UDC: 621.396.663(047)=20
COSATI: 17-03, 17-02

A generalized model of superposition of radio signals in a given frequency sub-band on the antenna array of arbitrary geometry

Miljko Erić, PhD (Eng)[1)]

A generalized spatial mathematical model of superposition of multiple incident radio signals in a given frequency sub-band on the antenna array of arbitrary volume non-uniform geometry is presented in this paper. All parameters of the model are normalized so that this model could be applied without restrictions to the modeling of signal superposition in different frequency ranges, as well as to the modeling of multi-user signal scenario with different multiple access schemes (such as TDMA, FDMA and CDMA). The mathematical model can also be applied to the modeling of signal superposition in multipath conditions.

Key words: **electronic reconnaissance, antenna arrays, signal superposition, multipath, spatio-time-frequency signal analysis.**

Introduction

THERE is a distinction between spatial models of narrow-band and wide-band signal superposition in the literature. Either models of narrow-band signal superposition or models of wide-band signal superposition are used. It is assumed, in both these cases, that superposed signals have both known and equal central frequencies and equal spectral bandwidths which is actually the restriction of these models [1,2,3]. Starting from the previous models which are widely used in the literature, a generalized spatial mathematical model of radio signal superposition is formulated and presented in this paper. It provides the modeling of simultaneous superposition of narrow-band and wide-band radio signals with different and arbitrary central frequencies, spectral bandwidths and other parameters of interest.

All parameters of the model are normalized so that this model could be applied without restrictions to the modeling of signal superposition in different frequency ranges, as well as to the modeling of multi-user signal scenario with different multiple access schemes (such as TDMA, FDMA and CDMA). The mathematical model can also be applied to the modeling of signal superposition under multipath conditions.

The structure of the paper is as follows.

The spatial model of narrow-band signal superposition is followed by the spatial model of superposition of wide-band signals.

After that, the generalized spatial model of superposition of radio signals in a given frequency sub-band is proposed and the generalized spatial model of radio signal superposition under multipath conditions precedes the results of simulation.

Spatial model of narrow-band signal superposition

Let us assume that the radio signal is received by the antenna array of arbitrary volume geometry (Fig.1) with L antenna elements non-uniformly located in the real 3-D space, at the points denoted with the set of vectors $\mathbf{r}_1, \mathbf{r}_2, .., \mathbf{r}_L$.

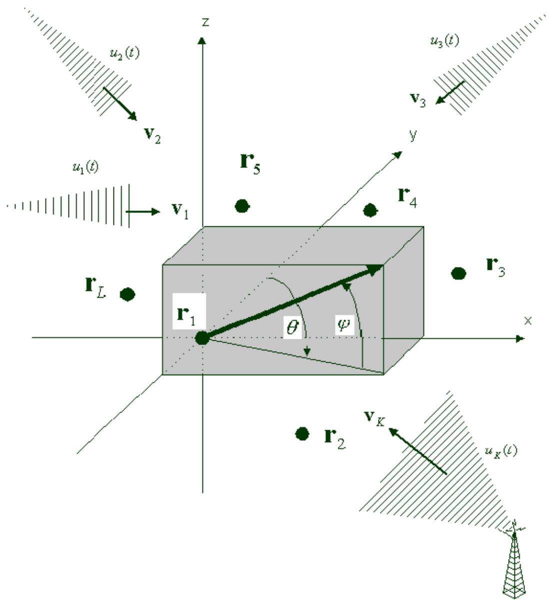

Figure 1. Superposition of radio signals on the antenna array of arbitrary volume non-uniform geometry

The spatial-discrete and time-continuous model of the k-th radio signal can be expressed as a function of the time-continuous variable t and the spatial-discrete variable $\boldsymbol{r}_l$ as

$$u_{lk}(t) = s_k(t-\tau_{lk})\exp[j\omega_c(t-\tau_{lk})] \tag{1}$$

where $\tau_{lk} = \mathbf{v}_k^T \mathbf{r}_l / c$ denotes the time delay of the k-th radio signal on the l-th antenna in relation to the origin of the

1) Military Technical Institute (VTI), Katanićeva 15, 11000 Beograd

coordinate system; $\mathbf{r}_l \in R^3$ is a vector of location of the l-th antenna in the real 3-D space, $\mathbf{v}_k \in R^3$ is a unit vector which represents the direction of the arrival of the k-th radio signal on the antenna array, which can be expressed in polar coordinates as a function of the direction of arrival – azimuth θ_k and the elevation φ_k, [4]:

$$\mathbf{v}_k = [\sin(\theta_k)\cos(\varphi_k) \quad \cos(\theta_k)\cos(\varphi_k) \quad \sin(\varphi_k)]^T \tag{2}$$

As previously mentioned, there is a distinction between spatial models of narrow-band and wide-band signal superposition in the literarure. The distinction is of great importance for the spatial modeling of superposition of radio signals.

A radio signal can be treated and spatially modeled as narrow-band if the time delays of its complex envelope along the antenna array can be neglected, or if the following approximation is valid $s_k(t-\tau_{lk}) \approx s_k(t)$ za l=1,2,...,L.

Starting from the previous approximation, the k-th radio signal on the l-th antenna can be expressed as

$$u_{lk}(t) = s_k(t)\exp[j\omega_c(t-\tau_{lk})] \tag{3}$$

Thus, the time delays of the narrow-band signals along the antenna array are approximated with the phase shifts.

The term "narrow-bandness" is connected not only to the normalized spectral bandwidth of the radio signal defined as $\Delta\omega_{BW}/\omega_c$ but also to the antenna aperture.

The radio signal of the given normalized spectral bandwidth can be at the same time modeled as narrow-band when the antenna aperture is small, and as wide-band if the antenna aperture is large. In the boundary case when the time delays of the radio signal along the antenna array are equal to zero, (which corresponds to the situation when the radio signal comes to the linear antenna array from the boresides), then the radio signal of any normalized spectral bandwidths can be modeled as narrow-band.

The assumption of the narrow-bandness of radio communication signals is almost always satisfied in practice.

The problem of wide-bandness is typical in modeling acoustic signal superposition (such as in sonars or microphone arrays).

Another definition of the radio signal narrow-bandness is used in the literature, and it results from the previous approximation. According to this definition, a radio signal can be modeled as narrow-band if the reciprocal value of the time delay of the radio signal along the antenna array is much greater than its spectral bandwidth.

Let us assume that the K radio signals, which are overlapped both in time and frequency, with the same spectral bandwidths $\Delta\omega_{BW}$ and with the same known carrier frequency ω_c, impinge on the antenna array with L antenna elements with the direction of arrivals $\Theta_1, \Theta_2, .., \Theta_K$, where Θ_K represents the azimuth θ_k and the elevation φ_k of the arrival of the k-th radio signal.

Let us assume for the sake of simplicity that the antenna elements have identical omnidirectional characteristics. The mathematical model can be formulated simply for the antenna array with antenna elements of different gains.

