UDC: 681.58:007.52(047)=20
COSATI: 14-07, 14-02, 09-03

One method of electromechanical actuator parameters identification

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This paper shows an engineering method of electromechanical actuator parameters identification. Descriptions of each of four identification procedures is given, as well as the comparison of the measured system responses with those obtained through simulation, for parameters determined by identification. Descriptions of experiments necessary for the identification procedure, and the experiment results for one actuator, are given as well.

Key words: **electromechanical actuator, system parameters identification, DC motor.**

Used marks and symbols

i – reducer speed ratio
I_m – armature current, A
L_m – armature inductivity, H
R_m – armature resistance, Ω
U_m – motor supply voltage, V
E_m – back electromotive force, V
J_m – moment of inertia of the DC motor rotor, kgm^2
J_g – total moment of inertia of the actuator, kgm^2
J_f – moment of inertia of the reducer, kgm^2
M_m – DC motor torque, Nm
M_a – active torque of the actuator, Nm
M_f – friction torque in motion, Nm
M_{f0} – static friction torque, Nm
$M_{f\omega}$ – speed-related component of the friction torque, Nm
$K_{f\omega}$ – speed-friction torque relation coefficient, Nm/rad
K_M – motor-torque constant, Nm/A
K_e – generator voltage constant, Vs/rad
T_e – electrical time constant, s
T_m – electromechanical time constant, s
ω_m – DC-motor rotation speed, rad/s
$\dot{\delta}$ – output shaft rotation speed, rad/s
η_G – efficiency factor of the reducer.

Introduction

ELECTROMECHANICAL servo systems are frequently used in all fields of technology. A problem of parameters identification often exists as a start point in control system synthesis. This is very significant in cases when existing servo systems are to be modernized, or when servo systems are composed of components which do not have reliable catalog data. In these cases, identification is the only way to obtain system parameters, which are to be used in compensator linear and nonlinear analysis and synthesis.

The main point of this identification procedure is a direct obtaining of parameters, one by one, where each parameter is obtained by a separate experiment. Comparing it with other identification procedures (identification of impulse, step response etc.), the advantage is that each parameter is obtained separately, from its own experiment, which makes the possibility of an error lower than in case of getting a conjoint value of a group of parameters from a single experiment. The disadvantages are the need for a number of experiments and larger work in measured data processing. There is also a limitation in electromechanical time constant and moment of inertia obtaining: it is necessary that the electrical time constant is negligibly small in comparison with the electromechanical constant, i.e.: $\frac{T_m}{T_e} \geq 100$.

This identification procedure comes from a presumption that the structure of a mathematical model of an electromechanical actuator is already known, and we only need to determine its parameters. For this purpose, two types of experiments are performed: with a stalled motor, and with a free, unloaded motor.

For every step of identification, as an example, the results of measurements performed are given for a rocket fins actuator consisting of a DC motor Maxon A-max 205071 and a reducer combined from a planetary reducer Maxon A-max 110859 with the speed ratio of 27, and a single stage cylindrical gear pair with speed ratio of 7.5. Total speed ratio of the reducer is 202. The nominal supply voltage for the motor is $U_m = 24\,V$.

Mathematical model of an electromechanical actuator

Fig.1 shows the block diagram of an electromechanical fin servo actuator.

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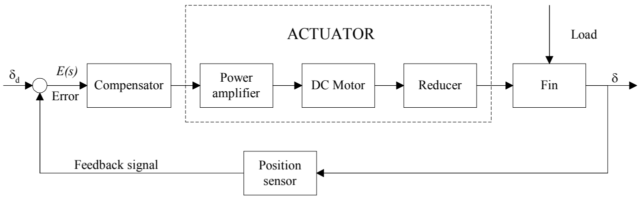

