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Stress concentration at points around elliptic openings in in-plane stressed multilayer orthotropic plates

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The paper discusses the calculation of stress at points along the elliptic opening contour in an in-plane stressed infinite orthotropic plate with two orthotropy axes. The stress dependence on material elasticity characteristics and a multi-layer plate stacking sequence as well as on a position of the ellipse semi-axes for prescribed stress conditions has been presented by numerical examples while the calculation results have been illustrated by diagrams and numeric data. The drawn conclusions pointing out a danger of stress concentration and orthotropy effects can be very useful in engineering practice.

Key words: **stress concentration, orthotropic plate, plane stress state, elliptic opening, multilayer plate.**

Introduction

AN opening in the stressed plate is a source of the stress concentration and it causes a local stress change at the points around the opening, unlike the stresses at the same points of a plate without an opening, which is stressed in the same way. The comparison of the stress states at the points of stressed plates with and without openings, leads to the conclusion that the local stress states at the points around the opening are considerably different, but the stress states at the points sufficiently far from the opening are almost the same.

The ratio of the maximum stress at the opening contour and the maximum normal stress that occurs at the points far from the opening represents the stress concentration coefficient/factor.

Stress concentration can be considerably more expressed at the points of stressed plates made of orthotropic materials than in case of plates made of isotropic materials, which depends on the relation between the elastic constants of the orthotropic material.

Besides the constructions made of classic orthotropic materials (veneer, for example), the problem of the stress analysis becomes particularly interesting in case of parts of modern constructions made of composite materials which, under specific conditions, behave as orthotropic ones.

For example, for a plate with a circular opening, which is subjected to extension, according to Ref. [1], p.290, the stress concentration factor is 3 for isotropic material, 4 for unidirectional glass/epoxy and about 6 for boron/epoxy, while for graphite/epoxy it goes up to 9.

Such high stress concentration factors influence the reduction of the static strength and fatigue resistance of the structure in question, because of the rapid propagation of the initial damage which occurs in the zone of the stress concentration source in the form of crack initiation and propagation.

Concept of orthotropy for the plane stress state

The plane (two-dimensional) stress state occurs in the bodies of which one dimension, for example, in the direction of z-axis, is very small in comparison with the other two dimensions, in the directions of the x and y axes. This is the case with thin plates subjected to the forces uniformly distributed along the plate thickness and parallel to the plate middle surface.

Since there are no forces parallel to the middle surface on the outer surfaces of the plate, it could be assumed that the components, σ_z, τ_{xz} and τ_{yz}, of the stress tensor, on these surfaces, equal zero. Taking the small plate thickness into consideration, it can be assumed that these stresses equal zero at the points of the cross section along the plate thickness, i.e.

$$\sigma_z = \tau_{xz} = \tau_{yz} = 0 \tag{1}$$

Because of the small plate thickness, the remaining components σ_x, σ_y and τ_{xy} of the stress tensor can also be considered constant along the plate thickness i.e. they do not depend on the coordinate z but only on the coordinates x and y.

Generally speaking, the generalized Hooke's law for the plane stress state has the following form

$$\{\sigma\} = [C]\ \{\varepsilon\} \tag{2}$$

where $[C]=[c_{ij}]$ is the plate stiffness matrix.

The second form of Hooke's law which gives the strain--stress relations for the plane stress state, is

$$\{\varepsilon\} = [S]\ \{\sigma\} \tag{3}$$

where $[S]$ is the compliance matrix inverse to the stiffness matrix, i.e.

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$$[S]=\left[s_{ij}\right]=[C]^{-1} \tag{4}$$

For stressed orthotropic plates, two mutually perpendicular directions (principal directions of orthotropy, 1 and 2) can be defined in the plate plane, for which there is no shear between layers of the plate, i.e. $\gamma_{12}=0$, if the plate is extended in these directions with the uniformly distributed forces.

In this case of stress, the principal directions of the orthotropy are the principal direction of stresses, as well.

It results from (3) that for such defined directions some coefficients of compliance matrix [S] must equal zero, i.e. $s_{16}=s_{26}=0$. Keeping in mind (4), the conclusion is that for the orthotropic plate some stiffness coefficients must equal zero, i.e. $c_{16}=c_{26}=0$.