The spatio-temporal model of the radio signal received by the antenna array can be expressed in the matrix form as [4]

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ . \\ x_L(t) \end{bmatrix} = \begin{bmatrix} \exp(j\omega_c\tau_{11}) & \exp(j\omega_c\tau_{12}) & .\ . & \exp(j\omega_c\tau_{1K}) \\ \exp(j\omega_c\tau_{21}) & \exp(j\omega_c\tau_{22}) & .\ . & \exp(j\omega_c\tau_{2K}) \\ \exp(j\omega_c\tau_{31}) & \exp(j\omega_c\tau_{32}) & .\ . & \exp(j\omega_c\tau_{3K}) \\ . & . & .\ . & . \\ \exp(j\omega_c\tau_{L1}) & \exp(j\omega_c\tau_{L2}) & .\ . & \exp(j\omega_c\tau_{LK}) \end{bmatrix} \begin{bmatrix} s_1(t) \\ s_2(t) \\ . \\ s_K(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ . \\ n_L(t) \end{bmatrix} \tag{4}$$

or in the short matrix form as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{5}$$

where

- $\mathbf{x}(t) \in C^{Lx1}$ - is a vector of the spatial samples of the radio signal on the antenna array,
- $\mathbf{A} \in C^{LxK}$ - is a matrix which models the antenna array response to the K superposed radio signals,
- $\mathbf{s}(t) \in C^{Kx1}$ - is a vector of the complex envelopes of the radio signals in the origin of the coordinate system,
- $\mathbf{n}(t) \in C^{Lx1}$ - is a noise vector of the antenna array.

The matrix $\mathbf{A}$ describes the transformation of the radio signal from the referent point in space (origin of the coordinate system) to the antenna elements.

The columns of the matrix $\mathbf{A}$ are so-called *steering vectors* which have the following general form

$$\mathbf{a}_k(\theta_k, \varphi_k) = \\ = [\exp(j\omega_c\tau_{1k}) \quad \exp(j\omega_c\tau_{2k}) \quad .. \quad \exp(j\omega_c\tau_{Lk})]^T \tag{6}$$

The steering vectors are the function of direction of arrivals (azimuth and elevation) and they describe the response of the antenna array to the k-th incident radio signal with the direction of arrival determined by the vector $\mathbf{v}_k$.

The value of the first element of the steering vector is equal to one if the referent antenna is placed in the origin of the coordinate system.

The origin of the coordinate system can be chosen arbitrarily, although the origin of the coordinate system is placed on one antenna which is the referent antenna in the antenna array.

Spatial model of wide-band radio signal superposition

The spatial model of wide-band radio signal superposition stands for the theoretical basis of the development of the generalized spatial mathematical model of superposition of radio signals in a given frequency sub-band, which will be later formulated and presented.

Let us suppose that the k-th radio signal $u_k(t) = s_k(t)\exp(\omega_c t + \phi_k)$ has the spectral banwidth $\Delta\omega_{BW}$ at the central frequency ω_c so its spectrum is located in the frequency sub-band $[(\omega_c - \omega_{BW}/2), (\omega_c + \omega_{BW}/2)]$.

Let us suppose that the signal is observed in the time interval $(t, t+T_0)$ where T_0 is the duration of the observation interval.

The compex envelope $s_k(t)$ of the radio signal $u_k(t)$ can be expressed in the Fourier series as

$$s_k(t) = \sum_{n=1}^{J} S_k(\omega_n)\exp(j\omega_n t); \quad t_0 \le t \le t_0 + T_0 \tag{7}$$

where $S_k(\omega_n)$ denotes the Fourier coefficients related to the complex envelope in the following way

$$S_k(\omega_n) = \frac{1}{T_0}\int_{t_0}^{t_0+T_0} s_k(t)\exp(-j\omega_n t) \quad (8)$$

where

$$\omega_n = 2\pi f_n = \frac{2\pi}{T_0}[n-(J+1)/2] \quad (9)$$

From the previous expressions it can be concluded that the spectrum of the complex envelope with the spectral bandwidth of $\Delta\omega_{BW}$ contains J spectral components symmetrically arranged about zero, with the distance among themselves equal to

$$\omega_{n+1} - \omega_n = \Delta\omega_{BW} / J = 2\pi / T_0 \quad (10)$$

The radio signal $u_k(t)$ can be expressed in the form of the Fourier coefficients in the following way

$$u_k(t) = \sum_{n=1}^{J} S_k(\omega_n)\exp[j(\omega_c+\omega_n)t] \quad (11)$$

The radio signal of the k-th emitter on the l-th antenna can be expressed as a function of the radio signal at the referent point in space as

$$\begin{aligned} u_{lk}(t+\tau_{lk}) &= s_k(t+\tau_{lk})\exp(j\omega_c(t+\tau_{lk})) = \\ &= \sum_{n-1}^{J} S_n(\omega_n)\exp[j(\omega_c+\omega_n)(t+\tau_{lk})] \end{aligned} \quad (12)$$

As previously said, the complex envelope $s_k(t)$ of the radio signal is provided by the *I-Q* demodulation of the radio signal $u_k(t)$. The complex envelope of the k-th radio signal on the l-th antenna can be expressed as

$$s_{lk}(t+\tau_{lk}) = \sum_{n-1}^{J} S_n(\omega_n)\exp[j(\omega_c+\omega_n)\tau_{lk}]\exp(j\omega_n t) \quad (13)$$

Let us suppose that the K radio signals which are overlapped both in time and frequency with the same spectral bandwidts $\Delta\omega_{BW}$ and central frequencies ω_c are received by the antenna array.

After the *I-Q* demodulation, the received signal on the l-th antenna, a result of the superposition of the K incident radio signals, can be expressed in the following way [4]

$$\begin{aligned} x_l(t) &= \sum_{k-1}^{K} s_k(t+\tau_{lk}) + n_l(t) = \\ &= \sum_{k-1}^{K}\sum_{n=1}^{J} S_{nk}(\omega_n)\exp[j(\omega_c+\omega_n)\tau_{lk}]\exp(j\omega_n t) + n_l(t) \end{aligned} \quad (14)$$

The vector of spatial samples of the superposed radio signals $\mathbf{x}(t) = [x_1(t)\ \ x_2(t)\ \ ...\ \ x_L(t)]^T$ can be expressed as [4]

$$\begin{aligned} \mathbf{x}(t) &= \sum_{n=0}^{J}[\mathbf{A}(\omega_c+\omega_0)\mathbf{S}(\omega_n)+\mathbf{N}(\omega_n)]\exp(j\omega_n t) = \\ &= \sum_{n=1}^{J}\mathbf{X}(\omega_n)\exp(j\omega_n t) \end{aligned} \quad (15)$$

where

$$\mathbf{X}(\omega_n) = \mathbf{A}(\omega_c+\omega_n)\mathbf{S}(\omega_n)+\mathbf{N}(\omega_n) \quad (16)$$

$$\mathbf{S}(\omega_n) = [S_1(\omega_n)\ S_2(\omega_n)\ ...\ S_K(\omega_n)]^T \quad (17)$$

$$\mathbf{N}(\omega_n) = [N_1(\omega_n)\ N_2(\omega_n)\ ...\ N_L(\omega_n)]^T \quad (18)$$

$$\begin{aligned} &\mathbf{A}(\omega+\omega_n) = \\ &= [\mathbf{a}_1(\omega_c+\omega_n)\ \mathbf{a}_2(\omega_c+\omega_n)\ ...\ \mathbf{a}_K(\omega_c+\omega_n)]^T \end{aligned} \quad (19)$$

The columns of the matrix $\mathbf{A}$ are the steering vectors, which for the k-th radio signal have the form

$$\begin{aligned} \mathbf{a}_k(\omega_c+\omega_n) &= \exp[(\omega_c+\omega_n)\tau_{1k}]\exp[j(\omega_c+\omega_n)\tau_{2k}]... \\ &...[\exp[j(\omega_c+\omega_n)\tau_{Lk}] \end{aligned} \quad (20)$$

From the above, it can be seen that the steering vectors of wide-band radio signals are dependent on frequency.

Generalized spatial mathematical model of superposition of radio signals in a given frequency sub-band

The previously presented spatial mathematical models of superposition of narrow-band and wide-band signals, are based on the assumption that the superposed radio signals are narrow-band or wide-band and that they have same spectral bandwidths and same frequency carriers.

