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On the natural laws and their formal description

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A brief survey of the development of the idea concerning the invariance of laws in physical theories is done. The tensor calculus is pointed out as an unsubstitutible calculus of invariants until now. A direct consequence of the adoption of the idea on invariance in the applied mechanics is mentioned, as well.

Key words: **natural laws, invariance (covariance), tensor calculus**.

Introduction

OUR aim is, without entering into the justification of the physical theories which shall be mentioned here, to expose briefly the way of developing the idea on the invariance of the laws in these theories and then to dwell upon some characteristics of the laws and the mathematical apparatus of the general theory of relativity.

Principles of relativity, natural laws

Since the Newtonian mechanics it has been known that its laws are formulated in the same way in inertial coordinate systems; this is included in **Galileo - Newton's principle of relativity**: *"If the laws of mechanics are correct in one coordinate system then they are also correct in any other system, moving rectilinearly and uniformly with respect to the first one."* ([1], t. IV, p. 456). However, more attention was paid to this characteristic only when a generalisation of this principle was attempted to all physical laws and it turned out that it cannot be applied to Maxwell's equations of electrodynamics. The problem was solved by Einstein in his special theory of relativity generalising the above-mentioned principle by a so-called **special principle of relativity**: *" ... the laws of physics are equal in all coordinate systems moving rectilinearly and uniformly one with respect to the other ... "* ([1], t. IV, p. 512) and supposing the constancy of the light velocity, allowing for entire physics the equivalence of inertial systems, but defined in a different way in this theory than in the prerelativistic physics.

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ut is uestion: *"Must the dependence of the laws of physics on the choice of the coordinate system be limited only to the systems moving rectilinearly and uniformly one with respect to the other?"* ([1], t. II, p. 717) and the answer was his conviction in the possibility of foundation of the physics applicable in all coordinate systems, expressed by the **general principle of relativity**: *"The laws of physics should be formulated in order to be applicable in coordinate systems moving arbitrarily."* ([1], t. I, p. 456). Of course, one should bear in mind how the coordinate systems are mathematically defined in this case.

Just in the time of the beginning of the theory of relativity, which has generally underlined the laws of physics in such a way, they started to be named **natural laws** ([1], t. I, p. 457) — the name present in the physical theories until today ([2], p. 13; [3], p. 472; [4], p. 46) — in order to point out that the laws which should be valid *always and everywhere* are in question, i.e. that they do not depend on *when and where* they are established. Hence the expression "laws of the nature" is often used in the general principle of relativity ([1], t. II, pp. 724). This understanding of natural laws is, in essence, only the expression of the ancient human aspiration to perceive something unchangeable in any phenomenon. One could say that the adoption, in physical theories, of a supposition included in one Engels' statement that a physical and natural law is performed *always and everywhere* where the causes of the consequences mentioned in the law are present ([5], pp. 160-161) is in question. By the way, one should bear in mind Einstein's ([1], t. II, p. 787) pointing out of the generality and objectivity of natural laws in the description of the nature (in sense of the independence of the coordinate system choice) is not deprived of metaphysically-realistic treatment of the nature as an *"objective reality"* ([6], p. 11). However, it seems that in the formulation of the contemporary theories it is not possible to avoid the use of, in the above manner introduced, laws of the nature, especially not in the expectations of some universal natural law.

A contribution to this is the fact that the general principle of relativity, which serves as the basis of these laws, is not in contradiction with any experiment. Because of that Einstein was convinced that this principle can not be replaced by any other more general one and that it will represent a *"necessary and effective tool in the solving of the unified*

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field problem" ([1], t. II, p. 662), i.e. in the establishing of the unified field theory, able to describe characteristics of the gravitational and electromagnetic field as well as the ones corresponding to the elementary particles, hence the entire physics. Without entering into the correctness of the statement that the unified field theory is, in this program, a'priori condemned to a failure (since, as it is known, to each of a great number of particles corresponds a field which should be embraced by this theory; [7], p. 63), we shall point out that this theory has also adopted the most general condition until now of the allowability of some physical theory and its laws: they should have the character of natural laws. Just this will be the subject further on.