Hence, strain-stress relations (3) for the principal directions of orthotropy for the plane stress state come in the following form

$$\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{Bmatrix}=\begin{bmatrix}s_{11} & s_{12} & 0\\ s_{12} & s_{22} & 0\\ 0 & 0 & s_{66}\end{bmatrix}\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \tau_{12}\end{Bmatrix} \tag{5}$$

The coefficients of the compliance matrix depend on the elastic constants of the plate material and this matrix is obtained as (see Ref. [1], p.38)

$$[S]=\begin{bmatrix}\dfrac{1}{E_1} & -\dfrac{\nu_{21}}{E_2} & 0\\ -\dfrac{\nu_{12}}{E_1} & \dfrac{1}{E_2} & 0\\ 0 & 0 & \dfrac{1}{G_{12}}\end{bmatrix} \tag{6}$$

where: E_1 and E_2– Young's moduli in the principal directions of orthotropy 1 and 2, respectively, G_{12}– shear modulus in 1 – 2 plane and ν_{12} and ν_{21}– Poisson's ratios for transverse strain which, due to the compatibility of strains have such values that $\dfrac{\nu_{12}}{E_1}=\dfrac{\nu_{21}}{E_2}$.

On the macroscopic level, a multilayer plate can be analysed as orthotropic, if the following conditions are fulfilled (see Ref. [1], p. 162 – 172):

- stacking sequence should be symmetric to the middle plane, which results in the coupled in-plane and bending stiffness matrix [B]=[0] and
- for each layer with $+\theta$ orientation, there should be a layer of the same thickness and properties with $-\theta$ orientation, which results in the fact that the in-plane stiffness matrix coefficients $A_{16}=A_{26}=0$.

Under these conditions, the multilayer plate can be considered as an orthotropic plate with the following equivalent elastic constants

$$E_1=\frac{A_{11}A_{22}-A_{12}^2}{A_{22}\cdot T};\; E_2=\frac{A_{11}A_{22}-A_{12}^2}{A_{11}\cdot T};\; G_{12}=\frac{A_{66}}{T};\; \nu_{12}=\frac{A_{12}}{A_{22}} \tag{7}$$

where A_{ij} (i,j = 1,2,6) are the in-plane stiffness matrix coefficients* and T is the thickness of the multilayer plate.

Basic stress state for the stressed plate with an opening

We are considering an infinitely thin orthotropic plate with an elliptic opening, subjected to shear and extension in two mutually perpendicular directions (Fig.1), which satisfies the conditions of plane stress states, assuming that:

1. The plate is a thin material surface, and there is no shear between the layers of the plate.
2. The contour of the elliptic opening is free from external stresses, which is the case with almost all constructive, technological and maintenance openings, except those for the fasteners, and
3. External loads do not cause instability of the plate, neither global nor local at the opening i.e. buckling of the plate and out-of-plane of points of the plate middle surface do not occur.

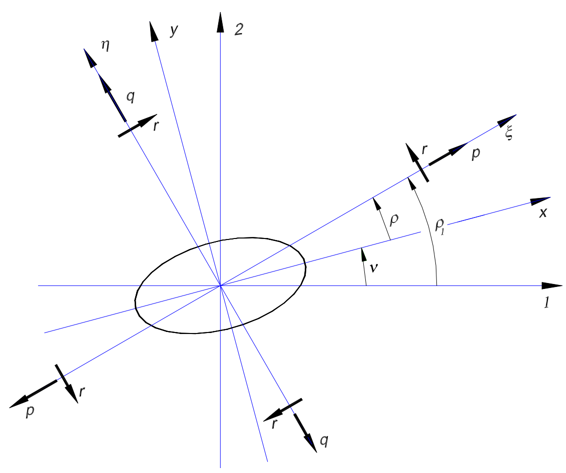

Figure 1. Load scheme of the plate with the elliptic opening

In relation to the principal material direction of the orthotropy 1, the larger semi-axis x of the elliptical opening makes an angle ν, and the perpendicular directions (ξ and η) of extension make the angles ρ_I and (ρ_I+90°).