Figure 1. Servo actuator with a position feedback

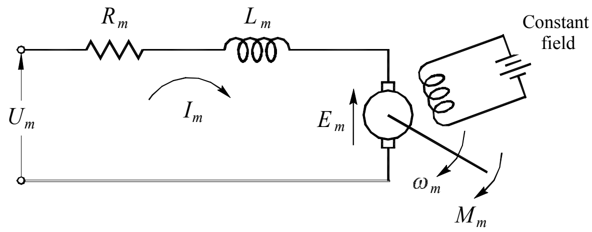

Figure 2. DC motor electric circuit

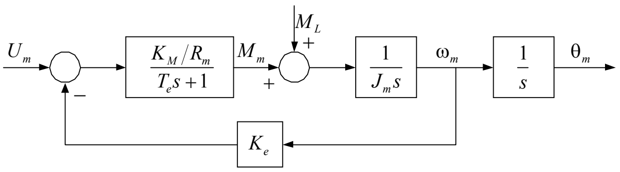

Figure 3. DC motor block diagram

The basic part of an electromechanical actuator is the DC motor, Fig.2. A complete mathematical model of the DC motor can be found in the literature [2], while only a brief presentation will be given here.

The DC motor torque is proportional to the magnitude of the exciting magnetic flux, and, therefore, to the armature current I_m

$$M_m = K_M I_m \tag{1}$$

where K_M is the motor-torque constant. The rotation of the motor armature develops a back electromotive force E_m. The back electromotive force is proportional to the speed of operation ω_m

$$E_m = K_e \omega_m \tag{2}$$

where K_e is the generator voltage constant.

The motor torque constant - K_M and the generator voltage constant are equal for an ideal motor. For real motors, they are slightly different.

The voltage equation describing the armature circuit is

$$U_m = R_m I_m + L_m \frac{dI_m}{dt} + E_m \tag{3}$$

Combining (1), (2) and (3), and applying the Laplace transform, the block diagram of the DC motor (Fig.3) is obtained, where

$$T_e = \frac{L_m}{R_m} \tag{4}$$

The motor is connected with an executive element by a reducer. The mathematical model of the reducer is

$$\dot{\delta} = \frac{\omega_m}{i} \tag{5}$$

where i is the reducer speed ratio. The total active torque of the actuator is equal to the motor torque lowered by losses in the system (friction).

$$M_a = \eta_G \cdot i \cdot M_m - M_f \operatorname{sgn}(\omega_m) \tag{6}$$

Considering an unloaded system (for the identification purpose) we find that the active torque is balanced only by the inertial load.

$$J_h \frac{d\dot{\delta}}{dt} = M_a \tag{7}$$

where J_h is the moment of inertia of the actuator, from the aspect of the output shaft.

$$J_h = J_g + i^2 J_m \tag{8}$$

The identification procedure determines the following parameters:

- $K_M\eta_G$ – multiplication of the motor-torque constant and efficiency factor of the reducer (which cannot be separated),
- K_e – generator voltage constant,
- R_m – armature resistance,
- L_m – armature inductivity,
- M_f – friction torque,
- J_h – total moment of inertia.

Determination of R_m, M_{f0} and $K_M\eta_G$

This experiment deals with a stalled motor. The active torque and the motor current are measured for various values of the motor supply voltage. The torque on the output shaft is

$$M_a = K_M \cdot \eta_G \cdot i \cdot I_m \tag{9}$$

and the torque we measure

$$M = M_a - M_{f0} = K_M \eta_G i \cdot I_m - M_{f0} \tag{10}$$

The torque can be measured with a torque transducer, or, in this case, with a force transducer shifted from the rotation axis for some distance.

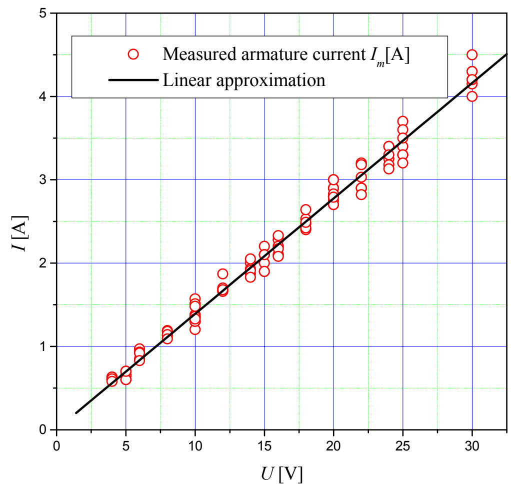

Figure 4. Measured values of the armature current

Fig.4 shows the measurement of the motor current. The obtained values are approximated with a straight line running through the coordinate system origin. The gradient of this line can be numerically obtained by the linear regression method and it represents the value of the motor armature resistance. It is obvious from Fig.4, that the measured values of the motor armature resistance have a vast dispersion.