Invariance

From all mentioned above, concerning natural laws, we can conclude: in order to be a natural law, some law of the physics must not depend on the choice of a coordinate system where it is described. Since laws are represented by mathe-matical relations, this means that the form of natural laws, i.e. of corresponding equations do not depend on the coordinate system where some law is formulated — they are **invariant** (unchangeable) with respect to the operations of the coordinate system substitution; it is said that these equations must be **covariant*** under the arbitrary transformations of the coordinate systems (general covariance) ([1], t. I, p. 459). In essence this **principle of general covariance** expresses mathematically the general principle of relativity. Similarly, Galileo-Newton's principle and the special principle of relativity might be expressed by the corresponding principles of covariance (in a narrow sense), but this is not of interest here because we speak only about the natural laws in the above-adopted sense.

The property of the invariance of natural laws is also called the **symmetry of laws** ([2], p. 13; [4], p. 39) and because of the preserved form of their equations during an (arbitrary) operation of the substitution of the coordinate system (called the operation of symmetry), it is clear that the symmetry in Weyl's sense ([2], p. 13) is in question and not the "common" symmetry of the objects. To say the truth, while pleading for "maximal" symmetry of the laws of physics which are expected to describe **natural phenomena**, one should maybe take into account the definitions of the notion of symmetry pretending to the universal generality ([9], pp. 16-17) but we do not see the way to impose some more general symmetries on natural laws, at least because of the necessity to point out to more general mathematical formalism corresponding to these symmetries.

The reason to pay such an attention to the property of invariance of natural laws lies in its role of the **criterion of naturalness** of any law of physics. In the case of nonexistence of this property, i.e. more exactly if it would not been perceived and explicitly formulated, hardly one law could be with assurance declared as a law of nature, except if these laws were introduced in another way; having been in essence postulated clearly the invariance of natural laws represents their contents, some **inherent property** of the structure of these laws ([4], p. 36) which obtains the character of a principle — the principle of invariance, the principle of symmetry — and can not be omitted during their formulation. This principle relates to the *"basic conditions which a physical theory should satisfy"* ([2], p. 13) because it represents a "test stone" for all natural laws ([4], p. 53) in this theory and therefore the condition of its allowability as well.

It should be noted that besides this invariance (in the sense of general relativity), there are, in other physical theories, to a certain extent similar properties which correspond to the principles of relativity, on the one side, and to the laws of these theories, on the other side. These others, "classical symmetries", are put into the category of the geometrical principles of invariance (by Wigner for example; [4], p. 23), which are not so focused to the "naturalness" of the laws as the invariance elaborated here and which have the character of a so-called dynamical symmetry, as it was pointed out by Fok ([4], p. 29); however, such symmetry is also represented by "new" invariances characterising so-called interactions, but they are not the invariances of all natural laws ([4], p. 40); (it is possible to speak about the role of the principle of invariance in quantum mechanics and the quantum field theory ([4], [10]), but there are attempts to find the quantum field theory as an extension of the general theory of relativity ([11])). However, there are also the opinions that a strict classification of the principles of invariance into geometrical and dynamical ones is not possible ([9], p. 18).

An attempt of a more profound analysis of both of these opinions will, without any doubt, surpass the extent as well as the level of this contribution, but it seems reasonable to point out the following: incompatibility arising when the character of invariance of the general theory of relativity should be approved certainly is the consequence of insufficiently explained role of the request for general covariance of the equations of natural laws in physics. Einstein himself at first (in 1916) underlines: "*It is clear that the physics which satisfies this principle also satisfies the general principle of relativity."* ([1], t. I, p. 459). Very soon (in 1918) he said that the laws of nature only "*find their natural expression in generally covariant equations."* ([1], t. I, p. 614) and in this way the role of the principle of general covariance is reduced to the question of the mathematical formulations of these laws. Later (in 1930) he pointed out that the general principle of relativity also *"represents a purely formal point of view and not some defined hypothesis on the nature"* ([1], t. II, p. 314). Fok asserts that the covariance of the equations of the general relativity is not conditioned by the nature of the (Riemannian) space in this theory ([12], p. 71), while Klein rejects the opinion that the physical meaning of the general covariance consists only of introducing a system of curvilinear coordinates ([13], p. 166). Although it is not easy to find a way in these and numerous other interpretations of the principle of general covariance, it could be said that the question of its true role in the formulation of the laws of nature probably is reduced to the always-present question on the physical meaning of the mathematical formalism of the general theory of relativity.