Load components are defined as unit forces through the plate thickness

$$p=\frac{N_x}{T};\, q=\frac{N_y}{T};\, r=\frac{N_{xy}}{T}=\frac{N_{yx}}{T} \tag{8}$$

where N_x and N_y are the extension forces and N_{xy} is the shear force per unit of the length of the outer plate edge.

For the plate without opening, which is loaded according to Fig.1, the stress tensor components have the constant values in all the points of the plate. This stress state in the opening-free plate is defined as the basic stress state $\left\{\sigma^o\right\}$.

For the coordinate system x - y, the basic stress state is obtained by using the transformation equations for expressing stresses in an x - y coordinate system in terms of stresses in a ξ - η coordinate system (according to Ref. [2], p.51)

* For the calculation of the in-plane stiffness matrix coefficients see Ref.[1], p.156.

$$\begin{aligned} \sigma_x^o &= p \cos^2 \rho + q \sin^2 \rho - r \sin 2\rho \\ \sigma_y^o &= p \sin^2 \rho + q \cos^2 \rho + r \sin 2\rho \quad \left(\rho = \rho_1 - \nu\right) \\ \tau_{xy}^o &= \frac{1}{2}\left(p - q\right)\sin 2\rho + r \cos 2\rho \end{aligned} \tag{9}$$

Additional components of stress tensor due to the plate opening

If an opening, depicted in Fig.1, is made in the plate, it causes the stress change at the points near the opening, so the new stress state of the loaded plate with the opening can be defined by the stress components

$$\begin{aligned} \sigma_x &= \sigma_x^o + \sigma_x^* \\ \sigma_y &= \sigma_y^o + \sigma_y^* \\ \tau_{xy} &= \tau_{xy}^o + \tau_{xy}^* \end{aligned} \tag{10}$$

where σ_x^*, σ_y^* and τ_{xy}^* are the additional stress components which are due to the presence of the opening in the plate.

Determining the stress components is reduced to defining Airy's stress function $U = U(x, y)$ (Ref. [3], p.23), so that

$$\sigma_x = \frac{\partial^2 U}{\partial x^2} \quad \sigma_y = \frac{\partial^2 U}{\partial y^2} \quad \tau_{xy} = \frac{\partial^2 U}{\partial x \partial y} \tag{11}$$

By introducing expressions (11) into the compatibility equation for the plane stress state

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \tag{12}$$

and by using relations (5), the basic equation of the theory of elasticity of the thin plate is obtained

$$s_{22} \frac{\partial^4 U}{\partial x^4} + \left(2s_{12} + s_{66}\right) \frac{\partial^4 U}{\partial x^2 \partial y^2} + s_{11} \frac{\partial^4 U}{\partial y^4} = 0 \tag{13}$$

The solution of the partial differential equation (13) can be assumed in the form (Ref. [4], p.111)

$$U(x, y) = F\left(x + \mu y\right) \tag{14}$$

By introducing expression (14) into eq.(13), the characteristic equation is obtained

$$s_{11}\mu^4 + \left(2s_{12} + s_{66}\right)\mu^2 + s_{22} = 0 \tag{15}$$

By introducing the relations of matrix coefficients [*S*] with material elastic constants (6), the characteristic equation takes the following form (Ref. [5], p.104)

$$\mu^4 + \left(\frac{E_1}{G_{12}} - 2\nu_{12}\right)\mu^2 + \frac{E_1}{E_2} = 0 \tag{16}$$

It is obvious that the roots of the characteristic equation (15), or (16), do not depend on the external load, but only on the elastic constants of the plate material.

