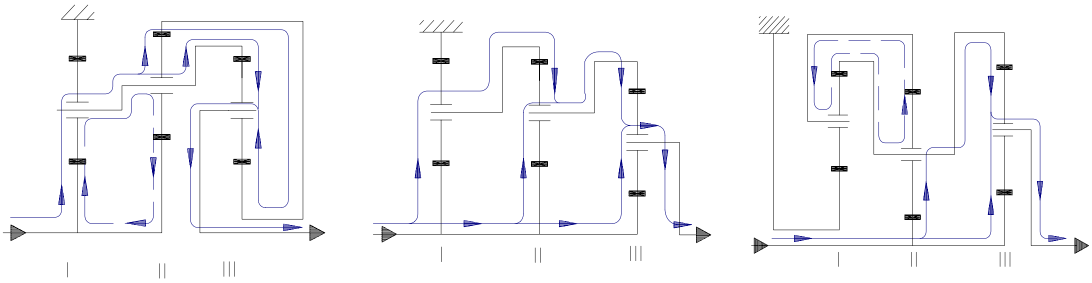

Figure 9. Three-step functional modules - kinematic schemes and power flows

Table 13

Module label	Power in gear mesh method	Krejnes method
TM1	$1-0.05\dfrac{k_1[(k_2k_3+k_2+1)(1+k_3)+(1+k_1)(1+k_3)+k_3(1+k_1)]}{(k_2k_3+k_2-k_1)(1+k_1)(1+k_3)}$	$\dfrac{(k_2k_3+k_2-k_1)(\eta_0+k_3)(1+k_1\eta_0)}{(1+k_3)(1+k_1)(k_2k_3+k_2\eta_0-k_1\eta_0^3)}$
TM2	$1-0.05\dfrac{k_1k_2k_3}{[(1+k_1)(1+k_2)+(1+k_1+k_2)k_3]}\left(\dfrac{1}{1+k_1}+\dfrac{1}{1+k_2}+\dfrac{k_3}{1+k_3}\right)$	$\dfrac{(1+k_1\eta_0)(1+k_2\eta_0)(1+k_3\eta_0)[(1+k_1)(1+k_2)+(1+k_1+k_2)k_3]}{(1+k_1)(1+k_2)(1+k_3)[(1+k_1\eta_0)(1+k_2\eta_0)+(1+k_1\eta_0+k_2\eta_0)k_3\eta_0]}$
TM3	$1-0.05\dfrac{K_2K_3[(1+2K_1)(1+K_3)+(1+K_1+K_2)]}{[(1+K_1)(1+K_3)+K_2](1+K_3)(1+K_1+K_2)}$	$\dfrac{[(1+k_1)(1+k_3)+k_2](1+k_3\eta_0)(\eta_0+k_1+k_2\eta_0^2)}{(1+k_3)(1+k_1+k_2)[(\eta_0+k_1)(1+k_3\eta_0)+k_2\eta_0^2]}$
RMK1	$1-0.05\dfrac{k_2[(k_3k_4+k_3+1)(1+k_4)+(1+k_2)(1+k_4)+k_4(1+k_2)]}{(k_3k_4+k_3-k_2)(1+k_2)(1+k_4)}$	$\dfrac{(k_3k_4+k_3-k_2)(\eta_0+k_4)(1+k_2\eta_0)}{(1+k_4)(1+k_2)(k_3k_4+k_3\eta_0-k_2\eta_o^3)}$
RMK2	$1-\dfrac{k_1}{k_3k_4+k_3-k_1}(0.09\dfrac{k_3k_4+k_3+1}{1+k_1}+0.05\dfrac{1+2k_4}{1+k_4})$	$\dfrac{(k_3k_4+k_3-k_1)(\eta_0+k_4)(1+k_1\eta_o)}{(1+k_1)(1+k_4)(k_3k_4+k_3\eta_o-k_1\eta_0^2)}$
RMK3	$1-k_1[0.03\dfrac{1}{1+k_1}+\dfrac{1}{k_2k_3+k_2-k_1}(0.06\dfrac{k_2(1+k_3)-2k_1-1}{1+k_1}+0.05\dfrac{k_3}{1+k_3}+0.02k_2)]$	$\dfrac{(1+k_1\eta_0)(\eta_0+k_3)(k_2k_3+k_2-k_1)}{(1+k_1)(1+k_3)(k_2k_3+k_2\eta_0-k_1\eta_0^3)}$

