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Mechanical and mathematical spatial modelling of the truck-crane telescope boom

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This paper analyzes the dynamics of the truck-crane telescope boom, using a relatively simple mechanical model consisting of elastic bodies and reduced to a mechanical system with three degrees of freedom. Such obtained theoretical results are verified by a numerical example in the case of variable force in the weight lifting rope. Finally, the results are compared with the results obtained by the static analysis of the same telescope boom.

truck-crane, telescope boom, deformation, dynamic model.

Introduction

IN former works referring to the field of telescopic boom dynamics, in most cases the boom construction is modeled as a system with a definite number of concentrated masses or as a system of elastic bodies. In the first case the model is simple and the results are of approximate value, while in the second case a mathematical model is considerably more complex and the results more precise. In order to get results that describe construction deformations more precisely, the boom should be modeled as a bearer with continually distributed masses considering solution possibilities of differential equations obtained.

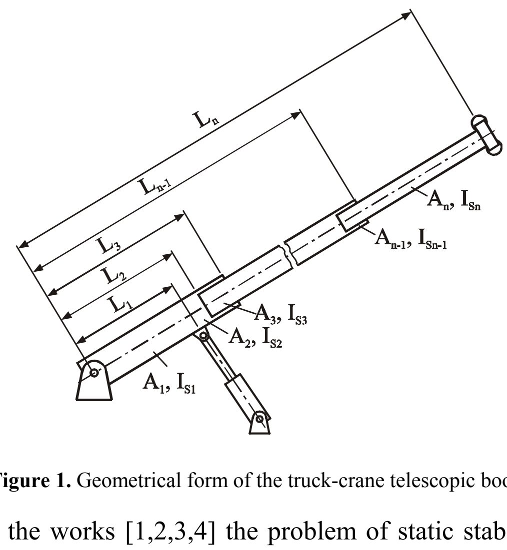


Figure 1. Geometrical form of the truck-crane telescopic boom

In the works [1,2,3,4] the problem of static stability of truck-crane lattice booms was discussed by means of an analysis of the basic differential equation of the boom neutral axis. The result produced expressions used to define the boom neutral axis in space under the effect of all types of load that might occur during its operation. Dynamic models with a greater number of degrees of freedom (system of concentrated masses), or with an infinite number of degrees of freedom (systems of elastic bodies) are given in the works [5,6,7]. The dynamic analysis of the truck-crane telescopic boom oscillations in the vertical plane for the case of its modeling with continually distributed masses is given in the work [8]. In this work the dynamic analysis of spatial oscillating of the truck-crane telescopic boom will be presented for the case of its modelling with continually distributed masses using a model reduced to a three-degrees-of-freedom system.

Mechanical model

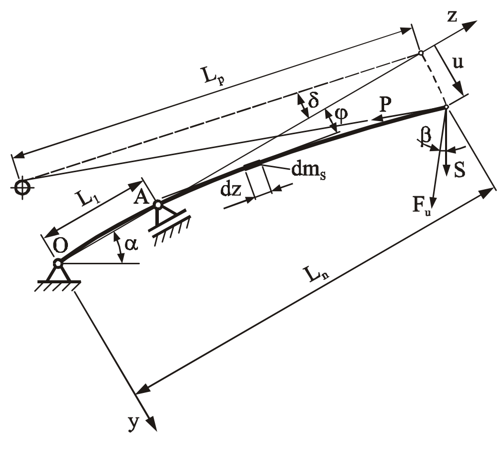


Figure 2. Scheme of the truck-crane telescopic boom load

The analysed mechanical system consists of a telescopic boom modeled as a beam bearer of a variable cross-section (Fig.1). The boom consists of segments (telescopes) fitting one into another, so some parts of the boom have increased cross-sections, moments of inertia and mass [9]. At its overhang (Fig.2) there acts the force within the load lifting rope , the force within the load lifting rope , and also

[1)] Faculty of Transport and Traffic Engineering, Vojvode Stepe 305, 11000 Beograd the force resulting from the action of the boom reduced mass (weight) onto its top

$$= \frac{}{\cdot\,\eta}, \qquad = \frac{\cdot}{3} \qquad (1)$$

where

number of pulley tackles,

η degree of pulley efficiency,

boom mass.