It is also assumed that the spectral bandwidths and frequency carriers of the superposed radio signals are known.

In a real situation, illustrated in Fig.2 many radio signals with different carrier frequencies, different spectral bandwidths, signal to noise ratios, etc., are superposed, without a priori information on the values of these parameters.

In a real situation, narrow-band and wide-band signals are superposed at the same time in a given frequency sub-band, as well.

Under multipath conditions, there is an additional superposition of many attenuated and delayed replicas of the same signal.

Since the superposed signals in a given frequency sub-band have different spectral bandwidths and carrier frequencies, the spatial mathematical model of signal superposition previously presented is not directly applicable, so its generalization is necessary.

Firstly, the generalization is related to the modeling of superposition of radio signals with different spectral bandwidths and frequency carriers.

Let us suppose that the K radio signals with different carrier frequencies $\omega_{ck}, k=1,..,K$ and spectral bandwidths $\Delta\omega_{BWk}, k=1,..,K$ are superposed in the observation interval ΔT_0 in the given frequency sub-band $\Delta\omega_{BW}$ with the central frequency ω_c (Fig.2).

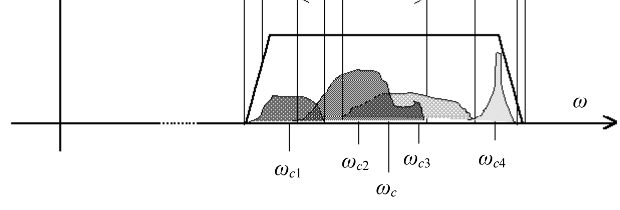

Figure 2. Superposition of radio signals in a given frequency sub-band

The received radio signal in the referent point in space (origin of the coordinate system), which is a result of the superposition of the K radio signals, can be expressed as [4]

$$u(t)=\sum_{k=1}^{K}u_k(t)=\sum_{k=1}^{K}s_k(t)\exp(j\omega_{ck}t)\;;\; t\in(t_0,t_{0+}\Delta T_0) \quad (21)$$

where $s_k(t)$ is the complex envelope of the *k*-th superposed radio signal.

Let us suppose that the radio signal $u(t)$ in the given frequency sub-band $\Delta\omega_{BW}$ is down translated by the *I-Q* demodulation. Mathematically, down translation is performed by multiplying the radio signal $u(t)$ with the complex sinusoid $\exp(-j\omega_c t)$, i.e., [4]

$$\begin{aligned} f(t)&=u(t)\exp(-j\omega_c t)=\\ &=\sum_{k=1}^{K}s_k(t)\exp[j(\omega_{ck}-\omega_c)t]=\sum_{k=1}^{K}f_k(t) \end{aligned} \quad (22)$$

where ω_c is the central frequency of the given frequency sub-band $\Delta\omega_{BW}$ and $f_k(t)=s_k(t)\exp[j(\omega_{ck}-\omega_c)]$ is the complex envelope of the *k*-th superposed radio signal which is frequency shifted in relation to the zero frequency by the $(\omega_{ck}-\omega_c)$ value (so in the further text it is refered to as '*shifted complex envelope*').

The complex envelope $s_k(t)$ of the radio signal can be expressed as a function of the Fourier coefficients as

$$s_k(t)=\sum_{h=-H/2}^{H/2}S_k(\omega_h)\exp(j\omega_h t)\;;\quad t_0\le t\le t_0+T_0 \quad (23)$$

where $S_k(\omega_h)$ denotes the set of the Fourier coefficients of the complex envelope $s_k(t)$ that can be expressed as

$$S_k(\omega_h)=\frac{1}{\Delta T_0}\int_{t_0}^{t_0+\Delta T_0}s_k(t)\exp(-j\omega_h t) \quad (24)$$

where

$$\omega_h=h\frac{\Delta\omega_{BW}}{H};\; h=-H/2,H/2 \quad (25)$$

The set of the Fourier coefficients $F_k(\omega)$ of the shifted complex envelope of the *k*-th radio signal can be expressed as

$$F_k(\omega_h)=\frac{1}{\Delta T_0}\int_{t_0}^{t_0+\Delta T_0}s_k(t)\exp[(\omega_c-\omega_{ck})t]\exp(-j\omega_h t) \quad (26)$$

The spectrum of the signal $f(t)$ is located around the zero frequency in the frequency sub-band $(-\Delta\omega_{BW},\Delta\omega_{BW})$ and it can be expressed in the spectral domain as

$$\begin{aligned} F(\omega_h)&=\sum_{k=1}^{K}F_k(\omega_h)=\\ &=\sum_{k=1}^{K}[S_k(\omega_h)*\delta(\omega_{ck}-\omega_c)]=\sum_{k=1}^{K}S_k[\omega_h-(\omega_{ck}-\omega_c)] \end{aligned} \quad (27)$$

where $S_k(\omega_h)$ denotes the set of the Fourier coefficients of the complex envelope of the *k*-th radio signal, $\delta(\omega_{ck}-\omega_c)$ denotes the Dirac function and the symbol $*$ denotes convolution.

The signal on the *l*-th antenna can be further expressed as [4]

$$f_l(t)=\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2}F_k(\omega_h)\exp[j(\omega_c+\omega_h)\tau_{lk}]\exp(j\omega_h t) \quad (28)$$

where $F_k(\omega_h)$ denotes the set of the Fourier coefficients of the shifted complex envelope of the *k*-th radio signal. The $\tau_{lk}=\mathbf{v}_k^T\mathbf{r}_l/c$ represents the time delay of the *k*-th radio signal on the *l*-th antenna related to the origin of the coordinate system. Thus [4]

$$f_l(t)=\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2}F(\omega_h)\exp[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^T\mathbf{r}_l}{c}]\exp(j\omega_h t) \quad (29)$$

As it can be seen, it was supposed that the central frequencies and spectral bandwidths $\Delta\omega_{BW}$ of the superposed radio signals are generally different, so eq.(29) can be used for mathematical modeling of an arbitrary multiple incident signal scenario in a given frequency sub-band.

Eq.(29) stands for the theoretical basis of the generalization of the spatial mathematical model of radio signal superposition in a given frequency sub-band. In order to form a generalized mathematical model that can be applied to modeling of radio superposition in any frequency sub-band, all parameters of the above spatial mathematical model have to be normalized, if possible.

The velocity of radio signal propagation *c* can be then expressed as

$$c=\lambda_A f_A=\frac{\lambda_A\omega_A}{2\pi} \quad (30)$$

The ω_A denotes the highest frequency in the spectrum, for which the Niquist condition for spatial sampling of wavefront is satisfied and it is called the *characteristic frequency* of an antenna array. The λ_A denotes the *characteristic wavelength* of an antenna array.