The armature resistance changes with heating. It also has a stochastic behavior, due to the fact that the contact resistance between the brushes and the rotor is a function of their position. The existence of the contact resistance makes measuring of resistance with an ohmmeter impossible.

Applying the linear regression method on the measured data, we get

$$U_m = 7.2011 \cdot I_m; \quad R_m = 7.2011\Omega \tag{11}$$

Fig.5 shows the actuator torque depending on the motor current. This dependence is described in eq.(9).

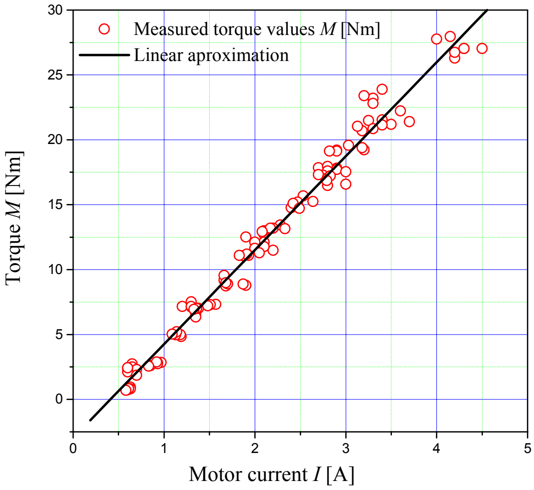

Figure 5. Measured torque

The measured torque values can be approximated with a straight line, running off the origin of the coordinate system. The parameters of the straight line can be numerically determined by the least squares method

$$M = aI_m + b \tag{12}$$

The comparison of eq.(9) and (12) gives the following parameters

$$K_M \eta_G = \frac{a}{i}; \quad M_{f0} = -b \tag{13}$$

In this case

$$K_M \eta_G = 0.03584 \frac{Nm}{A} \tag{14}$$

$$M_{f0} = 2.973 Nm \tag{15}$$

The measurement of electromechanical actuator characteristics, which implies the DC motor connected to the gear reducer, does not enable obtaining separate values for the parameters K_M and η_G, because the active actuator torque is measured behind the reducer. The exact value of the motor-torque constant could be obtained if the reducer efficiency factor were already known or measured.

The dissipation of the measured values, present due to the stochastic nature of the contact resistance between the motor brushes and the rotor, can be compensated by a large number of data. The influence of temperature can be reduced by a longer time period between successive experiments.

Determination of the electrical time constant of the DC motor T_e

Eq. (3) gives

$$L_m \frac{dI_m}{dt} = U_m - R_m I_m \tag{16}$$

with a presupposition that the transient process in the motor armature is ended before it gets a significant rotation speed, so that the back electromotive force $E_m = K_e\omega$ is negligible. This assumption is valid in most of the cases, because the experiment is performed with a stalled motor. Even when it is not possible to immobilize the rotor (gear reducer with a significant backlash), if the ratio $\frac{T_m}{T_e} \geq 100$ (see Chapter 6), at the end of the armature circuit transient process, when the motor current reaches its maximum value, the back electromotive force is up to 2% of the supply voltage, its maximum value.

The solution of eq.(16) in the time domain is

$$I_m(t) = \frac{U_m}{R_m}(1 - e^{-\frac{R_m}{L_m}t}) = \frac{U_m}{R_m}(1 - e^{-\frac{t}{T_e}}) \tag{17}$$

Introducing

$$b = \frac{U_m}{R_m}, \quad a = \frac{R_m}{L_m} = \frac{1}{T_e} \tag{18}$$

eq. (17) becomes

$$b - I_m(t) = be^{-at} \tag{19}$$

Applying the logarithm to eq.(19), we get

$$\ln(b - I_m) = \ln b - at \tag{20}$$

The coefficients a and b of the linear regression are obtained by the least squares method.

