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Reliability and availability analysis by grouping states of four telecommunication stations connected in a ring

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Models for reliability and availability of a telecommunication system comprising four telecommunication stations connected in a ring by duplex lines are derived. The variants with redundant and unredundant receiver-transmitters are analyzed. The models are developed based on the grouping of possible states of an analyzed system using the Markov models for transition probabilities between system groups of states. The derivation of transition probabilities between groups of states is presented as well as the results of system reliability and availability.

Key words: **reliability, availability, telecommunication system, grouping of states, Markov models.**

Introduction

RELIABILITY and availability are very important features of quality of any technical system, and telecommunication systeTj10.02 0 0 10.02 14 5284 465.4997 Tms as we(l)Tj10.02 0 0 10.021 84.734 465.4997 Tm(l)Tj10.02 0 0 10.02

led using the Markov theory [1,2,3]. In this paper we analyze reliability of a telecommunication system with four telecommunication stations connected in a ring by duplex lines.

Based on these models of reliability and availability, we can compare different variants of the system in order to choose an optimum one. But, a problem can arise when a system has a large number of states, when we must solve a large number of equations, which can be practically impossible. This problem can be surpassed by grouping the states of the system, and solving the relations for such a system, with smaller number of equations. But, now we must get conditional transition probabilities between particular groups of states, by similar procedure, but for a group of states which has a smaller number of states. By this method, in this paper, we have examined the reliability and availability of a telecommunication system comprising four telecommunication stations connected in a ring by duplex lines. The variants with redundant and unredundant telecommunication stations (receiver-transmitters) are analyzed. The models of reliability and availability for a system with unredundant stations have been derived in [4,5] , so far and the models of reliability for a system with redundant stations are shown in [6]. The complex models of availability for the system with redundant stations are presented as well as the results of the analysis of reliability and availability for both variants. d

Examination of the system states

We examine a telecommunication system consisting of four telecommunication stations (transmitter-receiver) connected in a ring by duplex lines (Fig.1). The telecommunication stations are connected by lines directly (for example, station 1 and station 2 by a duplex line) or via a transit station (for example, stations 1 and 4 by duplex lines 1 and 2 and via station 2 as a transit station).

There are other variants of connecting these four stations, for example, by six duplex lines, when communication can be established between stations without transit stations (for example, station 1 communicates directly with station 4 by a duplex line), which can be analyzed and modeled in a similar way.

The goal of examining reliability and availability of different variants is to get an optimum configuration of a system from the cost and reliability $R(t)$ and availability $A(t)$ criteria.

We analyze two variants:

1. system with unredundant stations (variant 1),
2. system where each station is redundant with its own station (variant 2).

Let x_i and $\overline{x}_i$ denote possible states of the i-th line (operating and failed respectively), and y_i and $\overline{y}_i$ are the possible states of the i-th station (i = 1 to 4). The system operates if all four stations are operable and connected. The system fails if at least one station fails or if all station are operable but there is no connection between them.

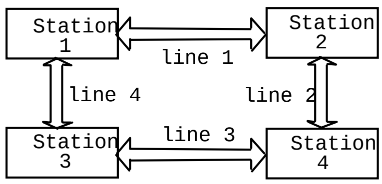

Figure 1. System of four telecommunication stations connected in a ring by duplex lines

Because any of lines and any of four stations can fail,

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this system has a large number of states, and it is not easy to derive a model for reliability and availability. In order to simplify the models, we can group all these states in five groups, called basis states, and the other states considered as substates of the basis states.

The basis states can be the following:

In variant 1 (stations are not redundant):

1. all lines and all stations are operating (state $x_1x_2x_3x_4 \wedge y_1y_2y_3y_4$). No substates. The system is operating.
2. one line failed and all stations are operating. Four substates. The system is operating.
3. all lines are operating and at least one station failed. Fifty substates. The system failed.
4. at least one line failed and at least one station failed. Sixty substates. The system failed.
5. at least two lines are failed. One hundred and seventy-six substates. The system failed.

In variant 2 (stations are redundant):

1. all lines and each station or its redundancy is operating. Sixteen substates. The system is operating.
2. one line failed and each station or its redundancy is operating. Sixty-four substates. The system is operating.
3. all lines are operating and at least one station or its redundancy failed. Sixty-five substates. The system failed.
4. at least one line failed and at least one station or its redundancy failed. Two hundred and sixty substates. The system failed.
5. at least two lines failed. Eight hundred and ninety-one substates. The system failed.

Reliability and availability diagrams for this system are presented in Fig.2. The circles with numbers stand for the system basis states.

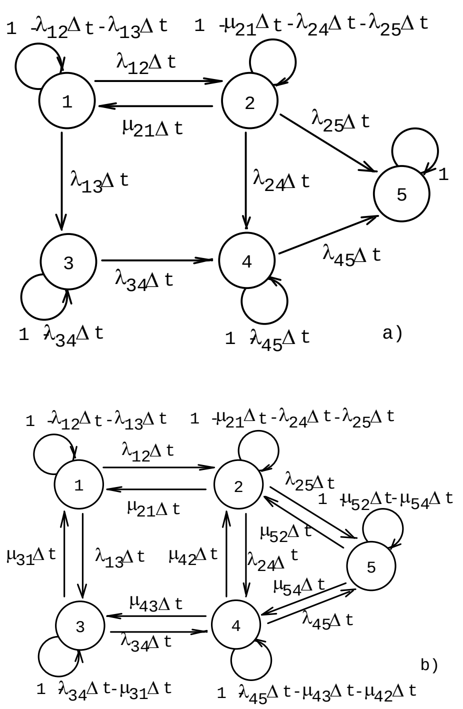

Figure 2. Reliability (a) and availability (b) diagram for the system in Fig.1

Let λ_{p1} and λ_{p2} be the failure rates of the stations due to jamming and physical failure respectively, and μ_{p1} and μ_{p2} be their repair rates.