If this equation is included in eq.(25), it follows [4]

$$\begin{aligned} f_l(t)&=\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2}F_k(\omega_h)\exp[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^T\mathbf{r}_l}{c}]\exp(j\omega_h t)=\\ &\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2}F_k(\omega_h)\exp[j(\omega_c+\omega_h)\mathbf{v}_k^T\mathbf{r}_l(\frac{2\pi}{\lambda_A\omega_A})]\exp(j\omega_h t)=\\ &\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2}F_k(\omega_h)\exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_l}{\lambda_A})]\exp(j\omega_h t) \end{aligned} \quad (31)$$

The spatial model given by eq.(27) is time-continuous and frequency-discrete. Let us suppose that the complex *I-Q* demodulated signal $f_l(t)$ is time-sampled with the sampling period

$$\Delta t=\frac{1}{2f_g}=\frac{\pi}{\omega_g}=\frac{2\pi}{\Delta\omega_{BW}} \quad (32)$$

In that case, the spectrum of the shifted complex envelope of the *k*-th signal can be estimated by the discrete Fourier transform as a function of the normalized frequency $\Omega_h=\dfrac{\omega_h}{\Delta\omega_{BW}}$ as [4]

$$\begin{aligned}F(\Omega_h) &= \sum_{n=1}^{N} s(n)\exp[j(\omega_c-\omega_{ck})n\Delta t]\exp(j\omega_h n\Delta t) = \\ &= \sum_{n=1}^{N} s(n)\exp[j(\omega_c-\omega_{ck})\frac{2\pi}{\Delta\omega_{BW}}n]\exp(j\omega_h\frac{2\pi}{\Delta\omega_{BW}}n) = \\ &= \sum_{n=1}^{N} s(n)\exp[j2(\frac{\omega_c-\omega_{ck}}{\Delta\omega_{BW}})n]\exp(j2\pi\frac{\omega_h}{\Delta\omega_{BW}}n) = \\ &= \sum_{n=1}^{N} s(n)\exp(j2\pi\Omega_{ck}n)\exp(j2\pi\Omega_h n) = \\ &= \sum_{n=1}^{N} s(n)\exp(j2\pi\Omega_{ck}n)\exp(j2\pi\frac{h}{H}n)\end{aligned} \tag{33}$$

The normalized frequency $\Omega_h = \dfrac{\omega_h}{\Delta\omega_{BW}} = \dfrac{h}{H}$ has its values in the range of $\Omega_h \in [-0.5, 0.5]$.

If eqs.(30) and (27) are included in eq.(29), we can obtain a discrete-spatial-time model of superposed radio signals on the l-th antenna

$$\begin{aligned}f_l(n) &= \\ &= \sum_{k=1}^{K}\sum_{h=-H/2}^{H/2} F_k(\Omega_h)\exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_l}{\lambda_A})]\exp(j\omega_h n\Delta t) = \\ &\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2} F_k(\Omega_h)\exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_l}{\lambda_A})]\exp[j2\pi(\frac{h\frac{\Delta\omega_{BW}}{H}}{\Delta\omega_{BW}})n] = \\ &\sum_{k=1}^{K}\sum_{h=-H/2}^{H/2} F_k(\Omega_h)\exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_l}{\lambda_A})]\exp[j2\pi\Omega_h n]\end{aligned} \tag{34}$$

where

$k = 1, K;$ - k is the ordinal number of a radio signal and K is the number of superposed signals,

$h = -H/2, H/2;$ - h is the ordinal number of a spectral component, H is the number of spectral components (Fourier coefficients),

$n = 1, N;$ - n is the ordinal number of a time sample, N is the number of time samples,

$l = 1, L;$ - l is the ordinal number of an antenna element in the antenna array, L is the number of antenna elements in the antenna array.

In eq.(31) $q_c = \dfrac{\omega_c}{\omega_A}$ denotes the *normalized central frequency* of the selected frequency sub-band. The values of the normalized central frequency are in the range of (0,1]. The $q_r = \dfrac{\mathbf{r}_l}{\lambda_A}$ represents the *normalized vector of location* of the l-th antenna. The $q_h = \dfrac{\omega_h}{\omega_A}$ denotes the *normalized circular frequency* of the translated radio signal, normalized with the characteristic frequency ω_A of the antenna array.

Since the circular frequency ω_h has its values in the range of $(\dfrac{-\Delta\omega_{BW}}{2}, \dfrac{\Delta\omega_{BW}}{2})$ it follows that $q_h = (\dfrac{\omega_h}{\omega_A})$ has its values in the range of $(\dfrac{-\Delta\omega_{BW}}{2\omega_A}, \dfrac{\Delta\omega_{BW}}{2\omega_A})$. The $\Omega_{ck} = (\dfrac{\omega_{ck}-\omega_c}{\Delta\omega_{BW}})$ represents a *normalized frequency shift of the central frequency* of the k-th radio signal in relation to the central frequency ω_c of the given frequency sub-band $\Delta\omega_{BW}$.

The spectrum of the superposed radio signal on the l-th antenna can be obtained by the Fourier transform

$$\begin{aligned}F_l(\Omega_h) &= \sum_{n=1}^{N} f_l(n)\exp(-jh\frac{2\pi}{H}n) = \\ &= \sum_{n=1}^{N} f_l(n)\exp(-j2\pi\Omega_h n)\end{aligned} \tag{35}$$

If the noise on the antenna elements is included, then the signal on the l-th antenna can be expressed as

$$x_l(n) = f_l(n) + n_l(n) \tag{36}$$

The vector of a signal on the antenna array in the time domain $\mathbf{x}(n) = [x_1(n) \;\; x_2(n) \;\; \dots \;\; x_L(n)]^T$ can be expressed as [4]

$$\begin{aligned}\mathbf{x}(n) &= \sum_{h=-H/2}^{H/2}[\mathbf{A}(\omega_c+\omega_h)\mathbf{F}(\Omega_h)+\mathbf{N}(\Omega_h)]\exp(j2\pi\omega_h n\Delta t) = \\ &= \sum_{h=-H/2}^{H/2}[\mathbf{A}(\omega_c+\omega_h)\mathbf{F}(\Omega_h)+\mathbf{N}(\Omega_h)]\exp(h\frac{j2\pi}{H}n) = \\ &= \sum_{h=-H/2}^{H/2}[\mathbf{A}(\omega_c+\omega_h)\mathbf{F}(\Omega_h)+\mathbf{N}(\Omega_h)]\exp(j2\pi\Omega_h n)\end{aligned} \tag{37}$$

The generalized model of the radio signal superposition on the antenna array in the frequency domain can be expressed in the following matrix form as [4]

$$\mathbf{X}(\Omega_h) = \mathbf{A}(\omega_c, \omega_h)\mathbf{F}(\Omega_h) + \mathbf{N}(\Omega_h) \tag{38}$$

where

$\mathbf{X}(\Omega_h) = [X_1(\Omega_h) \;\; X_2(\Omega_h) \;\; .. \;\; X_L(\Omega_L)]^T$ **-** is the vector of the spectral components of the superposed signals on the antenna array,

$\mathbf{F}(\Omega_h) = [F_1(\Omega_h) \;\; F_2(\Omega_h) \;\; .. \;\; F_K(\Omega_h)]^T$ - is the vector of the spectral components of the shifted complex envelopes in the referent point in space,

$\mathbf{N}(\Omega_h) = [N_1(\Omega_h) \;\; N_2(\Omega_h) \;\; .. \;\; N_L(\Omega_h)]^T$ -is the vector of the spectral components of the noise on the antenna array.

The spectrum $F_k(\Omega_h)$ of the shifted complex envelope of the k-th radio signal is calculated by eq.(32).

The matrix $\mathbf{A}(\omega_c, \omega_n)$ has the LxK dimension and a structure similar to the one in the case of the model of wide -band signal superposition, explained earlier in the paper. The columns of the matrix $\mathbf{A}(\omega_c, \omega_n)$ are the steering vectors of superposed radio signals which can be expressed in the following form [4]

$$\begin{aligned}\mathbf{a}(\omega_c,\omega_h) &= \left[\exp[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^T\mathbf{r}_1}{c}] \;\; \exp[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^T\mathbf{r}_2}{c}] \;\; \dots \;\; \exp[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^T\mathbf{r}_L}{c}]\right]^T = \\ &\left[\exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_1}{\lambda_A})] \;\; \exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_2}{\lambda_A})] \;\; .. \;\; \exp[j2\pi(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A})\mathbf{v}_k^T(\frac{\mathbf{r}_L}{\lambda_A})]\right]^T\end{aligned} \tag{39}$$

It is very important to notice that the transformation of the signals from the referent point in space to the antenna elements is performed in the frequency domain even in the case of superposition of narrow-band signals.