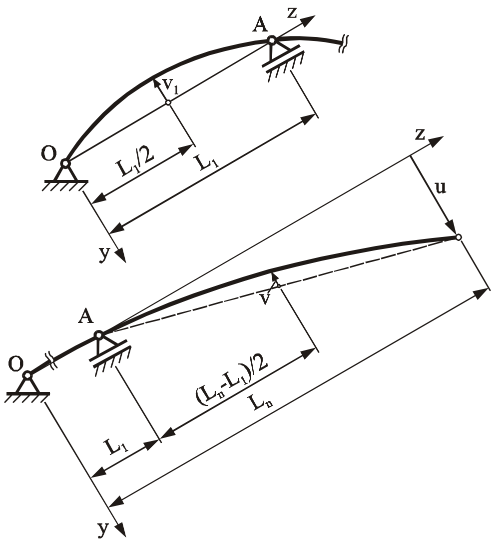


Figure 3. Form of the boom neutral axis in the vertical plane

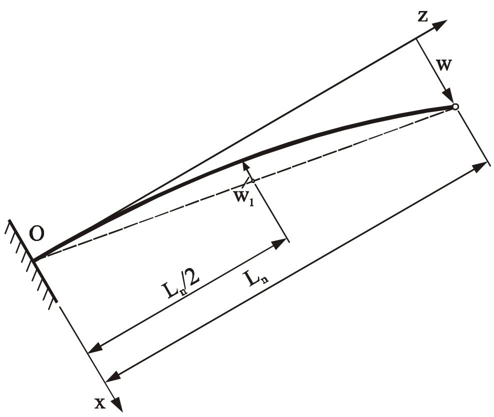


Figure 4. Form of the boom neutral axis in the horizontal plane

The established mechanical model takes into account angle oscillations - load swaying in both planes. It is assumed that the angles of swaying β in both planes are approximately equal.

The influence of the boom weight can be also treated as continual load which results into more complex final expressions. Their solving is very complicated even when applying the numerical method, and the obtained results, it is a realistic assumption, are only slightly more precise.

In a general case the discussed system has an infinite number of degrees of freedom. However, with regard to the task of investigation and introduced simplifications, this system can be substituted by a model with three degrees of freedom. The motion of such a system is defined by generalized coordinates

boom top deformation in the direction perpendicular onto the longitudinal, non-deformed boom axis in the vertical plane (Fig.3),

boom top deformation in the direction perpendicular onto the longitudinal, non-deformed boom axis in the horizontal plane (Fig.4),

midboom deformation in the direction perpendicular onto the longitudinal, non-deformed boom axis in the vertical plane [11].

Force within the rope for load lifting

Load lifting process can be modeled as a system of two masses [10] (Fig.5)

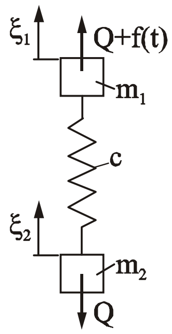


Figure 5. Dynamic model of load suspension

where

$_1$ mass of the engine rotor and the reduced mass of drive mechanisms on it,

$_2$ load mass,

rigidity of the load lifting rope,

load weight,

dynamic drive force.

Dynamic drive force occurs during the period of non-stationary motion and can be defined by the following expression

$$(\;) = \cdot[1-(-)^{0,5}] \qquad (2)$$

where

dynamic drive force for =0,

time of engine acceleration.