Table 14

Module label	Combination method
TM1	$1-\dfrac{2Am[(CmZ_{a_1}n_u\frac{k_1}{1+k_1})^n+(CmZ_{a_2}n_u\frac{k_1k_2}{(1+k_1)(1+k_1)})^n]+0.015P_u\frac{k_1k_2(2+k_1+k_2)}{(1+k_1+k_2)(1+k_1)(1+k_2)}}{c_r P_u}$
TM2	$1-\dfrac{2Am[(CmZ_{a_1}n_u\frac{k_1k_2}{k_1k_2-1})^n+(CmZ_{a_2}n_u\frac{k_2}{k_1k_2-1})^n]+0.03P_u\frac{k_1k_2}{k_1k_2-1}}{c_r P_u}$
TM3	$1-\dfrac{2Am[(CmZ_{a_1}n_u\frac{k_1k_2}{1+k_1+k_2})^n+(CmZ_{a_2}n_u\frac{k_2(1+k_1)}{1+k_1+k_2})^n]+0.015P_u\frac{1+2k_1}{1+k_1+k_2}}{c_r P_u}$
RMK1	$1-\dfrac{2Am[(CmZ_{a_2}n_u\frac{k_2}{1+k_2})^n+(CmZ_a3n_u\frac{k_2}{1+k_2})^n+(CmZ_a4n_u\frac{k_2k_4}{k_3(1+k_4)(1+k_2)})^n]+0.015P_u\frac{k_2}{k_3k_4+k_3-k_2}(1+\frac{k_4}{1+k_4}+\frac{k_3k_4+k_3+1}{1+k_2})}{c_r P_u}$
RMK2	$1-\dfrac{Am[(CmZ_a2n_u\frac{k_1}{1+k_1})^n(3+\frac{1}{0.995})+2(CmZ_a3n_u\frac{k_1}{1+k_1})^n+2(CmZ_a4n_u\frac{k_1k_4}{k_3(1+k_4)(1+k_1)})^n]+0.015P_u\frac{k_1}{k_3k_4+k_3-k_1}(1+\frac{k_4}{1+k_4}+\frac{k_3k_4+k_3+1}{1+k_1})}{c_r P_u}$
RMK3	$1-\dfrac{Am[4(CmZ_a2n_u\frac{k_1}{1+k_1})^n+2(CmZ_a3n_u\frac{k_1k_3}{(1+k_1)(1+k_3)k_2})^n]+P_uk_10.01\frac{1}{1+k_1}+\frac{1}{k_2k_3+k_2-k_1}[0.05k_2+0.02\frac{k_2(1+k_3)-2k_1-1}{1+k_1}+0.015\frac{k_3}{1+k_3}]}{c_r P_u}$

$$\left. \begin{array}{l} k_1 \in \left\{ k_{1\min}, \min[\dfrac{k_2(1+k_3)^2}{1+3k_3},\ k_{1\max}] \right\} \\ k_2 \in \left[k_{2\min}, k_{2\max} \right] \\ k_3 \in \left[k_{3\min}, k_{3\max} \right] \end{array} \right\} \qquad (6)$$

The values determined by expression (6) refer to the RMK1 module that is, in a functional way, identical to the TM1 module, with one difference concerning the indices of markings the basic gear ratios that are increased by one.

Since there is a relatively small amount of functional modules of this type, the grading according the average value of efficiency is not done, but the minimum and maximum values were determined along with the basic gear ratios (Table 15).

The data given in Table 15 show that for all functional modules there is a correlation between the Power method in gear mesh and the M. A. Krejnes method in respect to the efficiency and their basic gear ratios.

For the functional modules TM1 and TM3 the values of efficiency obtained by the Combination method are very similar to the values obtained by the M. A. Krejnes method. The value of efficiency for the functional module TM2 greatly differs in respect to the applied method. For example, the difference between the maximum values of efficiency obtained by the first two methods (Table 15) and by the Combination method is 0.16988, while the difference between the minimum values of efficiency is 0.3071.

For the functional modules of the RAVIGNEAUX type in combination (kinematic schemes and power flows are shown in Fig.10), the maximum and minimum values efficiency as well as the value of the corresponding basic gear ratios were determined (Table 16).

Moreover, in this kind of functional modules it is evident that there exists a correlation between the power in gear mesh method and the M.A. Krejnes method regarding the maximum and minimum values of efficiency. The values of efficiency determined by the Combination method, for both functional modules, greatly differ from the values obtained by the first two methods (Table 16). The difference is the greatest with the functional module RMK3 and the maximum value is 0.09111, while the minimum efficiency value is 0.44794.