Generalized spatial model of radio signal superposition under multipath conditions

Under multipath conditions, many attenuated and time delayed replicas of the same radio signal are superposed on the antenna array (Fig.3). Therefore, the k-th superposed radio signal can be mathematically modeled as

$$u_k(t) = \sum_{p=1}^{R_k} u_k^{(p)}(t) \tag{40}$$

where $u_k^{(p)}(t)$ denotes the p-th replica of the radio signal $u_k(t)$, the index p denotes the ordinal number of the replica and R_k denotes the number of superposed replicas of the k-th radio signal $u_k(t)$.

Under multipath conditions, the number of superposed signals on the antenna array increases. Since the replicas arrive on the antenna array from different directions, from the point of view of spatial modeling, each replica is treated as an independent superposed signal. From the point of view of parameter estimation of the spatial model, the aim is to estimate the arrival direction and other parameters of interest for each replica. It is not always possible. The superposition of the replicas of the same signal is in the theory treated as a *coherency* problem [1,3].

The replicas are attenuated, relatively time delayed among themselves and relatively frequency shifted among themselves (due to the Doppler effects). Multipath has a fading of the received signal as a consequence.

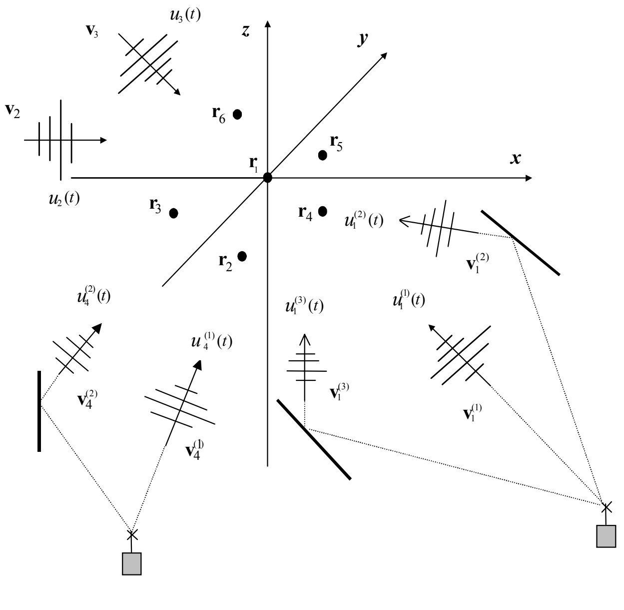

Figure 3. Superposition of radio signals under multipath conditions

The assumed model of the multipath is presented in Fig.4. This model stands for the theoretical basis of the modeling of signal superposition on the antenna array under mltipath conditions.

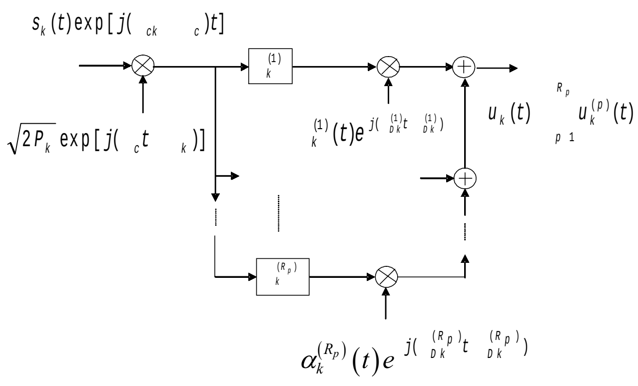

Figure 4. The multipath model

The mathematical model of radio signals under multipath conditions can be represented as [4]

$$\begin{aligned} u_k(t) &= \sum_{p=1}^{R_p} \alpha_k^{(p)}(t) s_k\left[t-\tau_k^{(p)}(t)\right]\cdot \\ &\cdot \exp\left[j(\omega_{ck}-\omega_c)t\right]\cdot \\ &\cdot \exp\left[j\left(\omega_c t+\omega_c \tau_k^{(p)}(t)+\omega_{Dk}^{(p)}(t)\right)t\right] = \\ &= u(t) = \sum_{p=1}^{R_k} \alpha_k^{(p)}(t) s_k\left[t-\tau_k^{(p)}(t)\right]\cdot \\ &\cdot \exp\left[j\omega_c \tau_k^{(p)}(t)\right]\exp\left[j(\omega_{ck}-\omega_c)t\right]\cdot \\ &\cdot \exp\left(\left[j\omega_{Dk}^{(p)}(t)t\right]\exp(j\omega_c t)\right. \end{aligned} \tag{41}$$

Let us suppose that time delays, frequency shifts and attenuations of replicas are constant inside the observation interval ΔT_0 so the following approximations are valid: $\tau_k^{(p)}(t) \approx \tau_k^{(p)}$ $\omega_{Dk}^{(p)}(t) \approx \omega_{Dk}^{(p)}$; $\alpha_k^{(p)}(t) \approx \alpha_k^{(p)}$.

The shifted complex envelope of the p-th replica of the k-th radio signal, $f_k^{(p)}(t)$ can be expressed in the following way

$$\begin{aligned} f_k^{(p)}(t) &= \alpha_k^{(p)} s_k[t-\tau_k^{(p)}]\exp[j\omega_c \tau_k^{(p)}]\cdot \\ &\cdot \exp[j(\omega_{ck}-\omega_c)t]\exp(j\omega_{Dk}^{(p)}t) \end{aligned} \tag{42}$$

The complex envelope of the p-th replica can be expressed in the following way

$$s_k(t-\tau_k^{(p)}) = \sum_{h=-H/2}^{H/2} S_k(\omega_h)\exp(j\omega_h \tau_k^{(p)})\exp(j\omega_h t) \tag{43}$$

Furthemore, it follows that the shifted complex envelope of the p-th replica of the k-th signal $f_k^{(p)}(t)$ on the referent antenna, provided by the I-Q demodulation of the signal $u_k(t)$, can be expressed in the following way [4]

$$\begin{aligned} f_k^{(p)}(t) &= \alpha_k^{(p)} \exp(j\omega_c \tau_k^{(p)})\exp[j(\omega_{ck}-\omega_c)t]\cdot \\ &\cdot \exp(j\omega_{Dk}^{(p)}t)\sum_{h=-H/2}^{H/2} S_k(\omega_n)\exp(j\omega_n \tau_k^{(p)})\exp(j\omega_h t) \end{aligned} \tag{44}$$

After time discretization with a sampling period of $\Delta t = \dfrac{1}{2f_g} = \dfrac{\pi}{\omega_g} = \dfrac{2\pi}{\Delta\omega_{BW}}$, the shifted complex envelope of the p-th replica of the k-th radio signal is provided in the discrete time domain [4].

$$f_k^{(p)}(n\Delta t) = \alpha_k^{(p)} \exp(j\omega_c \tau_k^{(p)}) \exp[j(\omega_{ck} - \omega_c) n\Delta t] \cdot$$
$$\cdot \exp(j\omega_{Dk}^{(p)} n\Delta t) \sum_{h=H/2}^{H/2} S_k(\omega_h) \exp(j\omega_h \tau_k^{(p)}) \exp(j\omega_h n\Delta t) \quad (45)$$