Anyhow, it is indisputable that the request for general covariance of the laws of the nature "*restricts the considered laws of the nature incomparably more than the special principle of relativity"* ([1], t. II, p. 756) and also more than the "news" invariances of the different interactions ([4], p. 40); in other words, the invariance of the general theory of relativity (kept in the unified field theory, too!) and covariance of the equations of its laws represent the contents of the most general principle of symmetry until now; therefore the symmetry of laws became "*essentially significant since the moment of beginning of the theory of relativity"* ([2], p. 20). This significance is reflected in the fact that the possi-

*) Although the notion of covariance is more general, it can be formally reduced to the one of invariance, using the **base vectors** in tensor calculus ([3], p. 54; [8], pp. 23-24).

bility of the **derivation** of natural laws from the principle of invariance is also pointed out in the theory of relativity ([4], p. 36); this possibility is also adopted by other physical theories ([2], p. 14; [4], p. 54; [9], p. 18).

Does the limit of the relativistic invariance exist? The answer should be contained in the fact that this invariance was postulated when it became evident that the "harmony of the nature" is not exhausted by the natural laws of the former theories, but also in the fact that this completeness is not yet achieved.

Mathematical formalism

The operation of the substitution of the coordinate system (contained in the principle of invariance) is performed, in mathematical sense, by the **general coordinate transformation***

$$\bar{x}^i = \bar{x}^i(x^j) \tag{1}$$

where x^i and $\bar{x}^i$ are the coordinates of the old and new coordinate system, respectively and i,j = 1,2,3,4, since the systems of coordinates in the space of four dimensions are in question (the fourth one corresponds to time). However, the mathematical apparatus used in prerelativistic physics and the special theory of relativity ([1], t. II, pp. 5-43) arose to be unable to describe the general covariance of the equations of natural laws (in the general theory of relativity), i.e. their covariance with respect to the transformations of the form (1), which should be the mathematical expression of the invariance of these laws requested by the general principle of relativity. In order to derive the mentioned equations in a generally covariant form a *"generalisation of the theory of invariants was necessary"* ([1], t. II, p. 50).

Namely, understanding the laws of the nature as some mathematical objects unchangeable under the coordinate transformations of the form (1), i.e. as **invariants** ([8], p. 12), one should bear in mind the possibility to determine these objects, with respect to the assumed coordinate systems, not only by a single one number or function (or, finally, by a scalar equation!), when we speak about **scalar invariant** ([1], p. 38), but also by a system or set of these quantities ([1], p. 37; [8], pp. 12-13) — the **components** of the object in the chosen coordinate system; namely, just in that case the invariance of objects with respect to the above coordinate transformation might not mean the invariance of their components with respect to this change — there is no reason to suppose or request this from them. How then these components should change under the transformation (1) and by which means their changes could be described? The answer to this question will, basically, determine a particular class of invariant objects (where we would like to classify the natural laws, too), because there is not only one way of transformation of their components and this gives a possibility to distinguish these objects ([3], p. 54; [8], p. 13).