Fig.6 shows the measured transient process of the actuator system in its initial part, where the transient process in the motor armature dominates. Fig.7 represents eq.20. The procedure is repeated several times for various values of the supply voltage, and each time the pairs of parameters a and b are obtained. From them, we have

$$T_e = \frac{L_m}{R_m} = \frac{1}{a} \tag{21}$$

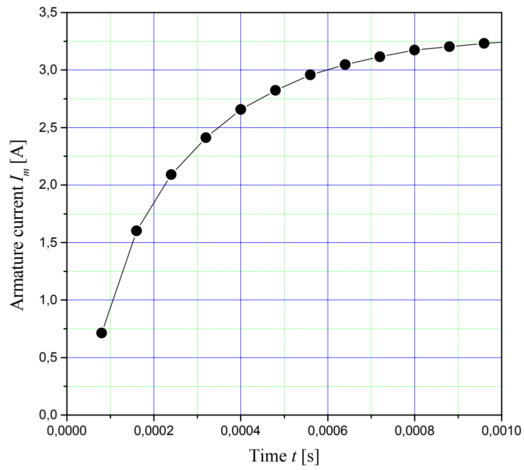

Figure 6. Experiment: transient process in the motor armature

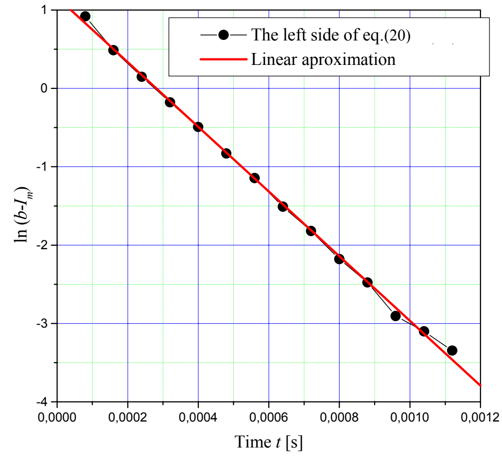

Figure 7. Equation (20)

For this experiment, it is necessary to provide a data acquisition system for the armature current with the sampling frequency of at least 10 KHz. Only the measured data from the initial part of the transient process are used for the analysis.

In this case,

$$T_e = 2.46\times10^{-4}\,\mathrm{s} \tag{22}$$

$$L_m = 0.001844\ \mathrm{H} \tag{23}$$

Determination of the parameters K_e and M_f

Considering eq.(3) after the ending of the transient process, we have $\frac{dI_m}{dt}=0$, if the influence of noise is neglected. This will be fulfilled if we consider average values of the parameters in a longer period of time (0.1 s and more). Applying the constant supply voltage U_m to the DC motor, the system reaches its maximum speed in 0.1 s, so that the interval of mediation can be $t\in(0.12, 0.22)$ s. Fig.8 shows parallel changes of the armature current and the output shaft rotation speed. Eq.(3) becomes

$$U_m = R_m\bar{I}_m + K_e\bar{\omega}_m \tag{24}$$

$$K_e = \frac{1}{\bar{\omega}_m}(U_m - R_m\bar{I}_m) \tag{25}$$

where the parameters with the upper-lined marks are average values of adequate parameters over the interval of mediation.

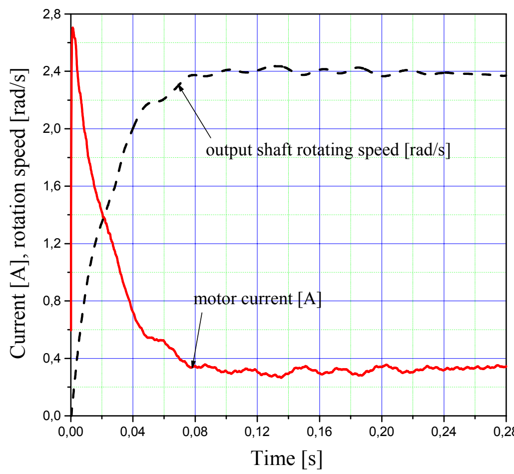

Figure 8. Motor current and rotation speed

This identification procedure uses experiments with a free rotating (unloaded) motor. Duration of the experiments must be at least $0.1s$ longer than the time necessary for the motor to reach its maximum speed. Several experiments are carried out for various values of the motor supply voltage.