Let us define the normalized time delay $\tau_{k,norm}^{(p)}$ [4] as

$$\tau_{k,norm}^{(p)} = \frac{\tau_k^{(p)}}{\Delta t} = \frac{\tau_k^{(p)} \Delta\omega_{BW}}{2\pi} \quad (46)$$

Then it follows [4]

$$f_k^{(p)}(n) = \alpha_k^{(p)} \exp(j2\pi \frac{\omega_c}{\Delta\omega_{BW}} \tau_{k,norm}^{(p)}) \cdot$$
$$\cdot \exp[j2\pi(\frac{\omega_{ck} - \omega_c}{\Delta\omega_{BW}}) n] \exp(j2\pi \frac{\omega_{Dk}^{(p)}}{\Delta\omega_{BW}} n) \cdot$$
$$\cdot \sum_{h=-H/2}^{H/2} S_k(\omega_h) \exp(j2 \frac{\omega_h}{\Delta\omega_{BW}} \pi\Omega_h \tau_{k,norm}^{(p)}) \exp(j2\pi \frac{\omega_h}{\Delta\omega_{BW}} n)$$
$$(47)$$

Since $\omega_h = h\frac{\Delta\omega_{BW}}{H}$; $h = -H/2, H/2$ and $\Omega_h = \frac{\omega_h}{\Delta\omega_{BW}} = = \frac{h}{H}$, it follows [4]

$$f_k^{(p)}(n) = \alpha_k^{(p)} \exp(j2\pi \frac{\omega_c}{\Delta\omega_{BW}} \tau_{k,norm}^{(p)}) \cdot$$
$$\cdot \exp[j2\pi\Omega_{ck} n] \exp(j2\Omega_{Dk\ h}^{(p)} n) \cdot \quad (48)$$
$$\cdot \sum_{h=-H/2}^{H/2} S_k(\Omega_h) \exp(j2\pi\Omega_h \tau_{k,norm}^{(t)}) \exp(j2\pi\Omega_h n)$$

The $\Omega_{Dk\ h}^{(p)} = \frac{\omega_{Dk}^{(p)}}{\Delta\omega_{BW}}$ represents the normalized Doppler shift of the p-th replica of the k-th radio signal.

From the point of view of the spatial modeling of signal superposition under multipath conditions, each replica has its own direction of arrival to the antenna array which is represented by the unit vector $\mathbf{v}_k^{(p)}$.

The superposed radio signal on the l-th antenna which is the result of superposition of K radio signals (every signal is a result of superposition of its R_k replicas) can be expressed in the following form [4]

$$f_l(n) = \sum_{k=1}^{K} \sum_{p=1}^{R_p(k)} f_k^{(p)}(n) =$$
$$= \sum_{k=1}^{K} \sum_{p=1}^{R_p(k)} \sum_{h=-H/2}^{H/2} F_k^{(p)}(\Omega_h) \exp[j2\pi(\frac{\omega_c}{\omega_A} + \frac{\omega_h}{\omega_A}) \mathbf{v}_k^T (\frac{\mathbf{r}_l}{\lambda_A})] \quad (49)$$

If we include the noise of the antenna elements, the signal on the l-th antenna can be expressed as

$$x_l(n) = f_l(n) + n_l(n) \quad (50)$$

The vector of the signal under multipath conditions on the antenna array $\mathbf{x}(n) = [x_1(n) \ \ x_2(n) \ \ ... \ \ x_L(n)]^T$ can be expressed in the matrix form as follows [4]

$$\mathbf{x}(n) =$$
$$= \sum_{h=-H/2}^{H/2} [\mathbf{A}(\omega_c + \omega_h)\mathbf{F}(\Omega_h) + \mathbf{N}(\Omega_h)] \exp(j2\pi\omega_h n\Delta t) =$$
$$= \sum_{h=-H/2}^{H/2} [\mathbf{A}(\omega_c + \omega_h)\mathbf{F}(\Omega_h) + \mathbf{N}(\Omega_h)] \exp(h\frac{j2\pi}{H} n) = \quad (51)$$
$$= \sum_{h=-H/2}^{H/2} [\mathbf{A}(\omega_c + \omega_h)\mathbf{F}(\Omega_h) + \mathbf{N}(\Omega_h)] \exp(j2\pi\Omega_h n)$$

The generalized model of radio signal superposition under multipath conditions can be expressed in the frequency domain in the matrix notation as [4]

$$\mathbf{X}(\Omega_h) = \mathbf{A}(\omega_c, \omega_h)\mathbf{F}(\Omega_h) + \mathbf{N}(\Omega_h) \quad (52)$$

It can be concluded that the structure of the above model is almost the same as the structure of the mathematical model of superposition of uncorrelated signal scenario, defined in the previous Chapter.

The difference is in the fact that the dimensionalities of the matrix $\mathbf{A}(\omega_c, \omega_h)$ and the vector $\mathbf{F}(\Omega_h)$ under multipath conditions are larger than their dimensionalities in the modeling of superposition of uncorrelated signals.

In the above expressions,
$\mathbf{X}(\Omega_h) = [X_1(\Omega_h) \ X_2(\Omega_h) .. X_L(\Omega_L)]^T \in C^{Lx1}$ represents the vector of spectral components of the signal on the antenna array.

The $\mathbf{F}(\Omega_h) = [F_1^{(1)}(\Omega_h) \ F_1^{(2)}(\Omega_h) \ F_1^{(R_1)}(\Omega_h) ... F_K^{(1)}(\Omega_h)$ $F_K^{(2)}(\Omega_h) .. \ F_K^{(R_K)} \ F_K^{(R_K)}(\Omega_h)] \in C^{\sum_{p=1}^{K} R_K x1}$ vector is the vector of the spectral components of the shifted complex envelopes of superposed radio signals. The dimension of this vector is equal to $(\sum_{k=1}^{K} R_k x1)$.

The spectrum of the shifted complex envelope of the p-th replica of the k-th radio signal in the referent point in space $F_k^{(p)}(\Omega_h)$ can be calculated by the Fourier transform of the shifted complex envelope $f_k^{(p)}(n)$ and it can be expressed as [4]

$$F_k^{(p)}(\Omega_h) = \sum_{n=1}^{N} f_k^{(p)}(n) \exp(-j2\pi\Omega_h n) =$$
$$\sum_{n=1}^{N} [\alpha_k^{(p)} \exp(j2\pi \frac{\omega_c}{\Delta\omega_{BW}} \tau_{k,norm}^{(p)}) \exp[j2\pi\Omega_{ck} n] \exp(j2\Omega_{Dk\ h}^{(p)} n) \cdot$$
$$\sum_{h=-H/2}^{H/2} S_k(\Omega_h) \exp(j2\pi\Omega_h \tau_{k,norm}^{(t)}) \exp(j2\pi\Omega_h n)] \exp(-j2\pi\Omega_h n)$$
$$(53)$$

The spectrum $F_k^{(p)}(\Omega_h)$ of the shifted complex envelope of the p-th replica of the k-th radio signal can be expressed as a function of the spectrum of the complex envelope $S_k(\Omega_h)$ as [4]

$$F_k^{(p)}(\Omega_h) = S_k(\Omega_h - \Omega_{ck} - \Omega_{Dk}^{(p)}) \exp(j\frac{\omega_c}{\Delta\omega_{BW}} \tau_{k,norm}^{(p)}) \quad (54)$$

The vector $\mathbf{N}(\Omega_h) = [N_1(\Omega_h) \ N_2(\Omega_h) .. N_K(\Omega_h)]^T$ is a vector of the spectral components of the noise on the antenna arrays.