Let λ_{v1} and λ_{v1} be the failure rates of the lines due to jamming and physical failure respectively and μ_{v1} i μ_{v2} be their repair rates.

In order to simplify the relations we denote

$$\lambda_v = \lambda_{v1} + \lambda_{v2};\ \lambda_p = \lambda_{p1} + \lambda_{p2}$$
$$\mu_v = \mu_{v2};\ \mu_p = \mu_{p2}$$

The transition rates between the basis states for the variant 1 of the system are

$$\lambda_{12} = \lambda_{34} = 4\lambda_v \qquad \mu_{21} = \mu_{43} = \min(\mu_{v1}, \mu_{v2}) \approx \mu_{v2} = \mu_v$$
$$\lambda_{13} = \lambda_{24} = 4\lambda_p \quad \mu_{31} = \mu_{42} = \mu_p^{*1}\left(\min(\mu_{p1}, \mu_{p2}) \approx \mu_{p2} = \mu_p\right)$$
$$\lambda_{25} = \lambda_{45} = 3\lambda_v \quad \mu_{52} = \mu_v^{*11} \quad \mu_{54} = \mu_v^{*12} \tag{1a}$$

where

- μ_p^{*1} - conditional repair rate for the station,
- μ_v^{*11} and μ_v^{*12} - conditional repair rates for the connection lines between the stations.

The transition rates between the basis states for the variant 2 of the system are

$$\lambda_{12R} = \lambda_{34R} = 4\lambda_v$$
$$\mu_{21R} = \mu_{54R} = \min(\mu_{v1}, \mu_{v2}) \approx \mu_{v2} = \mu_v$$
$$\lambda_{13R} = \lambda_{24R} = \lambda_{pR}^{*1}$$
$$\mu_{31R} = \mu_{42R} = \mu_{pR}^{*1} \tag{1b}$$
$$\lambda_{25R} = \lambda_{45R} = 3\lambda_v$$
$$\mu_{52R} = \mu_{vR}^{*11}$$
$$\mu_{43R} = \mu_{vR}^{*12}$$

The reliability and availability diagram for the telecommunication system variant 1 and variant 2 is obtained by replacing the transition rates in Fig.2 with relations 1a and 1b respectively.

In order to calculate the system reliability and availability we have to know the transition rates $\lambda_v, \lambda_p, \mu_v$ and μ_p and the conditional transition rates $\mu_v^*, \mu_p^*, \mu_{vR}^*$ and μ_{pR}^*. Since conditional transition rates are a function of transition rates, we have to determine this dependency first.

Determination of conditional transition rates

Variant 1

In order to determine the conditional transition rates between state 5 and 2; 5 and 4; 4 and 2; 3 and 1, in variant 1, for the system in Fig.2, we consider states 3 and 5 as isolated states (assuming that the system is a very short time Δt in this state), and derive the conditional transition rates. The diagram for the transition between the substates in state 3 is given in Fig.3.

Analyzing the third state we see that it has four groups of substates (one station failed - 4 substates; two stations failed - 6 substates; three stations failed - 4 substates; and all four stations failed - 1 substate), for which we can write four probabilities, the sum of which must equal unity when we consider this state as an isolated state. So we write the following set of equations

$$
\begin{aligned}
P_{(3)1} &= P_{(3)1}\left(1-3\lambda_p\Delta t\right)+P_{(3)2}3\mu_p\Delta t\\
P_{(3)2} &= P_{(3)1}2\lambda_p\Delta t+P_{(3)2}\left(1-2\lambda_p\Delta t-2\mu_p\Delta t\right)+P_{(3)3}3\mu_p\Delta t\\
P_{(3)3} &= P_{(3)2}3\lambda_p\Delta t+P_{(3)3}\left(1-\lambda_p\Delta t-3\mu_p\Delta t\right)+P_{(3)4}\mu_p\Delta t\\
P_{(3)4} &= P_{(3)3}4\lambda_p\Delta t+P_{(3)4}\left(1-4\mu_p\Delta t\right)\\
4P_{(3)1}&+6P_{(3)2}+4P_{(3)3}+P_{(3)4}=1
\end{aligned}
\qquad (2)
$$

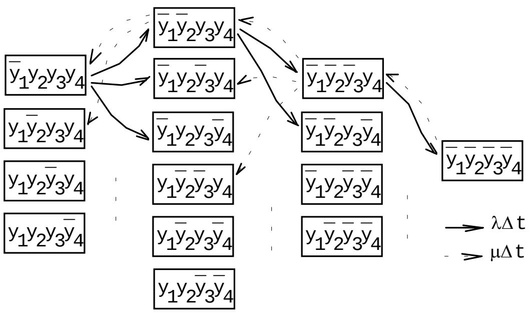

Figure 3. State 3 as an isolated state (for all substates we assume $x_1x_2x_3x_4$)

Solving set of equations (2) we can get probabilities $P_{(3)1}$, $P_{(3)2}$, $P_{(3)3}$ and $P_{(3)4}$

$$
\begin{aligned}
P_{(3)1} &= \frac{\mu_p^3}{4\mu_p^3+6\lambda_p\mu_p^2+4\lambda_p^2\mu_p+\lambda_p^3}\\
P_{(3)2} &= \frac{\lambda_p\mu_p^2}{4\mu_p^3+6\lambda_p\mu_p^2+4\lambda_p^2\mu_p+\lambda_p^3}\\
P_{(3)3} &= \frac{\lambda_p^2\mu_p}{4\mu_p^3+6\lambda_p\mu_p^2+4\lambda_p^2\mu_p+\lambda_p^3}\\
P_{(3)4} &= \frac{\lambda_p^3}{4\mu_p^3+6\lambda_p\mu_p^2+4\lambda_p^2\mu_p+\lambda_p^3}
\end{aligned}
\qquad (3)
$$