The final value of the parameter K_e we get as an arithmetic median of the values determined in each experiment.

At a constant speed, the DC motor produces the torque which is completely used for overcoming the motion resistances, i.e. friction. This is, actually, the value of the friction torque in the motion, M_f, while the static friction torque is M_{f0}. As well as K_e, it can be calculated using the average values of the motor current.

$$M_f = K_M i\eta_G\cdot\bar{I}_m \tag{26}$$

As seen in Fig.9, it is possible to determine a linear dependence of the friction torque and the motor rotation speed

$$M_f = M_{f\omega} + K_{M\omega}\cdot\omega_m \tag{27}$$

Approximating the measured results with a straight line

$$M_f = a + b\cdot\omega_m \tag{28}$$

we get

$$M_{f\omega} = a,\ K_{M\omega} = b \tag{29}$$

where a and b are obtained by the least squares method. In this case

$$K_e = 0.0363\,\mathrm{Vs} \tag{30}$$

$$K_{M\omega} = 0.01397\frac{\mathrm{Nms}}{\mathrm{rad}} \tag{31}$$

$$M_{f\omega} = 3.9601\,\mathrm{Nm} \tag{32}$$

$$M_f = 3.9601 + 0.01397\omega_m \tag{33}$$

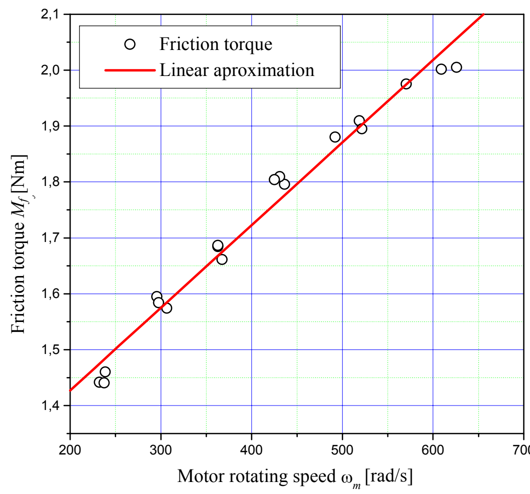

Figure 9. Dependence of the friction torque and the motor rotating speed

Determination of electromechanical time constant T_m and the total moment of inertia J_h

Starting with equations

$$U_m = R_mI_m + L_m\frac{dI_m}{dt} + K_e\omega_m \tag{34}$$

$$M_a = K_M i\eta_G I_m = J_h\frac{d\dot{\delta}}{dt} + M_f = \frac{J_h}{i}\frac{d\omega_m}{dt} + M_f \tag{35}$$

we get the following differential equation

$$T_e T_m \frac{d^2\omega_m}{dt^2} + T_m \frac{d\omega_m}{dt} + \omega_m = \frac{U_m}{K_e} - \omega_f \tag{36}$$

where

$$T_e = \frac{L_m}{R_m},\ \omega_f = \frac{R_m M_f}{K_e K_M i\eta_G} \tag{37}$$

$$T_m = \frac{R_m J_h}{K_e K_M i^2 \eta_G} \tag{38}$$

J_h is the total moment of inertia with respect to the output shaft. The parameter ω_f depends on the friction torque and represents its influence on the maximum rotation speed reducing.

The solution of differential eq.(36), under boundary conditions $\omega_m(0) = 0$ and $\frac{d\omega_m(0)}{dt} = 0$ is

$$\omega_m(t) = \frac{U_m - \omega_f K_e}{K_e} \cdot \left(1 - \frac{1}{2}\frac{\sqrt{T_m^2 - 4T_mT_e} + T_m}{\sqrt{T_m^2 - 4T_mT_e}} e^{\frac{-T_m + \sqrt{T_m^2 - 4T_mT_e}}{2T_mT_e}t} + \frac{1}{2}\frac{T_m - \sqrt{T_m^2 - 4T_mT_e}}{\sqrt{T_m^2 - 4T_mT_e}} e^{\frac{-T_m - \sqrt{T_m^2 - 4T_mT_e}}{2T_mT_e}t} \right) \tag{39}$$

Eq.(39) needs to be simplified. In order to make this possible, and the entire identification procedure applicable, it is necessary that $T_m \gg T_e$, i.e.

$$\frac{T_m}{T_e} \geq 100 \tag{40}$$

In order to predetermine the value of the electromechanical time constant T_m, we can use a preliminary value of the moment of inertia (from the catalogue, if available), or read the constant value directly from the rotation speed diagram (Fig. 8 or 10).