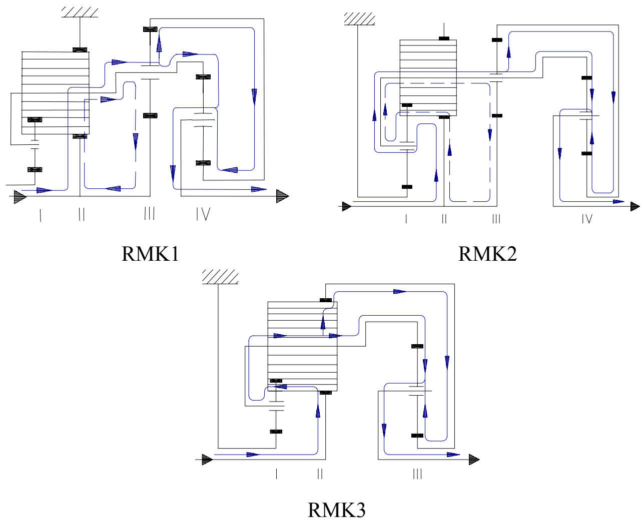

Figure 10. Functional modules of the RAVIGNEAUX type in combination – kinematic schemes and power flows

A significant difference in the efficiency values obtained by the Power in gear mesh method, M. A. Krejnes method and Combination method for these two functional modules and also in the three-step module TM2 shows that these functional modules do not function under nominal loads (in the first two methods, it is assumed that the gear functions under a nominal load [1]), which means that the input power (with the unchanged input speed), i.e. the input torque, should have a higher value. Because of this, during the analysis of the influence of other relevant parameters on efficiency the chosen input power was 1000 kW. In this analysis, the values of basic gear ratios of the planetary sets were assumed to be constant and their value was 1.5, and all the characteristics regarding the module TM1 are the same as in the module RMK1. Therefore, the following was concluded:

- With the increase of the input speed from 1500 to 3000 min^{-1} the efficiency of all functional modules decreases. The values of this decrease are in the range from 0.00342 (TM1) to 0.06984 (TM2), which means that the greatest influence of the input speed is in the functional module TM2. Comparing the efficiency of this functional module and the two-step module DM7, where there is also the greatest influence of the input speed on

Table 15

Module label	Power in gear mesh method						Krejnes method						Combination method					
	η_{max}			η_{min}			η_{max}			η_{min}			η_{max}			η_{min}		
	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3	k_1	k_2	k_3
TM1, RMK1				0.79000			0.96164			0.82558			0.96271			0.82618		
	1.5	4.5	1.5	4.5	1.5	4.5	1,5	4.5	1.5	4.5	1.5	4.5	4.5	1.5	4.5	1.5	4.5	4.5
TM2	0.96090			0.92844			0,983558			0.96702			0.81370			0.65992		
	1.5	1.5	1.5	4.5	4.5	4.5	1,5	1.5	1.5	4.5	4.5	4.5	4.5	4.5	1.5	1.5	1.5	4.5
TM3	0.98288			0.95821			0,98322			0.95857			0.96995			0.87146		
	4.5	1.5	4.5	1.5	4.5	4.5	4.5	1.5	4.5	1.5	4.5	4.5	1.5	4.5	1.5	4.5	1.5	4.5

Table 16

Module label	Power in gear mesh method						Krejnes method						Combination method					
	η_{max}			η_{min}			η_{max}			η_{min}			η_{max}			η_{min}		
	k_1	k_2^*	k_3^*	k_1	k_2^*	k_3^*	k_1	k_2^*	k_3^*	k_1	k_2^*	k_3^*	k_1	k_2^*	k_3^*	k_1	k_2^*	k_3^*
RMK2	0.96995			0.89318			0.98153			0.94774			0.90296			0.54298		
	0.5	1.5	4.5	1.0	1.5	1,5	0.5	4.5	4.5	1.0	1.5	1.5	1.0	1.5	4.5	0.5	4.5	4.5
RMK3	0.97000			0.94915			0.98060			0.93256			0.88949			0.48462		
	0.5	1.5	1.5	1.0	4.5	1.5	0.5	4.5	4.5	1.0	1.5	1.5	1.0	1.5	1.5	0.5	4.5	4.5

*For the RMK2 the indice of parameters are increased by one.

efficiency, it is evident, even though the value of the input torque is twice as large, that the efficiency values are lower. The reason for this is the fact that the number of gears in mesh in the functional modules TM2 is greater, and so are the losses.