Using the probabilities given by relations (3) we determine the probability of transition from state 3 to state 1 and from state 4 to state 2, which is equal to

$$\mu_{31}\Delta t=\mu_{42}\Delta t=4P_{(3)1}\mu_p\Delta t=\mu_p^{*1}\Delta t$$

and from there the conditional transition repair rate of the station is

$$\mu_p^{*1}=\frac{4\mu_p^4}{4\mu_p^3+6\lambda_p\mu_p^2+4\lambda_p^2\mu_p+\lambda_p^3} \qquad (4)$$

Analyzing the fifth state we conclude that it has 11 subgroups of states with three groups (I - two lines failed with stations in all possible states; II - three lines failed with stations in all possible states; and III - four lines failed with stations in all possible states). Group I has 6 subgroups, group II has 4 subgroups, and group III has one subgroup. The fifth state with three groups of substates is illustrated in Fig.4.

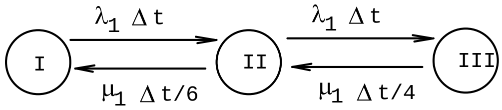

Figure 4. Fifth state with grouping substates in three group

Probabilities that the system is in one of the three groups (I, II or III) of the fifth state are

$$
\begin{aligned}
P_I &= \frac{\mu_v^2}{6\mu_v^2+4\lambda_v\mu_v+\lambda_v^2}\\
P_{II} &= \frac{\lambda_v\mu_v}{6\mu_v^2+4\lambda_v\mu_v+\lambda_v^2}\\
P_{III} &= \frac{\lambda_v^2}{6\mu_v^2+4\lambda_v\mu_v+\lambda_v^2}
\end{aligned}
\qquad (5)
$$

The system can transit from the fifth state to the fourth state only if the system is in group I of states and if at least one station is failed operating. Probability of that event (that at least one station failed) is determined in the same way as in previous case and the diagram is given in Fig.5.

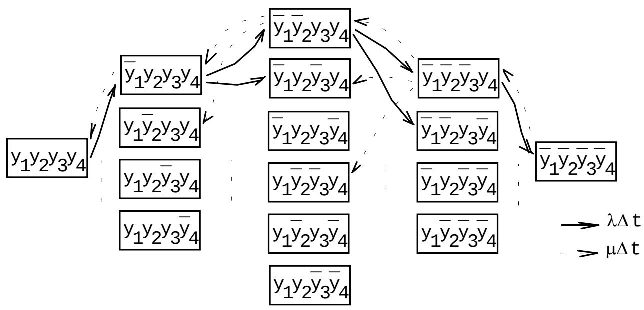

Figure 5. States of the telecommunication system in the first group of the fifth state

In the same way as we have done when examining the third state of the system, we can determine probabilities of five groups of the substate of the fifth state presented in Fig.5.

$$
\begin{aligned}
P_{(5)1} &= \frac{\mu_p^4}{\mu_p^4+4\lambda_p\mu_p^3+6\lambda_p^2\mu_p^2+4\lambda_p^3\mu_p+\lambda_p^4}\\
P_{(5)2} &= \frac{\lambda_p\mu_p^3}{\mu_p^4+4\lambda_p\mu_p^3+6\lambda_p^2\mu_p^2+4\lambda_p^3\mu_p+\lambda_p^4}\\
P_{(5)3} &= \frac{\lambda_p^2\mu_p^2}{\mu_p^4+4\lambda_p\mu_p^3+6\lambda_p^2\mu_p^2+4\lambda_p^3\mu_p+\lambda_p^4}\\
P_{(5)4} &= \frac{\lambda_p^3\mu_p}{\mu_p^4+4\lambda_p\mu_p^3+6\lambda_p^2\mu_p^2+4\lambda_p^3\mu_p+\lambda_p^4}\\
P_{(5)5} &= \frac{\lambda_p^4}{\mu_p^4+4\lambda_p\mu_p^3+6\lambda_p^2\mu_p^2+4\lambda_p^3\mu_p+\lambda_p^4}
\end{aligned}
\qquad (6)
$$

Then determining the probability to transit from the fifth to the second state

$$\mu_{52}\Delta t=P_IP_{52}\mu_v\Delta t=\mu_v^{*11}\Delta t$$

we get the conditional repair rate of the lines

$$\mu_v^{*11}=\frac{\mu_p^4}{\mu_p^4+4\lambda_p\mu_p^3+6\lambda_p^2\mu_p^2+4\lambda_p^3\mu_p+\lambda_p^4}\cdot \cdot\frac{6\mu_v^3}{6\mu_v^2+4\lambda_v\mu_v+\lambda_v^2} \qquad (7)$$

where $P_{52}=P_{(5)1}$.

By determining the probability to transit from the fifth to the fourth state

$$\mu_{54}\Delta t=P_IP_{54}\mu_v\Delta t=\mu_v^{*12}\Delta t$$

we get the conditional repair rate of the lines

$$\mu_v^{*12} = \frac{4\mu_p^3\lambda_p + 6\mu_p^2\lambda_p^2 + 4\mu_p\lambda_p^3 + \lambda_p^4}{\mu_p^4 + 4\lambda_p\mu_p^3 + 6\lambda_p^2\mu_p^2 + 4\lambda_p^3\mu_p + \lambda_p^4} \cdot \frac{6\mu_v^3}{6\mu_v^2 + 4\lambda_v\mu_v + \lambda_v^2} \quad (8)$$

where $P_{54} = 4P_{(5)2} + 6P_{(5)3} + 4P_{(5)4} + P_{(5)5}$.