In he general theory of relativity it is adopted to speak about invariant objects in the sense of the **absolute differential calculus** — the calculus independent with respect to the changes of coordinates, founded by Gauss and Riemann and further developed by Ricci and Levi-Civita ([1], t. II, p. 50) in the form close to today's **tensor calculus**. Such objects in the tensor calculus are **tensors** ([1], t. I, p. 462), i.e. **tensor fields**. Their components, given in one system of coordinates, change in a completely determined way if another system is introduced; for example, the transformation law for the components of the **covariant tensor of the second order** (designated by **w**), under the change of the variables (1), reads ([14], p. 47)

$$\bar{w}_{ij} = w_{mn} \frac{\partial x^m}{\partial \bar{x}^i} \frac{\partial x^n}{\partial \bar{x}^j} \tag{2}$$

where Einstein's summation convention is respected ([1], t. I, p. 464; [14], p. 4) and again i,j,m,n = 1,2,3,4, since four coordinates are still in question. Of course, the transformation law of the components of tensor objects is not an arbitrarily prescribed law, no matter how tensor calculus might seem arbitrary, but it could be said that it represents the natural generalisation of the corresponding rule for **vectors**, although we cannot concern us with it here.

Therefore, invariants in the general theory of relativity, i.e. the natural laws should have the tensor character. However, it will be certainly naive to believe that Einstein decided on tensors as invariant objects in his theory because of nonexistence in his time of another, well-suited mathematical theory. His several times expressed ([1], t. I, p. 452; t. II, p. 60), conviction that the tensor calculus should be considered as a *"mathematical apparatus necessary for the formulation of the laws of the general theory of relativity"* resulted after several years of doubt in the physical applicability of already existing results of this calculus ([1], t. II, p. 406). However, it seems that we can speak about the **necessity** of the tensor calculus for the foundation of the general theory of relativity only if we whish to point out that it has **fulfilled** the requests which this theory could impose to any other theory of invariant quantities (if this would be considered to be of interest), because this calculus appeared **sufficient** in the general theory of relativity and, on the other side, it is difficult to approve that this theory could not be build using another theory of invariants. After all, the problems in the development of the unified field theory, produced by the fact that *"the general principle of relativity could be applied in a satisfactory way only to the gravitational fields and not to the entire field"* ([1], t. II, p. 662), did not force Einstein to abandon this principle but to declare: "*Until now it is unknown to us what kind of mathematical apparatus should be applied for the description of the entire field in the space and what are the generally invariant natural laws to which this field is subordinated."* ([1], t. II, p. 662), thus raising the question of the **necessity** of the existing mathematical formalism, since this new theory should include the general theory of relativity as well. However, here it can be spoken only about the announced requests, which the tensor calculus fulfilled completely.

First of all, as the objects, which do not depend on the coordinate system where they are considered, the tensors could express some **natural** characteristics of the phenomenon in question ([14], p. 39), to which so much attention is devoted in the general theory of relativity. But the true reason to separate the tensor calculus from the other theories of invariants is in the fact that, due to the form of the transformation laws of the components of tensors, it has been achieved that the form of natural laws, described by the tensor equations, do not depend on the choice of coordinate systems, as it is requested by the principle of the general covariance. Namely, this law is such that, as it can be seen from (2), the components of any tensor with respect to the new coordinate system, are the **linear** and **homogeneous** functions of its components in the old system; hence, if all components of a tensor are equal to zero with respect

*) Which is supposed to be sufficiently "smooth".

to any coordinate system, they will be equal to zero as well with respect to all others systems and we then speak about the **zero tensor** ([14], p. 49). The significance of this result of the tensor calculus for the general theory of relativity is

[18] DRAŠKOVIĆ, Z. *Thin shell constitutive equations - a three-field approximations approach*. M.Sc. Thesis, Faculty of Mathematics, Belgrade, 1988. *(in Serbian)*

[19] DRAŠKOVIĆ,Z. *Thin shell field equations - an invariant approach*, Ph.D. Thesis, Faculty of Mathematics, Belgrade, 1990. *(in Serbian)*

[20] VUJIĆ,S., BERKOVIĆ,M., KUZMANOVIĆ,D., MILANOVIĆ,P., SEDMAK,A. and MIĆIĆ,M. *The Application of the Finite Element Method to Geostatic Analysis in Mining*. Faculty of Mining and Geology, Belgrade, 1991. *(in Serbian)*

[21] BERKOVIĆ,M., DRAŠKOVIĆ,Z. On the essential mechanical boundary conditions in two-field finite element approximations. *Computer Methods in Applied Mechanics and Engineering,* 1991, no.91, pp.1339-1355.