- With the increase of the gear module from 3 to 6 mm, the value of efficiency decreases negligibly. The minimum decrease is in the functional module TM1 (0.00565), and the maximum decrease is in the module TM2 with the value of 0.07595. In this case, there is also the greatest influence of the gear module on efficiency in the functional module TM2, which is the result of its kinematic scheme and chosen values for basic gear ratios.
- With the increase of the input power from 500 to 1000 kW (input speed is constant), the efficiency increases. The highest increase is in module TM2 (0.09204), and the lowest increase is 0.00450 in the TM1 module. If the efficiency of the two-step functional module DM7 is compared with the three-step module TM2 (where efficiency depends mostly on the input torque) efficiency it is evident (with the same values of other parameters) that a lower value is obtained in the three-step functional module. For example for the 500 kW input power, the efficiency of the functional module TM2 is 0.80092 and for the module DM7 – 0.92977. Lower efficiency values are obtained on the three-step functional module TM2 because there are more gears in mesh.
- With the increase of the lubrication factor (from 1.4 to 1.7) the efficiency decreases. The minimum decreases of efficiency is obtained in the module TM1 (0.00529), and the highest decrease is 0.083906 (TM2).

Duting the analysis of the influence of other parameters on efficiency of the functional modules of the RAVIGNEAUX type in combination, it was assumed that the values of basic gear ratios are k_1=0.5 and k_2=k_3=1.5. Therefore, the following was concluded:

- With the increase of the input speed from 1500 to 3000 min^{-1}, the efficiency decreases. The decrease is greater in the RMK2 module and its value is 0.00937, while the value of the module RMK3 is 0.00651. In these functional modules, in contrast to others, the characters of efficiency change are very similar and so are their values. For example, at the input speed of 1500 min^{-1}, the efficiency for the functional module RMK2 is 0.89512 and for the RMK3 module its value is 0.88853.
- With the increase of the gear module from 3 to 6 mm, the efficiency decreases from 0.01780 (RMK3) to 0.01550 (RMK2).
- With the increase of the input speed from 500 to 1000 kW, efficiency increases for both functional modules, from the value 0.00857 (RMK3) to 0.01130 (RMK2).
- The efficiency for both functional modules decreases when the lubrication factor increases from 1.4 to 1.7. For the module RMK2 the decrease is 0.01036, while for the module RMK3 the decrease is lower and its value is 0.00705.

During the analysis of the obtained values, it is evident that the dependence of efficiency on other influencing parameters, for both of the functional modules, is very similar (a slight change of efficiency value in a chosen range of values of influencing parameters). The values of efficiency of these functional modules, in respect to the three-step modules are lower by 0.08.

Conclusion

There is a correlation of the M. A Kreines method and the Power in gear mesh method in respect to the efficiency value in the considered functional modules, while their correlation with the Combination method exits only for some functional modules. The character of torque influencing the efficiency in gear mesh is that in the range of high values of torque, the efficiency change is smaller in comparison to the low values of torque. The absolute difference between the values obtained by the Combination method and the I. N. Kornilaev method is low. This difference appears because the Combination method disregards the influence of losses of one gear mesh on the efficiency in other gear mesh. The grade of functional modules according to the efficiency values is very significant in the creation of a correct kinematic scheme.

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Analiza stepena korisnosti funkcionalnih modula planetarnih prenosnika

Izvršena je analiza stepena korisnosti funkcionalnih modula planetarnih prenosnika čiji su rezultati veoma značajni pri projektovanju planetarnih prenosnika snage. Prikazani su analitički izrazi za određivanje stepena korisnosti za nekoliko postojećih metoda, utvrđena je korelacija razmatranih metoda s aspekta određenih vrednosti stepena korisnosti i sagledan je uticaj relevantnih parametara na stepen korisnosti funkcionalnih modula.

Ključne reči: **planetarni prenosnik snage, funkcionalni modul, stepen prenosa, stepen korisnosti.**

Analyse de l'efficacité des modules fonctionnels de l'engrenage planétaire

L'efficacité des modules fonctionnels de l'engrenage planétaire était le sujet d'une analyse dont les résultats sont très importants pendant la conception des trains d'engrenage planétaires. Les expressions analytiques pour la détermination de l'efficacité sont présentées pour quelques méthodes existantes et puis la correlation entre les méthodes traitées est effectuée de point de vue de leur efficacité. L'effet des paramètres pertinents sur l'efficacité de modules fonctionnels est également déterminé.

Mots-clés: **train d'engrenage planétaire, module fonctionnel, rapport d'engrenage, efficacité.**