Variant 2

In a similar manner, for variant 2, we get (for the following relations, because of simplicity, we use $\lambda \equiv \lambda_p$, $\mu \equiv \mu_p$)

$$\lambda_{pR}^{*1} = \frac{4\lambda^2}{\mu+\lambda}$$

$$\mu_{pR}^{*1} = \mu \frac{4\frac{\mu}{\lambda} + 12(1+E) + 4\frac{C+E(\lambda+D)}{\mu+\lambda}}{4G + \frac{\lambda}{\mu}F + 12(1+A+E+BE) + 6\left(C + DE + \lambda\frac{3-2E}{\mu}\right)}$$

$$\mu_{vR}^{*11} = \frac{\frac{\mu^2 - 3\mu\lambda - 4\lambda^2}{\mu\lambda} - 3k + w\left(6 + 3l + \frac{7\mu^2 + 11\mu\lambda + 5\lambda^2}{\mu\lambda}\right)}{N + wM + kO + lwO} \cdot \frac{6\mu_v^3}{6\mu_v^2 + 4\mu_v\lambda_v + \lambda_v^2}$$

$$\mu_{vR}^{*12} = \frac{6\Psi + 4\Gamma + 12\Theta + \frac{\lambda}{\mu}i + sk + w(j + sl)}{N + wM + kO + lwO} \cdot \frac{6\mu_v^3}{6\mu_v^2 + 4\mu_v\lambda_v + \lambda_v^2} \quad (9)$$

where

$$A = \frac{\lambda}{\mu^2}(2\mu+3\lambda) \quad B = -\frac{2\lambda}{\mu^2}(\mu+\lambda)$$

$$C = \frac{A(3\mu+2\lambda) - 3\frac{\lambda^2}{\mu} - A\frac{4\mu\lambda}{4\mu+\lambda}}{\mu + \frac{\mu\lambda}{4\mu+\lambda}}$$

$$D = \frac{B(3\mu+2\lambda) + 2\frac{\lambda^2}{\mu} - 2\lambda - B\frac{4\mu\lambda}{4\mu+\lambda}}{\mu + \frac{\mu\lambda}{4\mu+\lambda}}$$

$$E = \frac{2\lambda + \mu\left(2A + \frac{C}{\mu+\lambda}\right)}{3\lambda + \mu\left(2 - 2B - \frac{\lambda+D}{\mu+\lambda}\right)}$$

$$F = \frac{\lambda}{4\mu}\left[C + DE + \frac{\lambda(4A+C) + \lambda E(4B+D)}{4\mu+\lambda}\right]$$

$$G = \frac{\mu}{\lambda} + \frac{C+E(\lambda+D)}{\mu+\lambda} + F + \lambda\frac{4A+C+E(4B+D)}{4\mu+\lambda}$$

$$a = -\frac{3\mu^2 + 6\lambda\mu + 4\lambda^2}{\mu^2}$$

$$b = (\mu+\lambda)\frac{3\mu^2 + 6\mu\lambda + \lambda^2}{\lambda\mu^2}$$

$$c = \frac{a}{3\mu}(4\mu+3\lambda) + \frac{\lambda}{3\mu^2}(3\mu+4\lambda)$$

$$d = \frac{b}{3\mu}(4\mu+3\lambda) - \frac{3\mu^2 + 7\mu\lambda + \lambda^2}{3\mu^2}$$

$$e = \frac{\mu^2 + 3\mu\lambda + 12\lambda^2}{\mu^3}$$

$$f = -\frac{\mu^2 + 3\mu\lambda + 9\lambda^2}{\mu^2}$$

$$g = \frac{\lambda - a\mu}{2\mu} - \frac{\lambda^2}{\mu^3}(\mu+4\lambda)$$

$$h = \frac{\lambda}{\mu^2}(\mu+3\lambda) - \frac{\lambda + \mu(b+1)}{2\mu}$$

$$i = \frac{c(2\mu+\lambda) - \lambda(a+g)}{\mu}$$

$$j = \frac{d(2\mu+\lambda) - \lambda(b+h)}{\mu}$$

$$k = 2\frac{e\mu^2(\mu+\lambda) - g\mu^3 - \lambda^2(\mu+4\lambda)}{\mu^2(\mu+\lambda)(3\mu+7)}$$

$$l = 2\frac{f\mu(\mu+\lambda) - h\mu^2 + \lambda(\mu+3\lambda)}{\mu(\mu+\lambda)(3\mu+7)}$$

$$p = -\frac{(\lambda+\mu)(8\mu^2 + 4\mu\lambda + 3\lambda^2 - 4\lambda)}{\mu(3\mu+\lambda)}$$

$$s = -\frac{\mu+\lambda}{2\mu^2}(16\mu^2 + 17\mu\lambda + 6\lambda^2 + \lambda)$$

$$N = \frac{\mu}{\lambda} + \frac{\lambda}{\mu}i + 3\frac{4\lambda-\mu}{\mu} + 2(2a + 2i + 3e + 3c + 6g) + 4\frac{4g\lambda + i\mu}{3\lambda+\mu} + 12\frac{\lambda}{\mu^2}(\mu+4\lambda)$$

$$M = \frac{19\mu^2 + 47\mu\lambda + \lambda^2}{\mu\lambda} + \frac{\lambda}{\mu}j + 2(2b + 3d + 3f + 6h + 2j) + 4\frac{3h\lambda + j\mu}{3\lambda+\mu}$$