The previous model enables the modeling of a general multiple incident signal scenario when K uncorrelated signals and R_k multipath replicas of each uncorrelated radio signal are superposed on the antenna array.

The matrix $\mathbf{A}(\omega_c,\omega_n)$ has the dimension of $Lx\sum_{k=1}^{K} R_k$. The columns of this matrix are the steering vectors of superposed radio signals and their replicas, which can be expressed as [4]

$$\begin{aligned}\mathbf{a}(\omega_c,\omega_h) &= \left[\exp\left[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^{(p)^T}\mathbf{r}_1}{c}\right]\cdot\right.\\ &\left.\cdot\exp\left[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^{(p)^T}\mathbf{r}_2}{c}\right]...\exp\left[j(\omega_c+\omega_h)\frac{\mathbf{v}_k^{(p)^T}\mathbf{r}_L}{c}\right]\right]^T = \\ &=\left[\exp\left[j2\pi\left(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A}\right)\mathbf{v}_k^{(p)^T}\left(\frac{\mathbf{r}_1}{\lambda_A}\right)\right]\cdots\right.\\ &\cdot\exp\left[j2\pi\left(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A}\right)\mathbf{v}_k^{(p)^T}\left(\frac{\mathbf{r}_2}{\lambda_A}\right)\right]..\\ &\left.\cdot\exp\left[j2\pi\left(\frac{\omega_c}{\omega_A}+\frac{\omega_h}{\omega_A}\right)\mathbf{v}_k^{(p)^T}\left(\frac{\mathbf{r}_L}{\lambda_A}\right)\right]\right]^T\end{aligned} \tag{55}$$

where $\mathbf{v}_k^{(p)}$ is the unit vector that represents the arrival direction of the p-th replica of the k-th signal.

In the generalized spatial model of radio signal superposition on the antenna array in the given frequency sub-band given by eqs.(37), (38), (51) and (52) all the parameters of the model are normalized so the model is generally applicable to the modeling of signal superposition in any frequency sub-band. The properties of particular frequency sub-bands are specified by the values of the normalized parameters of the model.

In the formulated and presented generalized spatial model of radio signal superposition superposed radio signals can have different carrier frequencies, different spectral bandwidths and other parameters of interest. This mathematical model also describes the situation when narrow-band and wide-band signals are superposed at the same time on the antenna array in the given frequency sub-band.

It is important to notice that there is a distinction between the spatial model of wide-band signal superposition described earlier in the paper and the generalized model formulated here. In the spatial model of wide-band signal superposition it is assumed that superposed wide-band radio signals have the same carrier frequencies and spectral bandwidths which are already known. In the generalized model of signal superposition it is assumed that narrow-band and wide-band signals with different but unknown carrier frequencies and spectral bandwidths are superposed on the antenna array in the given frequency sub-band. In the process of spatio-frequency signal analysis these unknown parameters have to be estimated.

It can be noticed that the signals in the referent point in space can be described in the time or frequency domain, but the transformation of the signal from the referent point in space to the signal on the antenna elements is performed in the frequency domain.

Since the spectrum of the complex envelope of incoming radio signals is the input parameter to the generalized model, it can be concluded that the proposed model can be applied to the spatial modeling of multiuser FDM, TDM and CDMA signals.

Examples of numerical modeling

Two examples of spatial modeling of radio signal superposition on the antenna array in the given frequency sub-band are presented in this paper.

In the first example, the antenna array is a non-uniform volume with 10 antenna elements (Fig.5). The characteristic frequency of the antenna array f_A is 80 MHz. The antenna aperture is approximately equal to $6\lambda_A$. The normalized frequency sub-band $\Delta f_{BW} / f_A$ is equal to 100/80000 and the normalized central frequency of the frequency sub-band f_c / f_A is equal to 70/80. (in a real situation it is related to the frequency sub-band of 100 kHz at the central frequency of 70 MHz, or to the frequency sub-band of 1 MHz at the central frequency of 700 MHz).

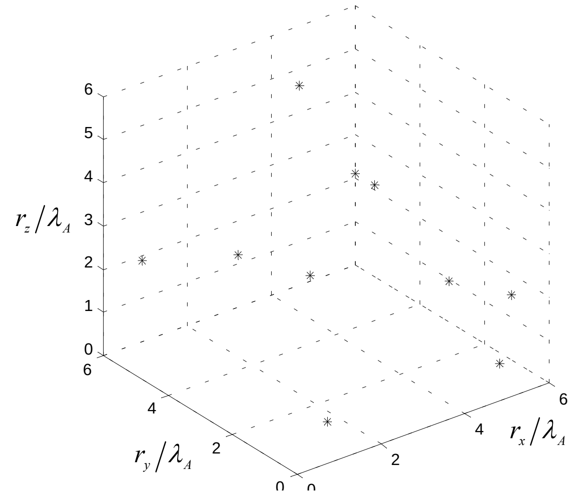

Figure 5. The antenna array geometry

Four radio signals (K=4) are superposed on the antenna array with the azimuths $\theta_1=120^\circ, \theta_2=130^\circ$, $\theta_3=-45^\circ$, $\theta_4=-150^\circ$ and the elevations of arrivals $\varphi_1=0^\circ, \varphi_2=45^\circ$, $\varphi_3=30^\circ$, $\varphi_4=20^\circ$. The first and the second radio signal are unmodulated carriers with the normalized frequencies -0.3 and 0.001. The third and the fourth signal have the normalized spectral bandwidths equal to 0.5 with the normalized central frequencies of 0 and 0.5. The simulated signals partially overlap in the frequency domain and fully overlap in the time domain.

The signal to noise ratio is 10 dB for the first and second signal, 15 dB for the third and fourth signal. The number of complex signal samples in the observation interval ΔT is N=4096.

The power spectral density of the superposed signal in the referent antenna is presented in Fig.6. The spatio-time samples of the superposed signal on the antenna array are presented in Fig.7.