The process of load lifting is carried out in two phases. In the first phase load lies on the ground, the consequence being that only mass oscillates under the action of the force + , which is described by the following differential equation

$${}_1\cdot\ddot{\xi}_1 + \cdot\xi_1 = + (\;) \qquad (3)$$

During this phase the rope strain (ξ) is equal to the generalized coordinate

$$\xi = \xi_1 \qquad (4)$$

The second phase starts at the moment when the force within the rope reaches the value of load weight. During this phase the rope strain is equal to the difference of the accepted generalized coordinates

$$\xi = \xi_1 - \xi_2 \qquad (5)$$

and its change is described by the following equation

$$\ddot{\xi} + \frac{{}_1 + {}_2}{{}_1 \cdot {}_2}\cdot \cdot\xi = \cdot\frac{{}_1 + {}_2}{{}_1 \cdot {}_2} + \frac{(\;)}{{}_1} \qquad (6)$$

The force within the rope for load lifting can be defined on the basis of expressions (4), (6) as

$$( ) = \cdot \xi( ) \tag{7}$$

Differential equation of the boom motion

The observed mechanical system is non-conservative and corresponding relations are holonomous and ideal. The boom bending is neglected.

The equation of the boom neutral axis is unknown. In the works [2,5,12,13,14] the neutral axis is assumed in the form of various functions

$$= {}_1 \cdot [1-\cos(\frac{\pi \cdot}{2 \cdot})] \tag{8}$$

$$= {}_1 \cdot \sin(\frac{\pi \cdot}{2 \cdot}) \tag{9}$$

$$= {}_0 + {}_1 \cdot + {}_2 \cdot {}^2 \tag{10}$$

where the unknown constants , are defined on the basis of limit conditions. In this work the neutral axis of the boom is assumed to be in the form of a polynomial.

The truck-crane boom is modelled as a beam with an overhang in the vertical plane. Its assumed equation of the neutral axis at the span of $_1$ is of the form

$$_2( ) = \frac{- {}_1}{- {}_1} + {}_0 + {}_1 \cdot + {}_2 \cdot {}^2 \tag{11}$$

where the unknown constants are defined on the basis of the limit conditions

$$\begin{aligned} &_2({}_1) = 0 \\ &_2( ) = \\ &_2(\frac{+ {}_1}{2}) = \frac{}{2} - \end{aligned} \tag{12}$$

On the basis of expressions (11) and (12) the final form of the assumed equation of the boom neutral axis is obtained

$$_2( ) = {}^2 {}_5 + ({}_1 + {}_4) + {}_2 + {}_3 \tag{13}$$

where

$$_1 = \frac{1}{- {}_1}, \quad _2 = -\frac{_1}{- {}_1}, \quad _3 = \frac{4 \quad _1}{(- {}_1)^2}$$

$$_4 = -\frac{4(+ {}_1)}{(- {}_1)^2}, \quad _5 = \frac{4}{(- {}_1)^2}$$

The assumed equation of the boom neutral axis at the span of $_1$ has the form

$$_1( ) = {}_0 + {}_1 \cdot + {}_2 \cdot {}^2 \tag{14}$$

where the unknown constants are defined on the basis of the limit conditions

$$\begin{aligned} &_1(0) = 0 \\ &_1({}_1/2) = - {}_1 \\ &_2({}_1) = 0 \end{aligned} \tag{15}$$

The boom displacement $_1$ can be defined in the function of the displacements and on the ground of the conditions

$$\frac{\partial {}_1}{\partial} = \frac{\partial {}_2}{\partial} \quad \text{for} \quad {}_1 \tag{16}$$