S. G. Lekhnitzki showed (Ref. 4, p.111–113) that the characteristic equation cannot have real roots but, depending on the ratios between the elastic constants, three cases are possible:

a) roots are imaginary, different and conjugate in pairs

$$\mu_1 = \beta i,\ \mu_2 = \delta i,\ \overline{\mu}_1 = -\beta i,\ \overline{\mu}_2 = -\delta i \ \left(\beta,\ \delta > 0\right) \tag{17}$$

b) two pairs of equal imaginary and conjugate roots

$$\mu_1 = \mu_2 = \beta i,\ \overline{\mu}_1 = \overline{\mu}_2 = -\beta i,\ \left(\beta > 0\right) \tag{18}$$

c) roots are complex and conjugate

$$\begin{aligned} &\mu_1 = \alpha + \beta i,\ \mu_2 = -\alpha + \beta i,\ \overline{\mu}_1 = \alpha - \beta i, \\ &\overline{\mu}_2 = -\alpha - \beta i \ \left(\beta > 0\right) \end{aligned} \tag{19}$$

where α and β are real constants and $i = \sqrt{-1}$ is an imaginary unit.

The roots μ_1 and μ_2 of the characteristic equation are usually called complex parameters.

For the given problem, it is necessary to recalculate the complex parameters for the local coordinate system $x - y$, and it is done according to the transformation formula (Ref. [4], p.50)

$$\mu_j' = \frac{\mu_j \cos\nu - \sin\nu}{\cos\nu + \mu_j \sin\nu} \quad (j = 1, 2) \tag{20}$$

In this way, the planes z_1 and z_2 are obtained, by an affine transformation, from the given complex plane $z = x + iy$ (Ref. [4], p.152)

$$\begin{aligned} z_1 &= x + \mu_1' y = x_1 + i \ y_1 \\ z_2 &= x + \mu_2' y = x_2 + i \ y_2 \end{aligned} \tag{21}$$

With this transformation, the given ellipse in the plane z is transformed into the corresponding ellipses in the planes z_1 and z_2.

If the contour of the opening is free from external stress the additional components of the stress tensor due to the opening, are defined by the following expressions

$$\begin{aligned} \sigma_x^* &= 2 \ \text{Re} \left[\mu_1'^2 \varphi_o'\left(z_1\right) + \mu_2'^2 \psi_o'\left(z_2\right)\right] \\ \sigma_y^* &= 2 \ \text{Re} \left[\varphi_o'\left(z_1\right) + \psi_o'\left(z_2\right)\right] \\ \tau_{xy}^* &= -2 \ \text{Re} \left[\mu_1' \varphi_o'\left(z_1\right) + \mu_2' \psi_o'\left(z_2\right)\right] \end{aligned} \tag{22}$$

where Re denotes the real parts of the expressions in brackets.

In the expression (22), the following designation is used

$$\varphi_o' = \frac{d\varphi_o\left(z_1\right)}{dz_1};\ \psi_o' = \frac{d\psi_o\left(z_2\right)}{dz_2} \tag{23}$$

where $\varphi_o\left(z_1\right)$ and $\psi_o\left(z_2\right)$ are analytical functions of their complex arguments, and they are determined from the boundary conditions on the edge of the opening.

Method of the function of a complex variable and conformal mapping for determining the stress state components at the points around an elliptic opening

For determining the analytical functions $\varphi_o\left(z_1\right)$ and $\psi_o\left(z_2\right)$ for anisotropic materials, G. N. Savin [3] adapted an original method of the function of a complex variable, developed by N. I. Muskhelishvili [6] for analysing the stress state in the in-plane loaded isotropic plates.

If an infinite plate, with the opening of any shape and dimension, is considered as a plane from which a corresponding part is removed, the aforementioned method is based on the conformal mapping of the given area on the inside (or the outside) of the unit circle.

If we know the analytical expression of the function

$$z=\omega(\zeta) \tag{24}$$

the solution of the problem is reduced to solving two functions

$$\begin{aligned}\varphi_o(z_1)&=\varphi_o\left[\omega_1(\zeta)\right]=\Phi_o(\zeta)\\ \psi_o(z_2)&=\psi_o\left[\omega_2(\zeta)\right]=\Psi_o(\zeta)\end{aligned} \tag{25}$$

A detailed and complete procedure of determining these analytical functions is given in Ref. [5], p.13–28. (For the explanation of the method, see Ref. [3], p.153–160 and [6]).

From the viewpoint of the stress concentration, the stress state at the contour points of the opening is most interesting, because the stresses are of the highest level at these points. In practice, the stresses are calculated at the discrete points along the opening contour.