$$O = -3 - 12(\mu+\lambda) - \frac{\mu+\lambda}{\lambda}\left(6\mu + 18\lambda - 18 + \lambda\frac{16\mu^2 + 17\mu\lambda + 6\lambda^2 + \lambda}{2\mu^2} + 4\frac{8\mu^2 + 4\mu\lambda + 3\lambda^2 - 4\lambda}{3\mu+\lambda} + 2\frac{6\mu^2 + 17\mu\lambda + 6\lambda^2 + \lambda}{\mu}\right)$$

$$z = 3\lambda\frac{\mu+\lambda}{\mu}(4\mu+3\lambda) + \frac{\lambda(\mu+\lambda)}{\mu(3\mu+\lambda)}(8\mu^2 + 4\mu\lambda + 3\lambda^2 - 4\lambda) - 2\frac{\mu+\lambda}{\mu^2}(16\mu^2 + 17\mu\lambda + 6\lambda^2 + \lambda)$$

$$w = \frac{(3\mu+\lambda)(3c\lambda - 4i\mu - kz) + \lambda(3g\lambda + i\mu)}{(3\mu+\lambda)(4j\mu + lz - 3d\lambda) + \lambda(3h\lambda + j\mu)}$$

$$\Psi = \frac{4\lambda}{\mu} + c + e + w(d + f - 3) + (k + lw)\frac{(\mu+\lambda)(\mu+3\lambda)}{\mu}$$

$$\Gamma = a+i+\frac{3\lambda g+i\mu}{3\mu+\lambda}+w\left(b+j+\frac{3\lambda h+j\mu}{3\mu+\lambda}\right)+$$
$$+(k+lw)(s+p-3\mu-3\lambda)$$
$$\Theta = g+\frac{\lambda}{\mu^2}(\mu+4\lambda)+w\left(h-\frac{\mu+3\lambda}{\mu}\right)+$$
$$+(k+lw)\left(1+3(\mu+\lambda)\frac{\mu+1}{2\mu}\right)$$

Models for system reliability and availability

Variant 1

On the basis of this analysis, using Fig.2a and relations (1a), we may write the following differential equations

$$\frac{dP_1(t)}{dt} = -(4\lambda_v+4\lambda_p)P_1(t)+\mu_v P_2(t)$$
$$\frac{dP_2(t)}{dt} = 4\lambda_v P_1(t)-(3\lambda_v+4\lambda_p+\mu_v)P_2(t)$$
$$\frac{dP_3(t)}{dt} = 4\lambda_p P_1(t)-4\lambda_p P_3(t) \tag{10}$$
$$\frac{dP_4(t)}{dt} = 4\lambda_p P_2(t)+4\lambda_v P_3(t)-3\lambda_v P_4(t)$$
$$\frac{dP_5(t)}{dt} = 3\lambda_v P_2(t)+3\lambda_v P_4(t)$$

Solving equations (10) as in references [1,2] we get the relation for the reliability as the sum of probabilities that the system is in operating states, $R(t) = p_1(t) + p_2(t)$

$$R(t)=\frac{7\lambda_v+4\lambda_p+\mu_v-r_1}{r_2-r_1}e^{-r_1 t}-\frac{7\lambda_v+4\lambda_p+\mu_v-r_2}{r_2-r_1}e^{-r_2 t} \tag{11}$$

where

$$r_1=\frac{7\lambda_v+8\lambda_p+\mu_v}{2}-\frac{\sqrt{(7\lambda_v+8\lambda_p+\mu_v)^2-4(12\lambda_v^2+16\lambda_p^2+28\lambda_v\lambda_p+4\lambda_p\mu_v)}}{2}$$

$$r_2=\frac{7\lambda_v+8\lambda_p+\mu_v}{2}+\frac{\sqrt{(7\lambda_v+8\lambda_p+\mu_v)^2-4(12\lambda_v^2+16\lambda_p^2+28\lambda_v\lambda_p+4\lambda_p\mu_v)}}{2}$$

In the same way, writing the system of differential equations for the diagram in Fig.2b for availability, and solving it, we may obtain the relation for the system availability. Since the relation for availability as a function of time is very complex in this case, we get the relation for the availability for the long duration of time as $A=p_1+p_2$. Finally we get

$$A=\frac{1+f}{1+a+c+e+f(1+b+d+e)} \tag{12}$$

where

$$a=4\frac{\lambda_v+\lambda_p}{\mu_p^{*1}}$$
$$b=-\frac{\mu_v}{\mu_p^{*1}}$$
$$c=\frac{a(4\lambda_v+\mu_p^{*1})-4\lambda_p}{\mu_v^{*11}}$$
$$d=b\frac{4\lambda_v+\mu_p^{*1}}{\mu_v^{*11}}$$
$$e=\frac{3\lambda_v}{\mu_v^{*11}+\mu_v^{*12}}$$
$$f=\frac{4\lambda_v+c\mu_p^{*1}+e\mu_v^{*11}}{4\lambda_p+3\lambda_v+\mu_v-d\mu_p^{*1}-e\mu_v^{*11}}$$

Variant 2

In a similar manner, using the diagram in Fig.2a and relations (1b), we obtain the relation for the reliability for variant 2

$$R(t)=\frac{7\lambda_v+\lambda_{pR}^{*1}+\mu_v-r_1}{r_2-r_1}e^{-r_1 t}-\frac{7\lambda_v+\lambda_{pR}^{*1}+\mu_v-r_2}{r_2-r_1}e^{-r_2 t} \tag{13}$$

where

$$r_1=\frac{7\lambda_v+2\lambda_{pR}^{*1}+\mu_v}{2}-\frac{\sqrt{(7\lambda_v+2\lambda_{pR}^{*1}+\mu_v)^2-4\left(12\lambda_v^2+(\lambda_{pR}^{*1})^2+7\lambda_v\lambda_{pR}^{*1}+\mu_v\lambda_{pR}^{*1}\right)}}{2}$$

$$r_2=\frac{7\lambda_v+2\lambda_{pR}^{*1}+\mu_v}{2}+\frac{\sqrt{(7\lambda_v+2\lambda_{pR}^{*1}+\mu_v)^2-4\left(12\lambda_v^2+(\lambda_{pR}^{*1})^2+7\lambda_v\lambda_{pR}^{*1}+\mu_v\lambda_{pR}^{*1}\right)}}{2}$$