On the basis of egs.(14), (15) and (16) it follows

$$_1( ) = {}^2({}_6 + {}_7) + ({}_8 + {}_9) \tag{17}$$

where

$$_6 = \frac{_1}{_1}, \quad _7 = \frac{_4 + 2 \; _5 \; _1}{_1}$$

$$_8 = - {}_1, \quad _9 = - {}_4 - 2 \; _5 \; _1$$

The truck-crane boom is in the horizontal plane modelled as a console bearer. Its assumed equation of the neutral axis is of the form

$$( ) = \frac{}{} + {}_0 + {}_1 \cdot + {}_2 \cdot {}^2 \tag{18}$$

where the unknown constants are defined on the basis of the limit conditions

$$\begin{aligned} &(0) = 0 \\ &( ) = \\ &(\frac{}{2}) = \frac{}{2} - {}_1 \end{aligned} \tag{19}$$

The boom displacement $_1$ can be defined in the function of the displacement on the ground of the conditions

$$\frac{\partial}{\partial} = 0 \quad \text{for} \quad 0 \tag{20}$$

On the basis of egs.(18), (19) and (20) the final form of the equation of the boom neutral axis in the horizontal plane is as follows

$$( ) = \frac{^2}{^2} \tag{21}$$

For the calculation of differential equations of motion, the Lagrange equations of the other kind will be used

$$\frac{}{}\left(\frac{\partial}{\partial \cdot}\right) - \frac{\partial}{\partial} + \frac{\partial \Phi}{\partial \cdot} + \frac{\partial \Pi}{\partial} = \tag{22}$$

$$1, 2, 3,$$

where Π Φ - are kinetic energy, potential energy, function of the boom dissipation (dissipative force), and corresponding generalized non-conservative force, respectively.

The kinetic energy of the boom is defined as

$$= + \tag{23}$$

where

kinetic energy of translation,

kinetic energy of boom rotation.

The kinetic energy of boom translation is of the form

$$= \frac{1}{2} \int_0 {}^2 \tag{24}$$

The boom is modelled as a beam bearer with a gradually variable cross-section (Fig.1). The mass of the elementary part of the boom is

$$= \rho \cdot \ (\ ) \cdot \qquad (25)$$

where

ρ density of the boom material,

cross-section surface of the elementary -part of the boom.

The speed of the elementary part is defined as follows

$$= \sqrt{\ ^2 + \ ^2} = \sqrt{\dot{}^2(\ ) + \dot{}^2(\ )} \qquad (26)$$

On the basis of egs. (24),(25) and (26) it follows

$$= \frac{\rho}{2}[\int_0 (\ ) \cdot \dot{}^2(\ ) \ + \int_0^1 {}_1 \cdot \dot{}_1^2(\ ) \ + \ + \int_1 (\ ) \cdot \dot{}_2^2(\ ) \ ] \qquad (27)$$

the solution of which offers the final form of the boom translation kinetic energy

$$= {}_1 \dot{}^2 + {}_2 \dot{}^2 + {}_3 \dot{}^2 + {}_4 \dot{}\dot{} \qquad (28)$$

where

$${}_1 = \frac{\rho}{10 \ ^4} \sum_{=1} (\ ^5 - \ ^5_{-1})$$

$${}_2 = \frac{\rho}{2}\{ \ {}_1 {}_1^3 (\frac{1}{5} {}_1^2 {}_6^2 + \frac{1}{2} {}_1 {}_6 {}_8 + \frac{1}{3} {}_8^2) + \sum_{=2} [\frac{1}{3} {}_1^2 (\ ^3 - \ ^3_{-1}) + \ + {}_1 {}_2 (\ ^2 - \ ^2_{-1}) + {}_2^2 (\ - \ _{-1})] \}$$

$${}_3 = \frac{\rho}{2}\{ \ {}_1 {}_1^3 (\frac{1}{5} {}_1^2 {}_7^2 + \frac{1}{2} {}_1 {}_7 {}_9 + \frac{1}{3} {}_9^2) + \sum_{=2} [\frac{1}{5} {}_5^2 (\ ^5 - \ ^5_{-1}) + {}_4 {}_5 (\ ^4 - \ ^4_{-1}) + \frac{1}{3}(2 {}_3 {}_5 + {}_4^2)(\ ^3 - \ ^3_{-1}) + {}_3 {}_4 (\ ^2 - \ ^2_{-1}) + {}_3^2 (\ - \ _{-1})] \}$$