For this reason, the parametric equations of the elliptic opening are used

$$\begin{aligned}x&=a\ \cos\theta\\ y&=b\ \sin\theta\end{aligned} \tag{26}$$

where θ is the angle parameter of the ellipse.

For practical applications, the following expressions for determining the additional stress state along the opening contour are very convenient (see Ref. [5], p.29 and [7])

$$\begin{aligned}\sigma_x^*&=\operatorname{Re}\left\{\frac{1}{\mu_1^{'}-\mu_2^{'}}\left[\mu_1^{'2}h_1g_2-\mu_2^{'2}h_2g_1\right]\right\}\\ \sigma_y^*&=\operatorname{Re}\left\{\frac{1}{\mu_1^{'}-\mu_2^{'}}\left[h_1g_2-h_2g_1\right]\right\}\\ \tau_{xy}^*&=-\operatorname{Re}\left\{\frac{1}{\mu_1^{'}-\mu_2^{'}}\left[\mu_1^{'}h_1g_2-\mu_2^{'}h_2g_1\right]\right\}\end{aligned} \tag{27}$$

where the following designations are introduced

$$\left.\begin{aligned}h_j&=\frac{1-i\mu_j^{'}k}{\delta_j\sqrt{\delta_j^2-1-\mu_j^{'2}k^2}+\delta_j^2-1-\mu_j^{'2}k^2}\\ \delta_j&=\cos\theta+\mu_j^{'}k\sin\theta\\ g_j&=ik\sigma_x^o-\mu_j^{'}\sigma_y^o-\left(1-i\mu_j^{'}k\right)\tau_{xy}^o\end{aligned}\right\}\ (j=1,\ 2) \tag{28}$$

and where $k=\dfrac{b}{a}$ is the parameter of the shape of the opening, i.e. the semi-axes ratio of the ellipse.

Based on expression (10), the components of the total tensor of the stress state at the points along the contour of the opening are obtained.

Expressions (27) become undefined for the complex parameters equal values, so they can not be applied in this case.

In cases such as this, expressions (27) are modified by the limiting values (see Ref. [5], p.29 and [8]).

The curvilinear rectangular coordinate system (Fig.2) is often used for the analysis of the stress state. The axes are set in the directions of the tangent and the normal at the points of the opening contour.

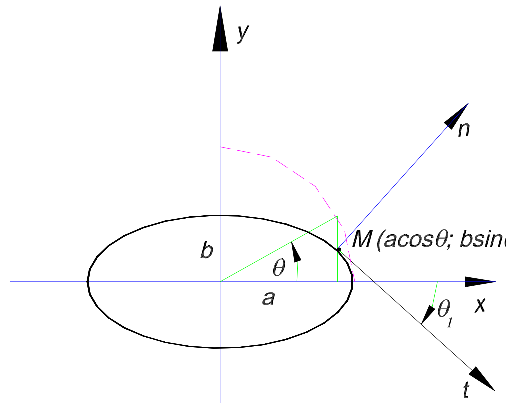

Figure 2. Curvilinear rectangular coordinate system for the analysis of the stress state along the opening contour

For the new coordinate system, the components of the stress state are obtained by transforming the coordinates

$$\begin{aligned}\sigma_t&=\sigma_x\ \cos^2\theta_1+\sigma_y\ \sin^2\theta_1+\tau_{xy}\ \sin 2\theta_1\\ \sigma_n&=\sigma_x\ \sin^2\theta_1+\sigma_y\ \cos^2\theta_1-\ \tau_{xy}\sin 2\theta_1\\ \tau_{tn}&=\frac{1}{2}\left(\sigma_y-\sigma_x\right)\sin 2\theta_1+\ \tau_{xy}\cos 2\theta_1\end{aligned} \tag{29}$$

where θ_1 is the angle between the tangent of the ellipse at the given point and the positive direction of the x-axis. From the analytic geometry, the following relation between the angle θ_1 and the angle parameter of ellipse θ is obtained

$$tg\ \ \theta_1=-k\cdot ctg\ \ \theta \tag{30}$$

For the opening boundary that is free from external stresses, the stress state on the opening edge is reduced to the state in which the component σ_t differs from zero, while the components σ_n and τ_{tn} equal zero.