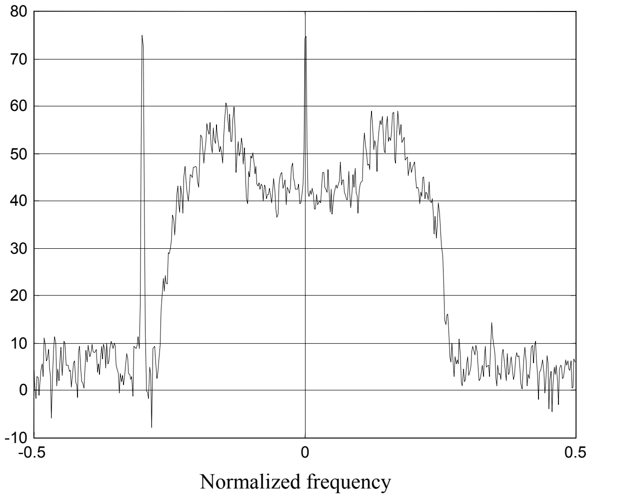

Figure 6. The power spectral density of the signal on the referent antenna

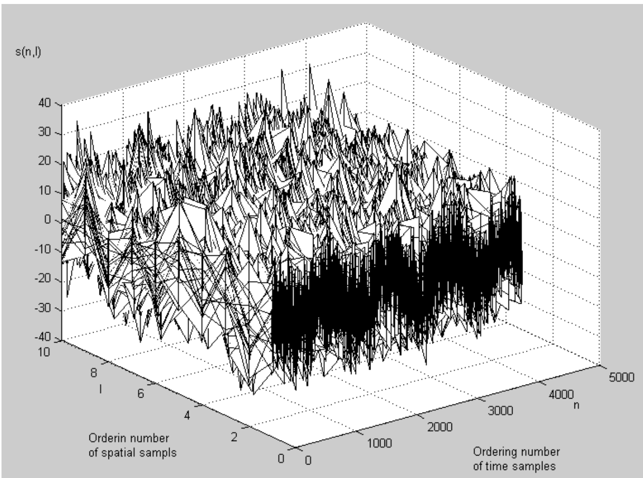

Figure 7. Spatio-time samples of the baseband signal on the antenna array

A multiple incident signal scenario under multipath conditions was simulated in the second example. The antenna array is linear with L=10 equidistant omnidirectional antenna elements. The normalized bandwidth of the selected frequency sub-band $\Delta f_{BW}/f_A$ is 1/30000 and the normalized central frequency f_c/f_A of the selected frequency sub-band is equal to 2/3 (it is related to the spectral bandwidth of 1 kHz at the central frequency of 20 MHz). The number of complex time samples in the observation interval ΔT is equal to $N = 8192$. Two replicas of the sinusoidal signal with the relative normalized Doppler shift $\Delta\Omega_{D1}^{(1,2)} = 0.0000001$ are superposed. In this case it is 0.0001 Hz, which is less than the reciprocal value of the observation interval, so the replicas of the simusoids cannot be resolved by the FFT in the spectral domain . It was supposed that the relative normalized time delay $\Delta\tau_{1norm}^{(1,2)}$ of the paths is equal to zero. The paths arrive on the antenna array from the azimuths of $\theta_1 = 5^0, \theta_2 = 15^0$ and the elevations of $\varphi_1 = \varphi_2 = 0^0$. The signal to noise ratio is the same for both replicas and is equal to 20 dB. The amplitude spectrum of the superposed signal on the referent antenna, calculated by the FFT algorithm, is presented in Fig.8. As it can be seen, the peaks from the replicas cannot be identified in the spectrum.

The first 10 x 256 spatio-time samples of the signal on the antenna array are presented in Fig.9. As it can be seen in Fig.9, the existence of a relative Doppler shift among the paths has a fading of the supposed signal as a consequence, both in time and space.

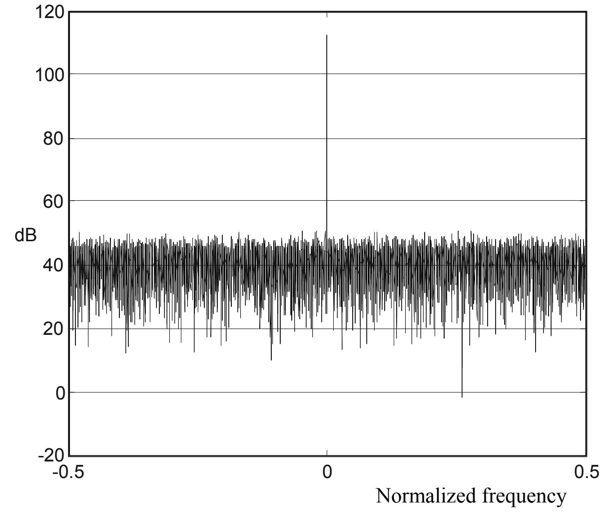

Figure 8. Amplitude spectrum of the baseband signal calculated by the FFT

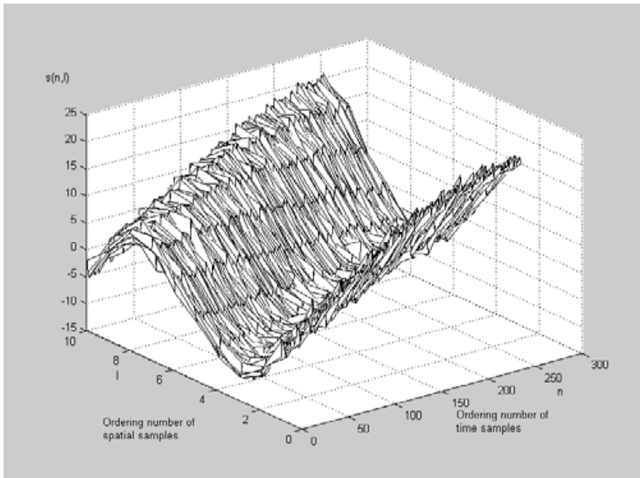

Figure 9. Spatio-time samples of the baseband signal on the antenna array

Conclusion

The generalized spatial mathematical model of the radio signal superposition on the antenna array of arbitrary geometry in a given frequency sub-band is formulated and presented in this paper. It enables the modeling of simultaneous superposition of narrow-band and wide-band signals with different carrier frequencies, different spectral bandwidths, signal to noise ratios and other parameters of interest. All the parameters of this model are normalized so that the presented model can be generally applied to the simulation of signal superposition in any frequency sub-band on an antenna array of arbitrary geometry.

The proposed generalized spatial model of signal superposition is formulated in the research into the methods of automatic radio-frequency spectrum monitoring based on the spatio-time-frequency analysis of superposed radio signals in a given frequency sub-band. The problem of the spatio-time-frequency analysis can be formulated as a problem of unknown parameter estimation of the generalized model of signal superposition. It is based on the available *LxN* spatio-time samples of superposed radio signals in a given frequency sub-band on the antenna array of arbitrary, already known geometry. The unknown parameters of the generalized model are: the number of superposed radio signals K, the carrier frequencies $\{\omega_{ck}\}, k = 1,..K$, the spectral bandwidths $\Delta\omega_{BWk}, k = 1,..,K$, and the direction of arrivals $\{\theta_k, \varphi_k\}; k = 1, K.$ of the superposed radio signals. It is also important to notice that the parameter estimation is performed in the space of normalized parameters.

The proposed normalized spatial model of signal superposition, presented in this paper, was the theoretical basis for the development of technical solutions of the direction finder RGK-2/3 [7]. It was verified in practice, by testing the performances of the RGK-2/3 in real conditions.

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Received: 8.7.2003

Generalizovani model superpozicije radio signala u zadatom frekvencijskom podopsegu na antenskom nizu proizvoljne geometrije

U radu je izložen generalizovani matematički model superpozicije više radio signala u zadatom frekvencijskom opsegu na antenskom nizu proizvoljne, prostorne neuniformne geometrije. Svi parametri generalizovanog modela su normalizovani tako da je model bez ograničenja primenljiv za različite frekvencijske opsege kao i za različite koncepte raspodele korisničkih kanala (TDMA, FDMA i CDMA). Model takođe omogućava modeliranje superpozicije radio signala u uslovima višestrukog prostiranja.

Ključne reči: **elektronsko izviđanje, antenski nizovi, superpozicija radio signala, višestruko prostiranje, prostorno-frekvencijsko-vremenska analiza radio signala.**

Modèle généralisé de la superposition des signaux radio dans une sous-bande de fréquence préalablement fixée sur le réseau d'antennes à géométrie arbitraire

Cet article donne un modèle mathématique généralisé de la superposition de plusieurs signaux radio dans une bande de fréquence préalablement fixée sur le réseau d'antennes à géométrie arbitraire, spatiale et non-uniforme. Tous les paramètres du modèle généralisé sont normalisés de facon à ce que le modèle puisse être appliqué sans restrictions pour les bandes de fréquence différentes et pour les conceptions différentes concernant la distributions des voies d'accès (TDMA, FDMA et CDMA). Le modèle peut être aussi utilisé pour la modellisation de la superposition des signaux radio en cas de la transmission par les voies multiples.

Mots-clés: **reconnaissance électronique, réseaux d'antennes, superposition des signaux radio, multivoie, analyse espace-temps-fréquence.**