For the availability for long duration of time, $A = p_1 + p_2$, for variant 2, we get

$$A=\frac{1+f}{1+a+c+e+f(1+b+d+e)} \tag{14}$$

where

$$a=4\frac{\lambda_v+\lambda_{pR}^{*1}}{\mu_{pR}^{*1}}$$
$$b=-\frac{\mu_v}{\mu_{pR}^{*1}}$$
$$c=\frac{a(4\lambda_v+\mu_{pR}^{*1})-4\lambda_{pR}^{*1}}{\mu_{vR}^{*11}}$$
$$d=b\frac{4\lambda_v+\mu_{pR}^{*1}}{\mu_{vR}^{*11}}$$
$$e=\frac{3\lambda_v}{\mu_{vR}^{*11}+\mu_{vR}^{*12}}$$
$$f=\frac{4\lambda_v+c\mu_{pR}^{*1}+e\mu_{vR}^{*11}}{4\lambda_{pR}^{*1}+3\lambda_v+\mu_v-d\mu_{pR}^{*1}-e\mu_{vR}^{*11}}$$

Analysis of reliability and availability

Because of complexity of the relations we obtain, it is not easy to conclude how reliability and availability depend on each of parameters. Therefore, we performed the quantitative analysis of reliability and availability for unredundant and redundant systems for assumed values, by varying one parameter within certain values, and keeping other parameters values constant. The combinations of input data for the analysis are given in tables 1 to 6, and the curves in diagrams are marked with corresponding numbers in the tables.

Table 1. Input data for the diagram in Fig.6

Input parameter	Mark for the curve on the diagram					
	1	2	3	4	5	6
λ_v [h^{-1}]x10^{-6}	50	100	50	50	100	50
λ_p [h^{-1}]x10^{-6}	10	10	20	10	10	20
μ_v	0.1	0.1	0.1	1	1	1

Table 2. Input data for the diagram in Fig.7

Input parameter	Mark for the curve on the diagram					
	1	2	3	4	5	6
λ_v [h^{-1}]x10^{-6}	50	100	500	500	500	500
λ_p [h^{-1}]x10^{-6}	10	10	10	10	100	100
μ_v	0,1	0,1	0,1	1	1	1
μ_p	0.1	0.1	0.1	0.1	1	0.1

Table 3. Input data for the diagram in Fig.8

Input parameter	Mark for the curve on the diagram		
	unredundant system/ redundant system		
	1/4	2/5	3/6
λ_v [h^{-1}]x10^{-6}	100	100	500
μ_v	0.1	1	1
μ_p	0.1	0.1	0.1

Table 4. Input data for the diagram in Fig.9

Input parameter	Mark for the curve on the diagram		
	unredundant system/ redundant system		
	1/4	2/5	3/6
λ_v [h^{-1}]x10^{-6}	10	10	50
μ_v	0.1	1	1
μ_p	0.1	0.1	0.1

Table 5. Input data for the diagram in Fig.10

Input parameter	Mark for the curve on the diagram		
	unredundant system/ redundant system		
	1/4	2/5	3/6
λ_p [h^{-1}]x10^{-6}	10	50	10
λ_v [h^{-1}]x10^{-6}	100	100	500
μ_p	0.1	0.1	1

Table 6. Input data for the diagram in Fig.11

Input parameter	Mark for the curve on the diagram		
	unredundant system/ redundant system		
	1/4	2/5	3/6
λ_p [h^{-1}]x10^{-6}	10	50	10
λ_v [h^{-1}]x10^{-6}	100	100	500
μ_p	0.1	0.1	1

The results we get for reliability versus time, for the unredundant system, are shown in Fig.6, and for the redundant system in Fig.7. Availability versus the failure rates of the lines λ_v, the failure rates of the stations (receiver-transmitter) λ_p, the repair rates of the lines μ_v and the repair rates of the stations μ_p, for the unredundant and the redundant system, are shown in Fig.8 to 11 respectively.