Stress state at the points on the contour of the opening for isotropic material

For the case of the isotropic material, the stress state at the points of the opening contour depends only on the shape of the opening (parameter $k=\dfrac{b}{a}$) and the position of the semi-axes of the opening in relation to the load directions (angle ρ in Fig.1).

The normal stress at the points of the opening contour in the direction of the tangent to the contour is

$$\sigma_{t\,(iz)}=\sigma_{t\,p}+\sigma_{t\,q}+\sigma_{t\,r} \tag{31}$$

where $\sigma_{t\,p}$, $\sigma_{t\,q}$ and $\sigma_{t\,r}$ are the components of the normal stress at the points of the opening contour in the direction of the tangent line to the contour of the elliptic opening, caused by the individual load components in Fig.1, and given by the following expressions

$$\sigma_{t\,p} = p\frac{(1+k)^2\sin^2(\rho-\theta)-\sin^2\rho-k^2\cos^2\rho}{\sin^2\theta+k^2\cos^2\theta}$$
$$\sigma_{t\,q} = q\frac{(1+k)^2\cos^2(\rho-\theta)-\cos^2\rho-k^2\sin^2\rho}{\sin^2\theta+k^2\cos^2\theta} \quad (32)$$
$$\sigma_{t\,r} = r\frac{(1+k)^2\sin 2(\rho-\theta)-(1-k^2)\sin 2\rho}{\sin^2\theta+k^2\cos^2\theta}$$

(Previous expressions are derived in Ref. [5], p.105, based on the procedure which is shown in Ref. [3], p.72–75).

Numerical examples

The described model of the stress analysis of the stressed plate with the opening will be illustrated by practical examples of the comparative analysis of the stress state at the discrete points along the contour of the elliptic opening for orthotropic and isotropic materials.

For the given shape (parameter $k=\frac{b}{a}$) and the position of the opening (angle ν), the calculation is made for the general case of plate loading, which is given by the ρ_l angle and the ratio between the load components $p:q:r=1:a_1:a_2$, or in particular cases, for pure shear loading ($p=q=0 \ \wedge \ r\neq 0$).

According to the described model, the reduced value of the normal stress $\overline{\sigma_t}$, for the relative stress state at the points along the opening contour in the direction of the tangent line to the opening contour in each calculation point, is obtained as a result of the calculation where

$$\overline{\sigma_t}=\frac{\sigma_t}{p}=f_1(\theta) \ \vee \ \overline{\sigma_t}=\frac{\sigma_t}{r}=f_2(\theta) \quad (33)$$

depending whether it is a general case of load or a special case of pure shear.

For that purpose, the author of this paper has made a FORTRAN PC program which uses the calculation step $\Delta\theta = 1^o$, for $0^o \le \theta \le 180^o$.

Based on the results obtained for both materials, the extreme values of dimensionless coefficients are determined. These coefficients are the maximum and minimum values of the reduced value of the normal stress for the relative stress state.

$$K_1=\left(\overline{\sigma_t}\right)_{\max} \quad \text{for } \overline{\sigma_t}\ge 0$$
$$K_2=\left(\overline{\sigma_t}\right)_{\min} \quad \text{for } \overline{\sigma_t}< 0 \quad (34)$$

The calculation step $\Delta\theta = 1^o$ provides very high calculation accuracy.

In a certain way, these coefficients are a measure of sensitivity of the material to the presence of the opening of the given shape and position and for the given external load.

As an illustration of this sensitivity two symmetrical stacking sequences of the composite plates, each with 8 layers satisfying the conditions of orthotropy, are analysed

- stacking sequence A [+45°/-45°/+45°/-45°/-45°/+45°/-45°/+45°]
- stacking sequence B [0°/90°/0°/90°/90°/0°/90°/0°]

The material is graphite/epoxy T300/5208 [7] with the following elastic constants:

$E_1 = 181000\frac{N}{mm^2}$ $E_2 = 10300\frac{N}{mm^2}$ $G_{12} = 7170\frac{N}{mm^2}$ $\nu_{12} = 0.28$
and layer thickness 0.125 mm.

The shape parameter of the elliptical opening is k = 0.5.