$${}_4 = \frac{\rho}{2}\{ \ {}_1 {}_1^3 [\frac{2}{5} {}_1^2 {}_6 {}_7 + \frac{1}{2} {}_1 ({}_6 {}_9 + {}_7 {}_8) + \frac{2}{3} {}_8 {}_9] + \sum_{=2} [{}_1 {}_5 (\ ^4 - \ ^4_{-1}) + 2 {}_2 {}_3 (\ - \ _{-1}) + \frac{2}{3}({}_2 {}_5 + {}_1 {}_4) \cdot (\ ^3 - \ ^3_{-1}) + ({}_1 {}_3 + {}_2 {}_4) \cdot (\ ^2 - \ ^2_{-1})] \}$$

where is the cross-section surface of the -part of the boom which is at the span of to (Fig.1).

The kinetic energy of rotation is defined on the basis of the expression

$$= \frac{1}{2} \int_0 \dot{\varphi}^2 \qquad (29)$$

The main central moment of inertia of the -part of the boom is of the form

$$= \ \cdot \ ^2 \qquad (30)$$

The rotation angles of the elementary -part of the boom in the Oyz and Oxz planes are defined in the following way

$$\varphi_1 \approx \frac{\partial \ _1}{\partial}, \quad \varphi_2 \approx \frac{\partial \ _2}{\partial}, \quad \varphi \approx \frac{\partial}{\partial} \qquad (31)$$

On the basis of egs.(29),(30) and (31) it is obtained

$$= \frac{\rho}{2}[\int_0 (\ ) \cdot \dot{\varphi}^2 \cdot \ ^2 \ + \ + \int_0^1 {}_1 \cdot \dot{\varphi}_1^2 \cdot \ ^2 \ + \int_1 (\ ) \cdot \dot{\varphi}_2^2 \cdot \ ^2 \ ] \qquad (32)$$

the solution of which offers the final form of kinetic energy of the boom rotation

$$= {}_1 \dot{}^2 + {}_2 \dot{}^2 + {}_3 \dot{}^2 + {}_4 \dot{}\dot{} \qquad (33)$$

where

$${}_1 = \frac{\rho}{10 \ ^4} \sum_{=1} (\ ^5 - \ ^5_{-1})$$

$${}_2 = \frac{\rho}{2}[\ {}_1 {}_1^3 (\frac{4}{5} {}_1^2 {}_6^2 + {}_1 {}_6 {}_8 + \frac{1}{3} {}_8^2) + \sum_{=2} \frac{1}{3} {}_1^2 (\ ^3 - \ ^3_{-1})]$$

$${}_3 = \frac{\rho}{2}\{ \ {}_1 {}_1^3 (\frac{4}{5} {}_1^2 {}_7^2 + {}_1 {}_7 {}_9 + \frac{1}{3} {}_9^2) + \sum_{=2} [\frac{4}{5} {}_5^2 (\ ^5 - \ ^5_{-1}) + {}_4 {}_5 (\ ^4 - \ ^4_{-1}) + \frac{1}{3} {}_4^2 (\ ^3 - \ ^3_{-1}) \}$$

$${}_4 = \frac{\rho}{2}\{ \ {}_1 {}_1^3 [\frac{8}{5} {}_1^2 {}_6 {}_7 + {}_1 ({}_6 {}_9 + {}_7 {}_8) + \frac{2}{3} {}_8 {}_9] + \sum_{=2} [\frac{2}{3} {}_1 {}_4 (\ ^3 - \ ^3_{-1}) + {}_1 {}_5 (\ ^4 - \ ^4_{-1}) \}$$

The potential energy of the observed mechanical system is defined as follows

$$\Pi = \Pi_1 + \Pi_2 + \Pi_3 \qquad (34)$$

where

Π potential energy of the elastic deformation due to boom bending,

Π potential energy of the boom pressure due to the axial force action ,

Π potential energy of the boom due to the transversal forces action i .