[22] BERKOVIĆ,M., DRAŠKOVIĆ,Z. A two-field finite element model related to the Reissner's principle. *Teorijska i primenjena mehanika,* 1994, no.20, pp.17-35.

[23] DRAŠKOVIĆ,Z. Stress-resultants in the shell theory - asymmetric or symmetric? *Teorijska i primenjena mehanika,* 1995, no.21, pp.19-28.

[24] MIJUCA,D. *Continual interpretation of the solid body FE stress state*, M.Sc. Thesis, Faculty of Mathematics, Belgrade, 1995. *(in Serbian)*

[25] DRAŠKOVIĆ,Z. Contribution to a more accurate nodal stresses determination in the classical finite element method, *Naučnotehnički pregled*, 1995, vol.XLV, no.9, pp.3-8. *(in Serbian)*

[26] DRAŠKOVIĆ,Z. Contribution to the invariant derivation of finite element equations of motion in curvilinear coordinates. *Facta Universitatis, Series "Mechanics, Automatic Control and Robotics",* 1995, vol.2, no.6, pp.25-32.

[27] BERKOVIĆ,M., DRAŠKOVIĆ,Z. *Mixed finite element model in the mechanics of solids*. Serbian Academy of Sciences and Arts, Scientific Meetings LXXXIII, Department of Technical Sciences, 1996, no.2, pp.69-76. *(in Serbian)*

[28] MIJUCA,D., DRAŠKOVIĆ,Z. and BERKOVIĆ,M. Stress recovery procedure based n the known displacements*. Facta Universitatis, Series "Mechanics, Automatic Control and Robotics",* 1997, vol.2, no.7/2, pp.513-523.

[29] MIJUCA,D. *Primal-mixed finite element approach in solid mechanics*, Ph.D. Thesis, Faculty of Mathematics, Belgrade, 1999. *(in Serbian)*

[30] BERKOVIĆ,M., MIJUCA,D. *On the main properties of the primal-mixed finite element formulation*. Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 1999, vol.2, no.9, pp.903-920.

[31] DRAŠKOVIĆ,Z. One of dangers in the approximation of tensor fields. *Naučnotehnički pregled,* 2000, vol.L, no.1, pp.35-41. *(in Serbian)*

[32] DRAŠKOVIĆ,Z.Contribution to a more accurate determination of equivalent nodal forces in the classical finite element method. *Naučnotehnički pregled,* 2001, vol.LI, no.3, pp.16-27. *(in Serbian)*

[33] DRAŠKOVIĆ,Z. *Numerical comparison of the scalar, pseudoinvariant and invariant approach in the derivation of finite element equations of motion in curvilinear coordinates.* Facta Universitatis, Series "Mechanics, Automatic Control and Robotics", 2002, vol.3, no.12, pp.351-357.

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O prirodnim zakonima i njihovom formalnom opisivanju

Dat je kratak pregled razvoja ideje o invarijantnosti zakona u fizikalnim teorijama. Ukazano je na tenzorski račun kao za sada nezamenljiv račun invarijanata. Pomenuta je i jedna neposredna posledica prihvatanja ideje o invarijantnosti u primenjenoj mehanici.

Ključne reči: **prirodni zakoni, invarijantnost (kovarijantnost), tenzorski račun.**

Sur les lois de la nature et leur description formelle

Le développement de l'idée sur l'invariance des lois dans les théories physiques est brièvement donné. Le calcul tensoriel est mis en relief comme le calcul des invariants irremplaçable jusqu'au présent. Une conséquence directe de l'acceptation de l'idée sur l'invariance en mécanique appliquée est aussi mentionnée.

Mots-clés: **lois de la nature, invariance (covariance), calcul tensoriel.**