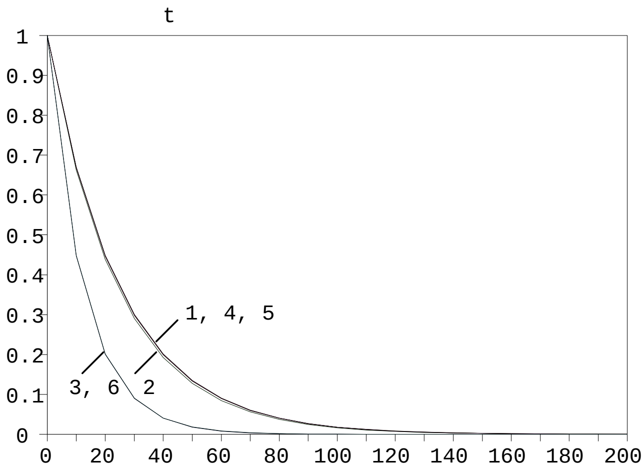

Figure 6. Reliability versus time for the unredundant system

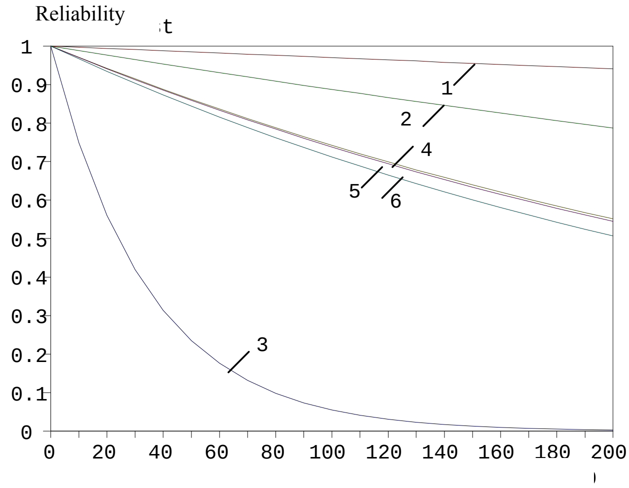

Figure 7. Reliability versus time for the redundant system

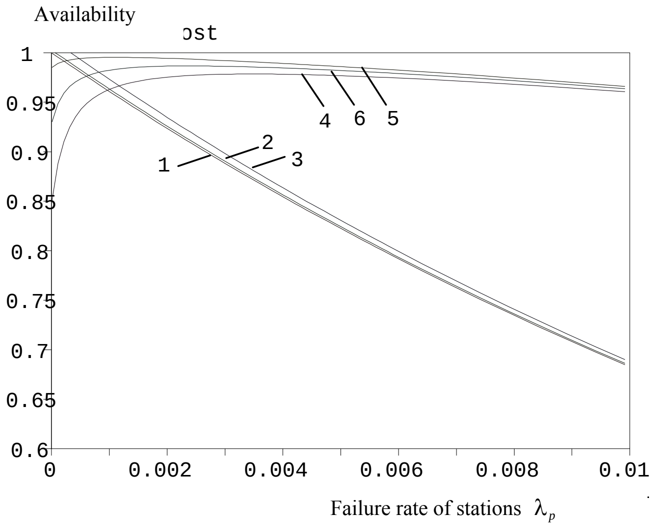

Figure 8. System availability versus the station failure rate

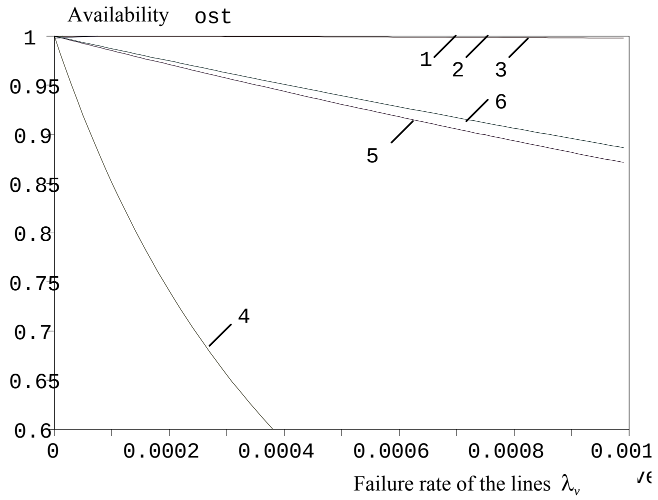

Figure 9. System availability versus the line failure rate

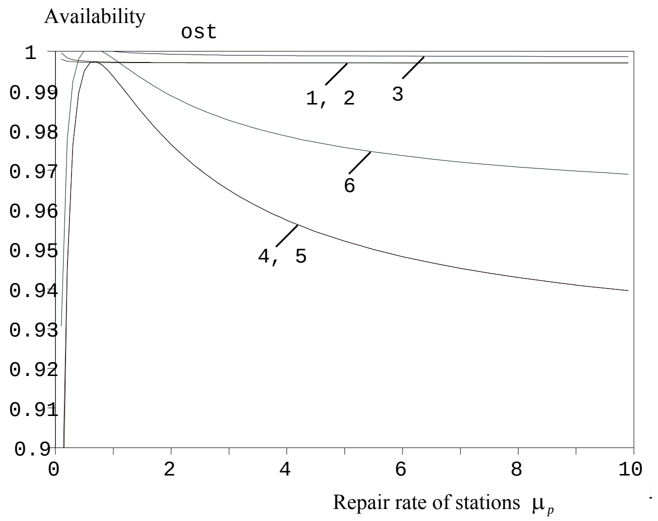

Figure 10. System availability versus the station repair rate

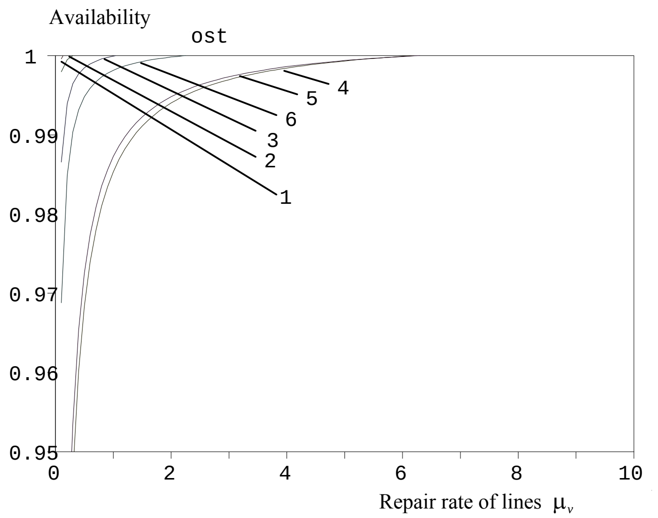

Figure 11. System availability versus the line repair rate

The results we get for the assumed data of reliability and repair rates show that by redunding stations we get higher reliability (Fig.7) in comparison with the unredundant system (Fig.6). We can see that the increase of the failure rate of the stations λ_p (2 times) in the unredundant system has greater effects on the change (decrease) of reliability (curves 1 and 6 in Fig.6) than the increase of the failure rate of the lines λ_v (curves 1 and 2 in Fig.6). The increase of the repair rate of the lines λ_v, for 10 times, is of less influence (curve 1 and 4 in Fig.6).