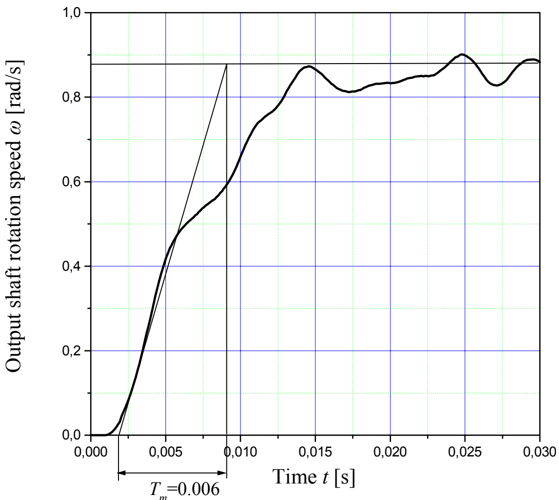

Figure 10. Preliminary determination of the electromechanical time constant

The electromechanical time constant represents the period of time in which the DC motor reaches its maximum speed if it runs with its maximum acceleration

$$\alpha_{\max} = \frac{M_{m,a}}{J_m} = \frac{K_M I_m \eta_G}{\frac{J_h}{i^2}} = \frac{K_M \eta_G i^2 U_m}{J_h R_m} \tag{41}$$

The maximum rotation speed that the DC motor would achieve if there were no friction is

$$\omega_{\max} = \frac{U_m}{K_e} \tag{42}$$

This makes

$$T_m = \frac{\omega_{\max}}{\alpha_{\max}} = \frac{J_h R_m}{K_e K_M i^2 \eta_G} \tag{43}$$

In the rotation speed–time diagram, the constant is visible as a fragment of the time axis formed by the tangent to the speed with the largest slope.

Fig.10 shows the preliminary determination of the electromechanical time constant for one actuator, for which the identification procedure is not applicable. For this actuator the ratio $\frac{T_m}{T_e} \approx 5$.

In our case

$$T_e = \frac{L_m}{R_m} = 2.46 \cdot 10^{-4} \text{ and } T_m \approx 2.39 \cdot 10^{-2}$$
$$\text{that is, } T_m = 97.2 \cdot T_e \text{, i.e. } T_m \approx 100 T_e \tag{44}$$

which fulfills the procedure applicability condition. Then

$$\sqrt{T_m^2 - 4T_mT_e} \approx 0.98 T_m \tag{45}$$

$$\frac{\sqrt{T_m^2 - 4T_mT_e} + T_m}{\sqrt{T_m^2 - 4T_mT_e}} \approx \frac{1.98T_m}{0.98T_m} \approx 2.02 \tag{46}$$

$$\frac{T_m + \sqrt{T_m^2 - 4T_mT_e}}{2T_mT_e} \approx \frac{0.99}{T_e} \tag{47}$$

$$\frac{-T_m + \sqrt{T_m^2 - 4T_mT_e}}{2T_mT_e} = \frac{1}{2T_e}\left(\sqrt{1 - \frac{4T_e}{T_m}} - 1\right) \approx -\frac{1}{T_m} \tag{48}$$

The last equation is obtained by developing the under root function in Maclaurin's polynomial of the first order and by neglecting the remainder. Now, the solution of differential equation (36) given in (39) can be expressed as

$$\omega_m(t) = \frac{U_m - \omega_f K_e}{K_e}\left[1 - 1.01 e^{-\frac{1}{T_m}t} + \frac{T_e}{0.98T_m} e^{-\frac{0.99}{T_e}t}\right] \text{ or} \tag{49}$$

$$1 - \frac{\omega_m(t) K_e}{U_m - \omega_f K_e} + \frac{T_e}{0.98T_m} e^{-\frac{0.99}{T_e}t} = 1.01 e^{-\frac{1}{T_m}t} \tag{50}$$

Applying the logarithm to eq.(50), we get

$$\ln\left(1-\frac{\omega_m(t)K_e}{U_m-\omega_f K_e}+\frac{T_e}{0.98T_m}e^{-\frac{0.99}{T_e}t}\right)= \quad (51)$$
$$=\ln 1.01-\frac{1}{T_m}t = A + Bt$$

The equation, transformed in this way, can be approximated by a straight line, and the linear regression coefficients A and B are determined by the least squares method.