Example 1. Load is defined by the ratio between the components p:q:r = 1:1:0 (extension in two mutually perpendicular directions) and by the angle ρ_l = 0° (directions of extension are the principal directions of orthotropy).

Changes in the reduced values of the normal stress $\overline{\sigma_t}$ along the opening contour for both stacking sequences, as well as for the isotropic material, are shown by diagrams in Fig.3.

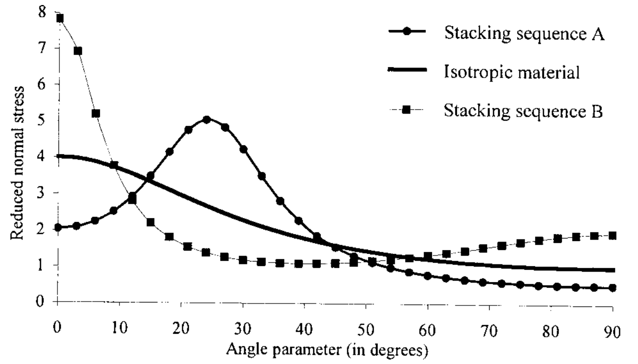

Figure 3. Changes in the values of $\overline{\sigma_t}$ at the points on the opening contour (for Example 1)

The calculation based on (34) shows that K_1=5.04246 (for θ= 24°) for the stacking sequence A, and K_1=7.83009 (for θ = 0°) for the stacking sequence B, while K_1=4.0 (for θ= 0°) for the isotropic material. That means that both stacking sequences are more sensitive to this source of the stress concentration than the isotropic material for the given loading conditions: stacking sequence A about 26%, and stacking B sequence even 96%. The stacking sequence B is about 55% more sensitive than the stacking sequence A.

Example 2. Load is defined by the ratio between the components p:q:r = 1:0:0 (unidirectional extension) and by the angle ρ_l = - 45°.

The changes in the reduced values of the normal stress $\overline{\sigma_t}$ along the opening contour for both stacking sequences, as well as for the isotropic material, are shown by diagrams in Fig.4.

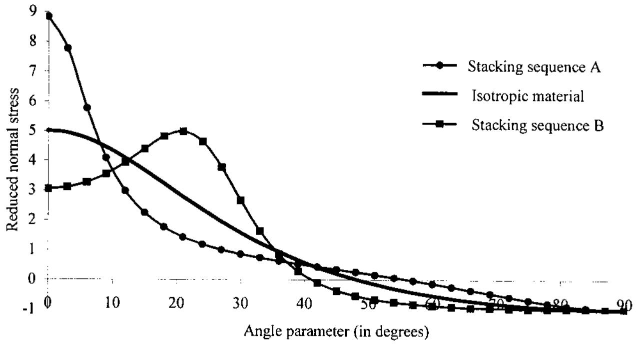

Figure 4. Changes in the values of $\overline{\sigma_t}$ at the points on the opening contour (for Example 2)

The calculation based on (34) shows that K_1 = 8.83010 (for θ= 0°) and K_2 = -1.0 (for θ= 90°) for the stacking sequence A and K_1 = 4.97594 (for $\theta = 21^o$) and K_2 = -1.0 (for $\theta = 90^o$) for the stacking sequence B, while K_1 = 5.0 (for $\theta = 0^o$) and K_2 = -1.0 (for $\theta = 90^o$) for the isotropic material. That means that the stacking sequence A is considerably more sensitive (about 77%), while the stacking sequence B is insignificantly less sensitive to this source of the stress concentration than the isotropic material for the given loading conditions.

The described procedure can also be applied to determine the best, as well as the worst, orientation of the opening for the given loading conditions.

The calculation of the coefficients K_1 and K_2 is made for each value from 0^o to 180^o of the angle ν with the step $\Delta\nu = 1^o$. The following relations are obtained

$$K_1 = f_1(\nu) \ \wedge \ K_2 = f_2(\nu) \qquad (35)$$

The best orientation of the opening is defined by the angle ν for which the minimum values of K_1 or $|K_2|$, are obtained. The worst orientation of the opening is defined by the angle ν for which the maximum values of K_1 or $|K_2|$, are obtained.