The potential energy of the elastic deformation due to boom bending is

$$\Pi_1 = \frac{}{2}\cdot\int_0 \;(\;)\cdot\left(\frac{\partial^2}{\partial^{\;2}}\right)^2 \; + \int_0^1 (\;)\cdot\left(\frac{\partial^2 \;_1}{\partial^{\;2}}\right)^2 \; + \int_1 (\;)\cdot\left(\frac{\partial^2 \;_2}{\partial^{\;2}}\right)^2 \tag{35}$$

resulting into

$$\Pi_1 = {}_1{}^2 + {}_2{}^2 + {}_3{}^2 + {}_4 \tag{36}$$

where

$${}_1 = \frac{}{2}\sum_{=1} 4 \;\frac{\;-\;_{-1}}{4}$$

$${}_2 = 2\;_{1\;1\;6}^{\;\;\;\;2}$$

$${}_3 = \frac{}{2}[4\;_{1\;1\;7}^{\;\;\;\;2} + \sum_{=1} 4\;_5^2(\;-\;_{-1})]$$

$${}_4 = 4\;_{1\;1\;6\;7}$$

where
 is the moment of inertia for the axis of the boom -part which is at the span of to (Fig.1)
 elasticity module.

The potential energy of the boom pressure due to the axial force action is

$$\Pi_2 = \frac{}{2}[\int_0 \left(\frac{\partial}{\partial}\right)^2 + \int_0^1 \left(\frac{\partial\;_1}{\partial}\right)^2 + \int_1 \left(\frac{\partial\;_2}{\partial}\right)^2 \;] \tag{37}$$

The axial force of pressure is defined on the basis of the expression

$$\begin{aligned} &= \cdot\sin(\alpha+\beta)\cdot\cos\beta + \\ &+ \cdot\sin\alpha + \cdot\cos\delta \end{aligned} \tag{38}$$

Finally, expression (37) reaches the following form

$$\Pi_2 = {}_1{}^2 + {}_2{}^2 + {}_3{}^2 + {}_4 \tag{39}$$

where

$${}_1 = \frac{2}{3\;_4}\sum_{=1} {}^3 - {}^3_{-1}$$

$${}_2 = [\frac{2}{3}\;_{1\;6}^{3\;2} + {}_{1\;6\;8}^{2} + \frac{1}{2}\;_{1\;8}^{\;2} + \sum_{=2}\frac{1}{2}\;_1^2(\;-\;_{-1})]$$

$${}_3 = [\frac{2}{3}\;_{1\;7}^{3\;2} + {}_{1\;7\;9}^{2} + \frac{1}{2}\;_{1\;9}^{\;2} + \sum_{=2}\frac{2}{3}\;_5^2(\;^3 - \;^3_{-1}) + {}_{4\;5}(\;^2 - \;^2_{-1}) + \frac{1}{2}\;_4^2(\;-\;_{-1})]$$

$${}_4 = [\frac{4}{3}\;_{1\;6\;7}^{3} + {}_1^2(\;_{6\;9} + \;_{7\;8}) + {}_{1\;8\;9} + \sum_{=2}\;_{1\;5}(\;^2 - \;^2_{-1}) + {}_{1\;4}(\;-\;_{-1})]$$

The potential energy of the boom due to the transversal force action is defined by applying the following formula

$$\Pi_3 = \frac{}{2}(\int_0^1 \frac{\;_1(\;)}{} + \int_1 \frac{\;_2(\;)}{}\;) + \frac{}{2}\int_0 \frac{(\;)}{} \tag{40}$$