Increasing the failure rate of the stations λ_p (10 times), is of less influence on reliability (curves 3 and 5 in Fig.7), and increasing the failure rate of the lines λ_v has more influence than in the unredundant system (curves 1 and 2 in Fig.7). Increasing the repair rate of the lines μ_v for 10 times, in this case, has much more influence (curves 3 and 4 in Fig.7) than in the unredundant system (redundant are only stations). The redundant system is also submitted to the influence of the repair rate of the stations μ_p, the increasing (10 times) of which increases reliability but not significantly (curves 4 and 5 in Fig.7). Obviously because of the redundant stations, the station failure and the repair rates have less influence on the reliability of the redundant system than on the reliability of the unredundant one.

The influence of each parameter on availability (for long duration of time) is presented in Figs.8-11. The first thing that can be noticed is that the availability of the redundant system has a certain maximum value as a function of the station failure and repair rates (Figs.8 and 10), in comparison with the unredundant system. This can be expected for a system with redundant stations.

Fig.8 shows that the increase of the station failure rate in the unredundant system results in significant decrease of availability (curves 1, 2 and 3). In the redundant system, availability has a maximum value for a certain value of the station failure rate and this maximum occurs at higher station repair rates if the line repair rate is higher, but this maximum is lower if the line repair rate is higher (curves 4, 5 and 6 in Fig.8).

This change of the line failure rate in the redundant system has much more influence on the availability of the system, as it can be seen from Fig.9. This influence is more significant if the line repair rate is lower (curves 4 and 5), while the change of the station failure rate has less significant influence (curves 5 and 6).

Fig.9 shows that the availability of the unredundant system quickly decreases when the station repair rate increases and the station repair rate is low (curves 1, 2 and 3), while in the redundant system it increases (curves 4, 5 and 6). The availability of the redundant system has its maximum (curves 4, 5 and 6) which is higher if the line failure rate is higher (curve 6), and the station repair rate has less influence (curves 4 and 5). The form of curves 4, 5 and 6 can be explained as follows: when the station repair rate is low the systems is more often in the state of failure because the element which failed is not repaired on time, and redundancy is not enough to compensate for the low station repair rate, but when the station repair rate is high, the system is more often repaired and availability is low again. This means that for an optimum availability we must choose proper values of the failure rate and the repair rate.

Availability is higher if the line repair rate is higher. This is obvious from Fig.11 for the unredundant system (curves 1, 2 and 3) and for the redundant system (curves 4, 5 and 6), but unlike the unredundant system where availability is less influenced by the increase of the line repair rate if the line failure rate is higher, in the redundant system the opposite occurs and availability is nearly the same as in the unredundant system (curve 3 and 6).

Conclusion

The presented analytical models for reliability and availability for two variants (with and without redundant stations) of a telecommunication system of four telecommunication stations connected in a ring by duplex lines, obtained by grouping the possible states of the system, are very complex. In spite of that, using these analytical models we showed that the system can be analyzed.

The results show that, in the redundant system, for certain values of some parameters, availability has a maximum value, so it can be optimized. The obtained analytical models for reliability and availability enable such an optimization. The cost of redundancy can be included in the optimization thus leading to optimum configurations of the system from the point of reliability (availability)/cost. By analyzing a great number of combinations of input data, more precise conclusions could be obtained.

In a similar way models for other variants of this system can be made, for example, when there are traverse lines between stations. Certainly, in this case, the analysis is even more complex due to a greater number of states, and obtained relations are more complex as well.

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Analiza pouzdanosti i raspoloživosti četiri u prsten povezane telekomunikacione centrale grupisanjem stanja

Izvedeni su modeli za pouzdanost i raspoloživost telekomunikacionog sistema koji sačinjavaju četiri telekomunikacione centrale međusobno povezane u prsten dupleks vezama. Razmatraju se varijante sa neredundovanim i redundovanim primopredajnicima. Modeli su razvijeni polazeći od grupisanja stanja u kojima se dotični sistem može naći i uz korišćenje modela Markova koji definišu verovatnoće prelaza između pojedinih stanja. Prikazan je način izvođenja uslovnih intenziteta prelaza između grupa stanja. Na osnovu predloženih modela razmatranog sistema kvantitativno je analizrana pouzdanost i raspoloživost i dati su rezultati.

Ključne reči: **pouzdanost, raspoloživost, telekomunikacioni sistem, grupisanje stanja, modeli Markova.**

Fiabilité et la disponibilité de quatre stations de télécommunication reliées en cercle – l'analyse à l'aide du groupement des états

L'article propose les modèles de la fiabilité et de la disponibilité d'un système de télécommunication composé de quatre stations de télécommunication reliées en cercle par des liaisons duplex. Les variantes avec les émetteurs-récepteurs redondants et non-rédondants sont traitées. Les modèles sont développés à partir du groupement des états dans lesquels le système analysé peut se trouver et à l'aide des modèles de Markov qui définissent les probabilités de transmission entre les états particuliers. La dérivation des probabilités de transmission entre les groupes des états est donnée aussi bien que les résultats de la fiabilité et de la disponibilité du système.

Mots-clés: **fiabilité, disponibilité, système de télécomunication, groupement des états, modèles de Markov.**