Relation (35) is illustrated on the aforementioned stacking sequences for two loading cases. This calculation step also provides very high calculation accuracy.

Example 3. Load is defined by the ratio between the components p:q:r = 1:0:0 (unidirectional extension) and by the angle ρ_l = - 30^o. The shape parameter of the opening is k=0.5.

The dependence of the coefficients K_1 and K_2, for both stacking sequences, on the angular orientation of the opening regarding the principal directions of orthotropy is given in Fig.5.

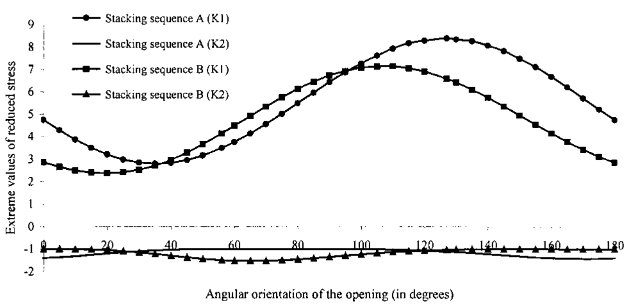

Figure 5. Dependence of the extreme values of $\bar{\sigma}_t$ on the angle of orientation of the opening (for Example 3)

The calculation shows that from the aspect of extension stresses for the stacking sequence A, the best orientation of the opening is defined by the angle $\nu = 35^o$ (K_1 = 2.805), and the worst by the angle $\nu = 127^o$ (K_1 = 8.389). From the aspect of the compressive stresses, the best orientation of the opening is defined by the angle $\nu = 56^o$ (K_2 = - 0.999) and the worst by the angle $\nu = 170^o$ (K_2 = - 1.428).

For the stacking sequence B, the best orientations of the opening are defined by the angle $\nu = 20^o$ (K_1 = 2.382), or ν = 145^o – 180^o (K_2 =-1.000 to – 1.003). The worst orientations of the opening are defined by the angle $\nu = 108^o$ (K_1 = = 7.156), or $\nu = 65^o$ (K_2 = - 1.533).

Example 4. Load is defined by the ratio between the components p:q:r = 1:0.3:0.8 and by the angle ρ_l = 50^o. The shape parameter of the opening is k = 2.2.

The dependence of the coefficients K_1 and K

applied to each layer, taking into consideration the strain compatibility between the layers.

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Koncentracija napona u tačkama oko eliptičnog otvora ravanski napregnute višeslojne ortotropne ploče

Prikazan je postupak proračuna napona u tačkama duž konture eliptičnog otvora ravanski opterećene beskonačne ploče, s dvema osama ortotropije. Zavisnost napona od karakteristika elastičnosti materijala i šeme slaganja višeslojne ploče, kao i položaja poluosa elipse otvora ploče za zadate posebne uslove opterećenja prikazani su na numeričkim primerima, a rezultati proračuna ilustrovani dijagramima i numeričkim podacima. Izvedeni su, za inžinjersku praksu, korisni zaključci o opasnosti koncentracije napona i uticaja ortotropnosti.

Ključne reči: **koncentracija napona, ortotropna ploča, ravansko stanje napona, eliptični otvor, višeslojna ploča.**

Concentration de la contrainte aux points autour de l’ouverture elliptique dans une plaque orthotrope multicouche à l’était plan de contrainte

Le papier donne le calcul de la contrainte aux points le long de l’arête d’une ouverture elliptique dans une plaque orthotrope infinie à l’état plan de contrainte à deux axes d’orthotropie. Les exemples numériques démontrent comment la contrainte dépend des caractéristiques de l’élasticité du matériau et de l’ordre de superposition de la plaque multicouche de même que de la position des demi-axes de l’ouverture elliptique en cas des conditions de contrainte particulières préalablement fixées. Les résultats de calcul sont illustrés par les diagrammes et les données numériques. Les conclusions dérivées sur le danger de la concentration de contrainte et des effets de l’orthotropie peuvent être utile en pratique d’ingénieurs.

Mots-clés: **concentration de la contrainte, plaque orthotrope, état plan de contrainte, ouverture elliptique, plaque multicouche.**