Transversal forces are defined on the basis of the expression (Fig.2)

$$\begin{aligned} &= \cdot\sin(\alpha+\beta)\cdot\sin\beta + \cdot\cos\delta, \\ &= \cdot\cos(\alpha+\beta) + \cdot\cos\alpha - \cdot\sin\delta \end{aligned} \tag{41}$$

The final form of the expression for potential energy (40) acquires the following form

$$\Pi_3 = {}_1 + {}_2 + {}_3 \tag{42}$$

where

$${}_1 = \frac{}{4}$$

$${}_2 = \frac{}{4}[\;_6\;_1^2 + 2\;_8\;_1 + 2\;_1(\;-\;_1) + 2\;_2\cdot\ln\frac{}{_1}]$$

$${}_3 = \frac{}{4}[\;_7\;_1^2 + 2\;_9\;_1 + {}_5(\;^2 - \;_1^2) + 2\;_4(\;-\;_1) + 2\;_3\cdot\ln\frac{}{_1}]$$

The function of dissipation (dissipative force) is defined by applying the following expression

$$\Phi = \frac{\varepsilon}{2\cdot}[\int_0 \dot{}^2(\;) + \int_0^1 \dot{}_1^2(\;) + \int_1 \dot{}_2^2(\;)\;] \tag{43}$$

the solution of which gives

$$\Phi = {}_1\dot{}^2 + {}_2\dot{}^2 + {}_3\dot{}^2 + {}_4\dot{}\dot{} \tag{44}$$

where ε is the coefficient of boom damping, and also the changes are introduced

$${}_1 = \varepsilon/10$$

$${}_2 = \frac{\varepsilon}{2}[\;_1^3(\frac{1}{5}\;_{1\;6}^{2\;2} + \frac{1}{2}\;_{1\;6\;8} + \frac{1}{3}\;_8^2) + \frac{1}{3}\;_1^2(\;^3 - \;_1^3) + {}_{1\;2}(\;^2 - \;_1^2) + {}_2^2(\;-\;_1)]$$

$${}_3 = \frac{\varepsilon}{2}[\;_1^3(\frac{1}{5}\;_{1\;7}^{2\;2} + \frac{1}{2}\;_{1\;7\;9} + \frac{1}{3}\;_9^2) + \frac{1}{5}\;_5^2(\;^5 - \;_1^5) + {}_{4\;5}(\;^4 - \;_1^4) + \frac{1}{3}(2\;_{3\;5} + \;_4^2)(\;^3 - \;_1^3) + {}_{3\;4}(\;^2 - \;_1^2) + {}_3^2(\;-\;_1)]$$

$$_4 = \frac{\varepsilon}{2}\{ \ _1^3[\frac{2}{5} \ _1^2 \ _6 \ _7 + \frac{1}{2} \ _1(\ _6 \ _9 + \ _7 \ _8) +$$
$$+ \frac{2}{3} \ _8 \ _9] + \ _1 \ _5(\ ^4 - \ _1^4) +$$
$$+ \frac{2}{3}(\ _2 \ _5 + \ _1 \ _4)(\ ^3 - \ _1^3) +$$
$$+ (\ _1 \ _3 + \ _2 \ _4)(\ ^2 - \ _1^2) +$$
$$+ 2 \ _2 \ _3(\ - \ _1) \}$$

The generalized force is of the form

$$= \ , \quad = \ , \quad = 0 \tag{45}$$

When into the Lagrange equations of the other type (22) the expressions for kinetic (23), (28) and (33) and potenti[redacted] energies (34), (36), (39) and (42) as well as the functi[redacted] dissipation (44) are introduced, three differential e[redacted] of the second order are obtained in the following[redacted]

$$\ddot{} = \ (\ , \ , \ , \ , \alpha, \beta, \delta, ...)$$
$$\ddot{} = \ (\ , \ , \ , \ , \alpha, \beta, \delta, ...)$$
$$\ddot{} = \ (\ , \ , \ , \ , \alpha, \beta, \delta$$ [redacted]

which can be solved numeri[redacted] introducing the changes [redacted]

In[redacted] neutral axis in space for an arbitrary moment of time are obtained.