This identification procedure uses free (unloaded) motor speed measurements, as in the previous procedure. The left side of eq.(51) is calculated for the measured motor speed values, while for computing $\frac{T_e}{0.98T_m}e^{-\frac{0.99}{T_e}t}$, for T_m we use the preliminary obtained value. This is completely valid, because, when (44) is fulfilled, $\frac{T_e}{0.98T_m}\approx 0.0105$, and $e^{-\frac{0.99}{T_e}t}$ rapidly tends to zero, because $T_e \ll T_m$, and its influence is negligible after few measured points.

The values of the electromechanical time constant and the total moment of inertia are obtained from

$$T_m = -\frac{1}{B} \quad (52)$$

$$J_h = \frac{T_m K_e K_M i^2 \eta_G}{R_m} \quad (53)$$

Fig.11 shows the step response of the motor rotation speed in an analysis appropriate time period. Fig.12 shows the left side of eq.(51) calculated for the measured values of the motor rotation speed ω_m at that period of time, as well as the linear approximation of these values.

In this case

$$B = -44.28 \quad (54)$$

$$T_m = -\frac{1}{B} = 0.02258\,\text{s} \quad (55)$$

$$J_h = \frac{T_m K_e K_M i^2 \eta_G}{R_m} = 0.178\ \text{kgm}^2 \quad (56)$$

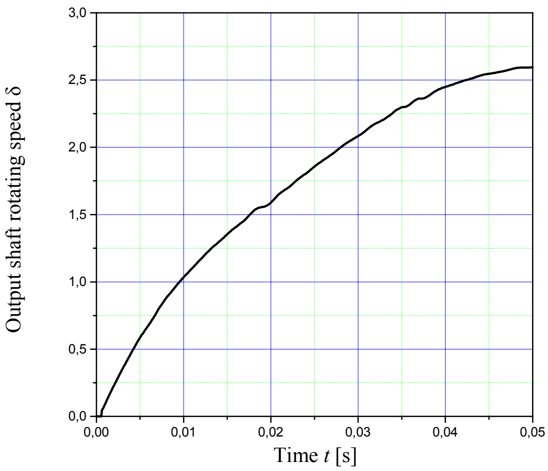

Figure 11. Experiment: Output shaft rotating speed for U_n=24V

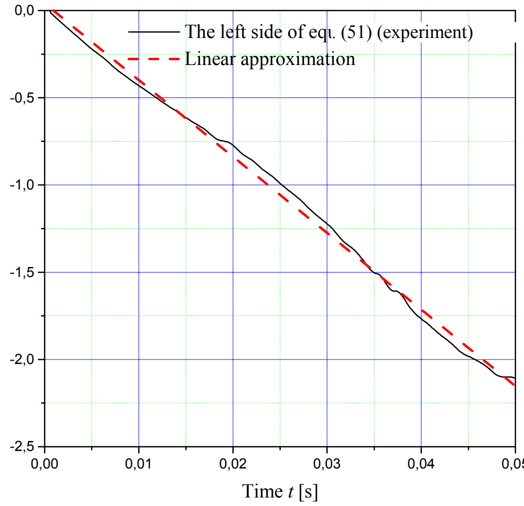

Figure 12. Transformed system response by eq.(51)

The following table contains comparative values of the actuator parameters obtained from the catalogue and by identification.