Numerical example and the result analysis

The obtained theoretical results are discussed on a numerical example of the truck-crane telescopic boom of maximum deadweight = 16 t, designed at Ivo Lola Ribar - Železnik (Belgrade).

A change in rope strain is of elasticity type in the course of time and depends on the characteristics of lifting drive, rope rigidity, rotor mass of the engine, drive mechanisms and load (Fig.6).

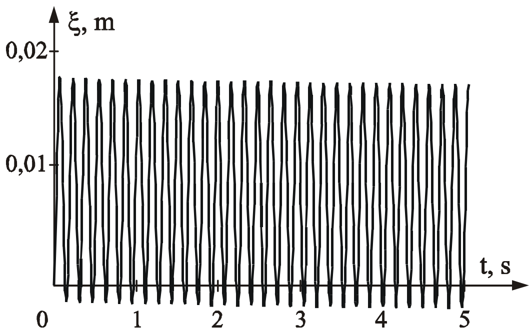
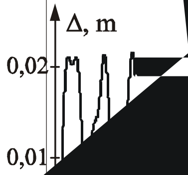


Figure 6. Change in rope strain in the function of time

By solving numerically non-linear differential equations (47), the curve of the boom top displacement Δ depend-ence on time is obta[redacted] the occurrence of da[redacted]

Δ = [redacted]



[redacted]

Figure 9. Forms of the boom neutral axis in various moments of time

For the accepted parameters of the truck-crane boom motion, deformations in the vertical plane are more critical than deformations that occur in the horizontal plane. This does not mean that deformations in horizontal plane should be neglected, for they gain higher values in case when the truck-crane upper machine rotates. The deflection $_1$ (Fig.3) in comparison with the deflection of the boom top is insignificant (≤ 0.1). From the performed analysis it appears that the dynamic deflections of the boom top are of oscillatory type. Such deflections can be influenced by optimum an appropriate, but also by drive reduction, i.e. by an appropriate engine characteristic of the load lifting mechanism.

Conclusion

The task of this work was to analyse the dynamic behaviour of the truck-crane telescopic boom in space, particularly of its neutral axis by modelling the boom as a bearer with continually distributed masses. By the application of the Lagrange equations of another type, differing from the static analysis usually applied in practice, the results that correspond better to real boom operation conditions are achieved. The formulae obtained define a spatial form of the boom neutral axis in the course of time. With slighter modifications, the established model can be applied to other similar constructions.

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Prostorno mehaničko-matematičko modeliranje teleskopske strele auto-dizalice

Analizira se oscilovanje teleskopske strele auto-dizalice u prostoru, korišćenjem kombinovanog dinamičko-statičkog modela sastavljenog od elastičnih tela koji se svodi na mehanički sistem sa tri stepena slobode kretanja. Dobijeni teorijski rezultati se verifikuju numeričkim primerom za slučaj promenljive vrednosti sile u užetu za podizanje tereta, a zatim se rezultati porede sa rezultatima dobijenim pri statičkoj analizi iste strele.

č č **auto-dizalica, strela, prostor, deformacija, dinamički model.**

Modélisation mécanique-mathématique et spatiale du flèche télescopique de camion-grue

Les oscillations du flèche teléscopique de camion-grue sont analysées dans l'espace en utilisant un modèle dynamique-statique combiné et composé de corps élastiques - un modèle reduit au système mécanique à trois degrés de liberté. Les résultats théoriques obtenus sont vérifiés par un exemple numérique pour le cas d'une force variable dans la corde pour lever des poids et puis comparés aux résultats obtenus dans l'analyse statique du même flèche.

: **camion-grue, flèche, espace, déformation, modèle dynamique.**