Table 1. Comparison of identified and catalogue values

Parameter	Mark	Identified value	Catalogue value
Armature resistance, R_m	Ω	7.2011	7.13
Static friction torque M_{f0}	Nm	2.973	-
Multiplication of the motor-torque constant and the efficiency factor $K_M\eta_G$	$\frac{Nm}{A}$	0.03584	0.02865
Armature inductivity, L_m	H	0.001844	0.00105
Electrical time constant, T_e	s	0.000246	0.000147
Generator voltage constant, K_e	Vs	0.0363	0.0389
Friction torque in motion, M_f	Nm	$3.9601+0.01397\omega_m$	
Electromechanical time constant, T_m	s	0.02258	0.02681
Total moment of inertia with respect to the output shaft, J_m	kgm^2	0.178	0.171

Verification of the mathematical model

The described procedure gave us the parameters of the system, with an assumed mathematical model structure. The verification can be performed by including the identified parameters in an electromechanical servo actuator simulation program. The results of the simulation can be compared with the measured ones.

Fig.13 and 14 show the comparative diagrams of the transient processes obtained by measuring and by simulation with identified parameters. They confirm the validity of the mathematical model and the identification procedure.

Fig.13 shows the transient process of the actuator rotation speed i.e. its response to the step function $U_m(t)=24\,\text{V}\cdot h(t)$. It shows the influence of the friction torque, generator voltage and motor-torque constant of the motor and the moment of inertia.

Fig.14 shows the transient process in the DC motor armature, influenced by the armature resistance and inductivity.

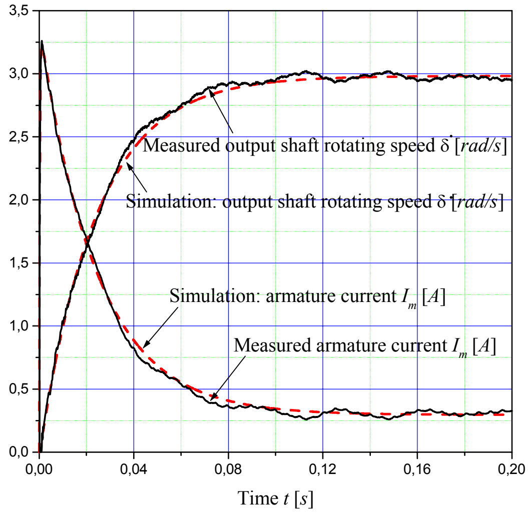

Figure 13. Comparative results at the time interval 0÷0.2 s

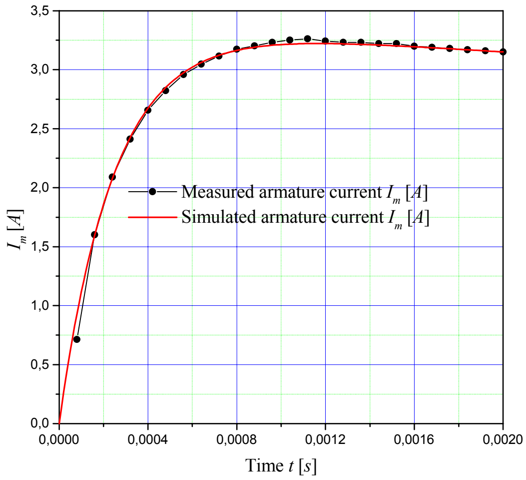

Figure 14. Comparative transient processes in the motor armature

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Received: 3.12.2003

Jedan metod identifikacije parametara elektromehaničkih aktuatora

Prikazan je jedan inženjerski metod identifikacije parametara elektromehaničkih aktuatora. Dat je opis svakog od četiri postupka identifikacije kao i poređenje izmerenih odziva sistema sa odzivima dobijenim simulacijom za parametre određene postupkom identifikacije. Dat je opis eksperimenata potrebnih za identifikaciju, kao i eksperimentalni rezultati za jedan aktuator.

Ključne reči: **elektromehanički aktuator, identifikacija parametara sistema, DC motor.**

Une méthode de l'identification des paramètres des servo-mécanismes électromécaniques

Une méthode d'ingénieurs pour l'identification des paramètres des servo-mécanismes électromécaniques est présentée. Chacun de quatre procédés d'identification est decrit et les reponses mesurés d'un système sont comparées avec les reponses obtenues par la simulation pour les paramètres déterminés par l'identification. Les essais nécessaires pour l'identification sont donnés aussi bien que les résultats expérimentaux pour un servo-mécanisme particulier.

Mots-clés: **servo-mécanisme électromécanique, identification des paramètres du système, moteur de courant continu.**